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Scalar Fields of Low Dimension: Evidence from Electromagnetic Mass Differences*

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The size of the mass splittings within isomultiplets suggests that (1) the usual strong-interaction Hamiltonian needs to be modified, or (2) there are scalar fields of dimension ≤ 4 belonging to $SU(3)\times SU(3)$ representations other than $(1,1)$ and $(\bar{3},3)+(3,\bar{3})$, or (3) there is a large electromagnetic renormalization of the strong interactions.

In Wilson's approach to broken scale invariance,[1] attention is focused on operator-product expansions. For example, the product of two currents is written as

$$j_\mu^\alpha(x)j_\nu^\beta(0)=\sum_n C_{n\,\mu\nu}^{\alpha\beta}(x)O_n(0)\,, \tag{1}$$

where the O_n are a set of operators, and the C_n are c-number functions. The singularities of the C_n at small x are given by dimensional analysis. If d_n is the effective dimension of O_n, then $C_n \sim (x)^{-6+d_n}$ at small x. The most singular C_n's are thus related to the O_n of small dimension.

Wilson[1] has assumed that the only Lorentz-scalar operators of dimension ≤ 4 are the identity, an $SU(3)\times SU(3)$ scalar, and a $(\bar{3},3)+(3,\bar{3})$ representation of $SU(3)\times SU(3)$. He has pointed out that this implies (1) the C_n's for the scalars have a simple dependence on symmetry breaking parameters (described more specifically below) and (2) the logarithmic divergence of the electromagnetic self-mass is related to the C_n's for the scalars. In this note, we discuss some consequences of these properties. In particular, we suggest the need to augment the list of low-dimension scalars, to modify the strong Hamiltonian, or to assume there are large renormalizations of the strong interactions due to electromagnetism.

It is usually assumed that the strong Hamiltonian may be written as[1,2]

$$\Theta_{00}=\bar{\Theta}_{00}+\epsilon_w w+\epsilon_\alpha u_\alpha\,. \tag{2}$$

Here Θ_{00} is scale-invariant and $SU(3)\times SU(3)$-invariant [perhaps also $U(3)\times U(3)$-invariant]; w breaks scale invariance [and $U(3)\times U(3)$, if that is a symmetry of $\bar{\Theta}_{00}$]; u breaks $SU(3)\times SU(3)$. For the scheme of broken scale invariance to make sense, w and u must have dimensions <4. They must therefore be included among the scalars of low dimension. There may be other scalars of low dimension which happen not to occur in the Hamiltonian; a minimal scheme[1] assumes no more.

It is usually further assumed[1-4] that u belongs to the $(\bar{3},3)+(3,\bar{3})$ representation of $SU(3)\times SU(3)$, so that $\epsilon_\alpha u_\alpha$ may be written

$$\epsilon_\alpha u_\alpha=\epsilon_0 u_0+\epsilon_8 u_8+\epsilon_3 u_3 \tag{3a}$$

$$=\epsilon_s u_s+\epsilon_\sigma u_\sigma+\epsilon_3 u_3 \tag{3b}$$

$$=\epsilon_s u_s+\epsilon_+ u_+ +\epsilon_- u_-\,, \tag{3c}$$

where

$$u_s=(u_0-\sqrt{2}\,u_8)/\sqrt{3}\,, \tag{4a}$$

$$u_\sigma=(\sqrt{2}\,u_0+u_8)/\sqrt{3}\,, \tag{4b}$$

$$u_\pm=(u_\sigma\pm u_3)/\sqrt{2}\,. \tag{4c}$$

Here u_s leaves $SU(2)\times SU(2)$ invariant, but u_σ does not. ϵ_σ is expected[1-4] to be considerably smaller than ϵ_s. ϵ_3 is usually taken as zero, but is included here for reasons which will become clear below.

Now we consider the contribution of the u's to the expansion of the product of two currents. Wilson[1] has pointed out that C_u must be $\sim(x)^{-2}$ at small x. The argument is as follows: We initially expect C_u to be $\sim(x)^{\Delta-6}$, where Δ is the dimension of u. But in the skeleton theory (the scale-invariant theory governed by $\overline{\Theta}_{00}$) u cannot appear in the expansion of two currents. This is so since j_μ belongs to the $(1,8)+(8,1)$ representation of $SU(3)\times SU(3)$. Since $SU(3)\times SU(3)$ is a symmetry of $\overline{\Theta}_{00}$, the expansion must respect this symmetry and only include operators belonging to representations which can occur in the product of $(1,8)+(8,1)$ with itself. But $(\overline{3},3)+(3,\overline{3})$ is not such a representation. We must now expand, using a spurion ϵ_α, with dimension $4-\Delta$. The u's can appear in the first order in ϵ. They are thus promoted to an *effective* dimension[5] $\Delta+(4-\Delta)=4$, and $C_u \sim (x)^{4-6} = x^{-2}$.

We also learn that $C_u \sim \epsilon$. But we can go even further. Consider the case $\epsilon_s \neq 0$, $\epsilon_\pm = 0$ [$SU(2)\times SU(2)$ symmetry]. u_s belongs to the $(1,1)$ representation of $SU(2)\times SU(2)$; $u_\pm$ to the $(2,2)$ representation. This latter representation cannot occur in the product of electromagnetic or weak currents as long as $\epsilon_\pm = 0$. Therefore $C_+ = C_- = 0$ in the case of $SU(2)\times SU(2)$ symmetry, i.e., there are no terms $\sim\epsilon_s$ in $C_\pm$. If we consider the case $\epsilon_+ \neq 0$, $\epsilon_s = \epsilon_- = 0$, we find a *different* $SU(2)\times SU(2)$ subgroup is conserved; u_+ is a scalar in this subgroup, while u_s and u_- transform as $(2,2)$. Hence $C_s = C_- = 0$ in this case. We can finally consider the case $\epsilon_- \neq 0$, $\epsilon_s = \epsilon_+ = 0$ and learn that $C_s = C_+ = 0$. We thus conclude that[6]

$$C_s \sim \epsilon_s, \quad C_+ \sim \epsilon_+, \quad C_- \sim \epsilon_- . \tag{5}$$

Now we turn to the question of experimentally detecting the scalar terms in the expansion. First we consider deep-inelastic ep scattering. The scaling limit of F_L measures the contribution of a scalar of effective dimension two. The less singular contribution of a scalar of effective dimension four vanishes in the scaling limit and its value must be sought in the *approach* to the scaling limit.[7] This is not likely to be measured soon.

However, C_u is "measured" by the electromagnetic mass differences. The contribution of any operator O_n to the e^2 self-mass is determined by

$$\int d^4x\, C_{n\mu\nu}(x) D^{\mu\nu}(x) , \tag{6}$$

where $C_{n\mu\nu}$ is the coefficient of O_n in the expansion of the time-ordered product of two electromagnetic currents and $D^{\mu\nu}$ is the photon propagator. Lorentz invariance implies that the integral will vanish unless O_n is a scalar.[8] For $O_n = u$, with effective dimension four, the integral will be logarithmically divergent.[9] (For fields of higher dimension, the integral will be finite.) Similarly, the weak self-mass of order G will have quadratic[10] and logarithmic divergences related to the C's in the expansion of the product of two weak currents.

We now identify the $\Delta I = 1$ part of these divergences (i.e., the terms proportional to u_3) with the electromagnetic tadpole.[11] We *assume* that notwithstanding their apparently divergent nature, they are genuinely of order e^2. (This is discussed further below.)

The observed tadpole will be $\epsilon_3' = \epsilon_3 + \delta\epsilon_3$, where $\delta\epsilon_3$ arises from the electromagnetic and weak self-mass. By the above assumptions $\delta\epsilon_3 \sim e^2 C_3$ and therefore[12] $\delta\epsilon_3 \sim e^2\epsilon_\sigma$, $e^2\epsilon_3$. The observed ϵ_3' is numerically of the order $e^2\epsilon_8$.[11,13] In the $(\overline{3},3)+(3,\overline{3})$ model with $\epsilon_3 = 0$ and ϵ_σ small compared with ϵ_s, and hence ϵ_8, we therefore *cannot have*[14] $\epsilon_3' = \delta\epsilon_3$.

Thus at least one of our assumptions must be false. We consider several possibilities:

(1) The strong Hamiltonian is not as assumed above:

(a) $\epsilon_3 \neq 0$; i.e., there is isospin breaking in the strong Hamiltonian.[15]

(b) ϵ_σ is not small compared to ϵ_s; i.e., $SU(2)\times SU(2)$ breaking is not small.

(c) There are terms belonging to other representations of $SU(3)\times SU(3)$. This is a special case of (2), discussed below.

(2) The operator expansion is not as assumed, i.e., there are scalar fields belonging to at least one other representation of dimension ≤ 4. Since the tadpole is predominantly octet,[11] the $SU(3)$ content of the additional low-dimension scalar is presumably octet and singlet. This rules out, e.g., $(8,8)$ but allows, e.g., $(1,8)+(8,1)$. Also in this case, the tadpole is not a member of the same octet as u_8, responsible for $SU(3)$-breaking mass differences. This can be tested by comparing the ratio of the isospin-breaking tadpole to $SU(3)$ mass splittings for *different* $SU(3)$ multiplets, e.g., baryons and mesons. Existing calculations[11] tend to favor the operators' being members of the same octet, although the evidence is not overwhelming. A more serious objection to this possibility arises from nonleptonic decays. These processes will now have a tadpole contribution, related to the occurrence of the strangeness-changing members of the new multiplet in the expansion of two weak currents. This is likely to result in an excessive enhancement of these decays.[16]

(3) The above assumption that the "divergences" are of order e^2 is incorrect, and $\delta\epsilon_3$ is not $\sim e^2 C_3$, but significantly larger. That is, the strong interactions are strongly renormalized by the weak and electromagnetic interactions. This implies that all of ϵ_s, ϵ_+, and ϵ_- have large and, in general, different renormalizations so that the values of these parameters without electromagnetic and weak interactions cannot be determined. We note

one exception to this statement. U-spin invariance of the electromagnetic current implies[7]

$$\frac{\delta^{\text{em}}\epsilon_-}{\epsilon_-}=\frac{\delta^{\text{em}}\epsilon_s}{\epsilon_s}. \tag{7}$$

If the renormalizations are due solely to the electromagnetic divergences, then Eq. (7) implies

$$\frac{\epsilon_-'}{\epsilon_s'}=\frac{\epsilon_-}{\epsilon_s}, \tag{8}$$

so that this ratio is not renormalized. If both weak and electromagnetic contributions are important, no such simple relations hold and no conclusions can be drawn about the unrenormalized values of the ϵ's. We also note that this possibility might lead to large radiative corrections to β decay.

Finally, we note two other consequences of the dependence of the C's on the ϵ's:

(1) The effort of Cabibbo and Maiani[10,17] to calculate the Cabibbo angle by canceling the weak and electromagnetic divergences is not valid in the present theory. This is so because they assume that the electromagnetic contributions to $\delta\epsilon$ satisfy

$$\delta^{\text{em}}\epsilon_-=\delta^{\text{em}}\epsilon_s. \tag{9}$$

In the case of $SU(3)$ symmetry ($\epsilon_3=\epsilon_8=0$ or $\epsilon_s=\epsilon_+=\epsilon_-$), Eqs. (7) and (9) are identical. However, in the case of small ϵ_σ (the case considered in Refs. 10 and 17), (9) is not a good approximation to (7).

(2) One cannot estimate $\delta\epsilon_3$ from the observed mass splittings by the use of the Dashen sum rule,[18] as has been attempted.[17] This is so because this sum rule neglects terms of order $e^2\epsilon$, and $\delta\epsilon_3$ is precisely of this order.

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[1]K. G. Wilson, Phys. Rev. 179, 1499 (1969).

[2]M. Gell-Mann, in *Particle Physics*, edited by W. A. Simmons and S. F. Tuan (Western Periodicals, Los Angeles, 1970).

[3]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[4]S. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

[5]If $\bar{\theta}_{00}$ conserves axial baryon number, but w does not. A similar argument implies that w is also promoted to an effective dimension of four. K. G. Wilson, Stanford Linear Accelerator Center Report No. SLAC-PUB-737, 1970 (unpublished).

[6]The result $C_3\sim\epsilon_\sigma, \epsilon_3$ was noted by Wilson (Ref. 1).

[7]S. G. Brown (unpublished).

[8]This is so if the nonscalar operators are *traceless* tensors of higher rank. Clearly, if we have a general tensor, we can write it as a traceless tensor, plus its trace; we are free to include the trace in the list of scalar operators, and include only the traceless part in the list of tensors. That the mass difference is sensitive only to the trace of a tensor has been shown for a particular case of equal-time commutators by J. M. Cornwall, Phys. Rev. D2, 578 (1970).

[9]This argument is given in Ref. 1. One may explicitly calculate the combination of moments of the structure functions in inelastic ep scattering which is proportional to C_nO_n for O_n = scalar of effective dimension four (Ref. 7). The result is the *same* quantity which, according to R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D2, 2473 (1970), gives the logarithmic divergence of the self-mass.

[10]The quadratic divergence has been studied by R. Gatto, G. Sartori, and M. Tonin, Phys. Letters 28B, 128 (1968), and N. Cabibbo and L. Maiani, *ibid.* 28B, 131 (1968).

[11]S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964); R. Socolow, *ibid.* 148, B1221 (1965).

[12]A similar conclusion has been reached by J. C. Bane and K. T. Mahanthappa, Phys. Rev. Letters 25, 779 (1970). Their dynamical framework is quite different from ours, but the basis of their result appears to be the same group-theoretical property that leads to our result.

[13]A lower bound on the tadpole is given by the n-p mass difference, since all the usual nontadpole calculations give the wrong sign of this difference. Thus we do not need to believe the details of the tadpole calculations to conclude $\epsilon_3'\gtrsim e^2\epsilon_8'$.

[14]In fact, the estimates of ϵ_3' and ϵ_σ' are numerically comparable: $\epsilon_3'\sim -0.02$, $\epsilon_\sigma'\sim 0.07$. See Ref. 2.

[15]This has been advocated for other reasons by the authors of Ref. 10.

[16]This enhancement does not occur if the only low-dimensional scalars are in the u multiplet, since its strangeness-changing members are the divergences of the corresponding currents and so cannot contribute to decays. See Ref. 1 and references cited therein.

[17]N. Cabibbo and L. Maiani, Phys. Rev. D1, 707 (1970).

[18]R. Dashen, Phys. Rev. 183, 1245 (1969).