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Allowed Domains in Theories of Broken Chiral Symmetry*

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We consider theories in which the explicit breaking of the chiral symmetry $SU_3\times SU_3$ of the energy density Θ_{00} is of the form $\epsilon_0 u_0 + \epsilon_8 u_8$, where ϵ_0, ϵ_8 are real constants and u_0 and u_8 are scalar densities in the representation $(3,\overline{3})+(\overline{3},3)$. We propose an extension of this picture , in which the $SU_3\times SU_3$-invariant part of Θ_{00} has the form $\overline{\Theta}_{00}+\epsilon_9 u_9$, where $\overline{\Theta}_{00}$ is $U_3\times U_3$ invariant and the $SU_3\times SU_3$-invariant scalar u_9 breaks $U_3\times U_3$ in a specific way. Specifically, it is assumed that the ninth axial charge F_0^5 transforms u_9 into a pseudoscalar v_9 according to $i[F_0^5,u_9]=\kappa v_9$, where $i[F_0^5,v_9]=-\kappa u_9$. (The parameter κ labels the representation.) Given this group structure, the analysis of Okubo and Mathur may be extended to find allowed domains for the symmetry-breaking parameters ϵ_0, ϵ_8, ϵ_9 and the vacuum expectation values $\langle u_0\rangle$, $\langle u_8\rangle$, and $\langle u_9\rangle$. The positivity conditions now restrict $\langle u_9\rangle$ so that the allowed domains occupy certain volumes in a three-dimensional space.

I. INTRODUCTION

Many authors have attempted to understand how the approximate hadron symmetry $SU_3\times SU_3$ is broken. In addition to the conventional problem of assigning correct group properties to operators which explicitly violate the symmetry, there is the problem of describing the "spontaneous breakdown" of the symmetry. Apparently the dynamics underlying the low-mass spectrum is such that even when the explicit symmetry-breaking terms are removed, the solutions do not belong to representations of the chiral group, but rather to some subgroup, usually supposed to be SU_3. In this picture the vacuum is not chiral invariant even in the symmetry limit and the associated massless pseudoscalar octet is the principal manifestation of the symmetry.

Perhaps the most promising model[1-3] (rather, class of models) ascribes the explicit symmetry breaking to scalar components (u_0 and u_8) of the representation $(3,\overline{3})+(\overline{3},3)$. Introducing coupling parameters ϵ_0 and ϵ_8, we may write the energy density $\Theta_{00}(x)$ in the form

$$\Theta_{00}(x)=\hat{\Theta}_{00}(x)+\epsilon_0 u_0+\epsilon_8 u_8, \tag{1.1}$$

where $\hat{\Theta}_{00}(x)$ is invariant under $SU_3\times SU_3$. The representation $(3,\overline{3})+(\overline{3},3)$ is usefully described by the nonet of Hermitian scalar densities u_i and pseudoscalar densities v_i ($i=0,1,\ldots,8$) which obey the (equal-time) commutation relations

$$\begin{aligned}[][F_i,u_j]&=if_{ijk}u_k,\\ [F_i,v_j]&=if_{ijk}v_k,\\ [F_i^5,u_j]&=-id_{ijk}v_k,\\ [F_i^5,v_j]&=id_{ijk}v_k.\end{aligned} \tag{1.2}$$

In order to compare the solutions of (1.1) with their counterparts in the symmetric limit, it is necessary to have some understanding of the analytic behavior of the solutions as a function of the parameters ϵ_0 and ϵ_8. In particular, we wish to be able to turn on the symmetry-breaking terms adiabatically without drastic alteration of the nature of the solution in the symmetric limit. As emphasized by Dashen,[4] this really means that we have to study the solution as we *turn off* the symmetry-breaking terms, i.e., certain "symmetric" solutions (among infinitely many degenerate ones) are selected by this limiting process.

It turns out that not all values of the parameters ϵ_0 and ϵ_8 are allowed, even if one does not give a detailed specification of the field-theoretic structure of the operators u_0 and u_8. This result was discovered by Okubo and Mathur,[5] who pointed out that only certain domains of the ratios

$$a=\epsilon_8/\sqrt{2}\,\epsilon_0, \quad b=\langle u_8\rangle/\sqrt{2}\langle u_0\rangle \tag{1.3}$$

are compatible with positivity requirements for the longitudinal spectral weight functions of the vector and axial-vector propagators. It will be noted that the quantities ϵ_8 and ϵ_0 may be regarded as "external fields" whose variation changes the values of the physical quantities $\langle u_8\rangle$ and $\langle u_0\rangle$.

In simple meson models the positivity conditions essentially amount to positivity of m^2. Hence the physical significance of the positivity criterion seems to be one of stability. This interpretation will be discussed elsewhere.[6] (It is also possible to sharpen the domain boundaries in these explicit models.)

In the present paper we propose that Θ_{00} violates $U_3\times U_3$ symmetry by means of a scalar operator u_9 which transforms in a simple way under the group $U_1\times U_1$ whose generators are composed of the baryon number F_0 and the axial baryon number F_0^5. We write

$$\Theta_{00}=\overline{\Theta}_{00}+\epsilon_9 u_9, \tag{1.4}$$

which is a generalization of models considered in a previous work.[7] $\overline{\Theta}_{00}$ is $U_3\times U_3$ invariant and may be (or may not be) "scale invariant," i.e., it may have dimension four. This generalizes specific models of the type discussed in Ref. 7.

The assumed group properties involve the pair of operators (u_9, v_9) which are $SU_3\times SU_3$ invariant but transform under F_0^5 as

$$\begin{aligned} i[F_0^5, u_9]&=\kappa v_9,\\ i[F_0^5, v_9]&=-\kappa u_9. \end{aligned} \tag{1.5}$$

The parameter κ serves to label the particular representation of $U_1\times U_1$ under consideration. We note the relation

$$e^{-i\lambda F_0^5}(u_9\pm i v_9)e^{i\lambda F_0^5}=e^{\pm i\lambda\kappa}(u_9\pm i v_9). \tag{1.6}$$

The question of the existence and the normalization of the operator F_0^5 and its associated current $\mathfrak{F}_{0\mu}^5$ is an important question on which little progress has been made. There are few observable quantities presently related to this current, although some speculations have been made[7] which relate the breaking of scale invariance to F_0^5. It is also possible[8] that the properties of certain of the heavy mesons may be related to matrix elements of $\mathfrak{F}_{0\mu}^5$. A second question concerns the normalization of the current $\mathfrak{F}_{0\mu}^5$. It will be noted that the value of κ depends on the normalization of F_0^5. (It is possible that a unique normalization follows[6] from a nonlinear commutation rule relating F_0^5 and $\partial^\mu F_{\mu 0}^5$ to Θ_μ^μ, as discussed elsewhere.) In the present paper, we suppose such a definition exists and explore abstractly the consequences of (1.5). In models it is easy to construct explicit examples of operators (u_9, v_9). For example, in Ref. 7 canonical meson fields S and P were introduced, which satisfy Eq. (1.5) with $\kappa=1$. It was also noted that from the $(3,\overline{3})+(\overline{3},3)$ meson fields (σ_i,ϕ_i), which are special cases of Eq. (1.2), we can construct[9] the $SU_3\times SU_3$-invariant coupling $u_9=(\det\mathfrak{M}+\text{H.c.})$. Normalizing F_0^5 so that $i[F_0^5,u_i]=v_i$, we find that (1.5) is satisfied with $\kappa=3$ and $v_9=(\det\mathfrak{M}-\text{H.c.})/i$.

The standard difficulty[10] with theories involving a current $\mathfrak{F}_{\mu 0}^5$ which is conserved in the limit $\Theta_{00}\to\hat{\Theta}_{00}$ (so that $\hat{\Theta}_{00}$ has $U_3\times U_3$ symmetry) is that one expects a ninth massless pseudoscalar meson in this limit. This meson would have the quantum numbers of the η and the lightest candidate is the (heavy) particle $\eta'(958)$. There are two common responses to this situation: (1) to claim that F_0^5 does not exist and (2) to say that F_0^5 is nonconserved in the limit $\Theta_{00}\to\hat{\Theta}_{00}$. The first position is common (though rarely stated) among advocates of nonlinear Lagrangian techniques. These people effectively say that if η' exists (or F_0^5), it does not belong to the world of their Lagrangian.

Adherence to the second point of view often goes with an acceptance of η' (along with the pseudoscalar octet and some scalars) and requires some description of F_0^5 nonconservation. If one admits nonpolynomial interactions[11] involving $SU_3\times SU_3$ invariants, the decomposition (1.4) is not often useful, and in particular the scale-invariant limit is not usually characterized by $U_3\times U_3$ invariance. In the case of old-fashioned polynomial interactions it is natural to identify the scale-invariant limit with that of F_0^5 conservation. In fact, our ulterior motive has been to explore the consequence of the latter circumstance. If there is no limit in which F_0^5 is conserved it may be futile to attempt to ascribe any physical significance to it.

II. DERIVATION OF STABILITY CONDITIONS

In addition to the assumptions (1.1) and (1.4) we suppose that the densities u_i $(i=0,\ldots,9)$ satisfy Gell-Mann's divergence relations[1]

$$i\partial^\mu G_\mu=[G,\mathcal{H}_{\text{SB}}], \tag{2.1}$$

where $\mathcal{H}_{\text{SB}}=\epsilon_0 u_0+\epsilon_8 u_8+\epsilon_9 u_9$, G_μ is one of the currents $\mathfrak{F}_{i\mu}$, $\mathfrak{F}^5_{i\mu}$, and $G=\int G_0 d^3x$.

Following Okubo and Mathur,[5] we now use the spectral representations

$$\langle[\mathfrak{F}^5_{\mu i}(x), \mathfrak{F}^5_{\nu j}(x')]\rangle_0=\int_0^\infty dm^2\left[\rho^5_{ij}(m^2)\left(g_{\mu\nu}+\frac{\partial_\mu\partial_\nu}{m^2}\right)-\frac{\sigma^5_{ij}(m^2)}{m^2}\partial_\mu\partial_\nu\right]\Delta(x-x';m^2),$$
$$\langle[\mathfrak{F}_{\mu i}(x), \mathfrak{F}_{\nu j}(x')]\rangle_0=\int_0^\infty dm^2\left[\rho_{ij}(m^2)\left(g_{\mu\nu}+\frac{\partial_\mu\partial_\nu}{m^2}\right)-\frac{\sigma_{ij}(m^2)}{m^2}\partial_\mu\partial_\nu\right]\Delta(x-x';m^2), \tag{2.2}$$

where ρ_{ij} and σ_{ij} are transverse and longitudinal weight functions. Taking the divergence of Eq. (2.2) and integrating on d^3x' gives, assuming that no subtraction is necessary,

$$K_{ij}\equiv\int_0^\infty dm^2\sigma_{ij}(m^2)=i\langle[\partial^\mu\mathfrak{F}_{\mu i}, F_j]\rangle_0,$$
$$I_{ij}\equiv\int_0^\infty dm^2\sigma^5_{ij}(m^2)=i\langle[\partial^\mu\mathfrak{F}^5_{\mu i}, F^5_j]\rangle_0. \tag{2.3}$$

Equation (2.1) permits the evaluation of the divergence when we utilize the assumed transformation properties expressed by Eqs. (1.2) and (1.5). This allows one to express K_{ij}, I_{ij} in terms of the three nonvanishing vacuum expectation values $\langle u_0\rangle$, $\langle u_8\rangle$, and $\langle u_9\rangle$. The only change from the results of Okubo and Mathur occurs in the quantity I_{00}. Our result for I_{00} is

$$I_{00}=-\tfrac{2}{3}(\epsilon_0\xi_0+\epsilon_8\xi_8+\epsilon_9\xi_9),$$
$$\xi_0=\langle u_0\rangle,\quad \xi_8=\langle u_8\rangle,\quad \xi_9=\tfrac{3}{2}\kappa^2\langle u_9\rangle. \tag{2.4}$$

A convenient parametrization is obtained by making, in addition to Eq. (1.3), the definitions

$$a'=\epsilon_9/\epsilon_0,\quad b'=\xi_9/\xi_0,$$
$$\gamma=-\tfrac{2}{3}\epsilon_0\xi_0,\quad \zeta=a'b'. \tag{2.5}$$

Now the nonvanishing quantities I_{ij}, K_{ij} are summarized by

$$I_{00}=\gamma(1+2ab+a'b'),$$
$$I_{08}=I_{80}=\sqrt{2}\,\gamma(a+b-ab),$$
$$I_{88}=\gamma[1-(a+b)+3ab],$$
$$I_{33}=\gamma(1+a)(1+b), \tag{2.6}$$
$$I_{44}=\gamma(1-\tfrac{1}{2}a)(1-\tfrac{1}{2}b),$$
$$K_{44}=\tfrac{9}{4}\gamma ab,$$

where we have not written explicitly those quantities equal to the listed ones by isospin invariance. It should be noted that a' and b' enter these equations only in the combination $a'b'$.

If one writes out the weight functions σ_{ij}, σ^5_{ij}, the following positivity conditions are satisfied for any choice of the nine constants C_i:

$$\sum_{i,j=0}^{8} C_i^* I_{ij} C_j \geq 0,$$
$$\sum_{i,j=0}^{8} C_i^* K_{ij} C_j \geq 0. \tag{2.7}$$

We first conclude that every diagonal quantity I_{ii} or K_{ii} satisfies $I_{ii}\geq 0$, $K_{ii}\geq 0$. The remaining condition, which takes I_{08} into account, is

$$|C_0|^2 I_{00}+|C_8|^2 I_{88}+2\,\mathrm{Re}(C_0^*C_8)(I_{08})\geq 0. \tag{2.8}$$

This inequality is maintained provided

$$I_{00}I_{88}\geq I_{08}{}^2. \tag{2.9}$$

[To make contact with Eqs. (21) of Okubo and Mathur,[5] we note that Eq. (2.9) amounts to $I_{-1-1}\times I_{-2-2}\geq I_{-1-2}{}^2$ which reduces to $I_{-1-1}=I_{33}\geq 0$, $I_{-2-2}\geq 0$ if their evaluations of I_{00}, I_{88}, and I_{08} are used.]

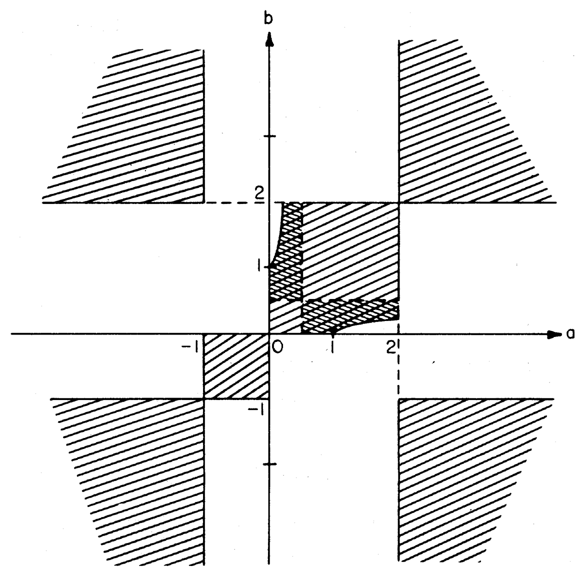

FIG. 1. The allowed domains for a and b are indicated by the shaded regions for the case that the theory has $SU_3\times SU_3$ symmetry when the explicit breaking terms are turned off. If the symmetry is increased to $U_3\times U_3$ in that limit (or if the quantity ζ vanishes), the double-hatched domains are forbidden.

Substitution of Eq. (2.6) into (2.9) gives the inequality

$$(1-2a)(1-2b) \geqslant -a'b' \frac{1+3ab-a-b}{(1+a)(1+b)}, \qquad (2.10)$$

where the right-hand side was zero in the treatment of Ref. 5.

Now we are in a position to reanalyze the inequalities due to the positivity conditions (2.7).

III. ALLOWED DOMAINS FOR THE VACUUM EXPECTATION VALUES OF THE SYMMETRY-BREAKING OPERATORS

We now consider the implications of the positivity conditions just derived. As a preliminary, we recall the results of Okubo and Mathur for the case in which $\hat{\Theta}_{00}$ has $SU_3 \times SU_3$ symmetry instead of $U_3 \times U_3$ symmetry. If F_0^5 does not exist, or if it does exist but has an unknown effect on $\hat{\Theta}_{00}$, we have to drop the quantities I_{00}, I_{08} from consideration. In that case I_{88}, I_{33}, I_{44}, and K_{44} must be non-negative. (Here $I_{88} \geqslant 0$ replaces $I_{-2-2} \geqslant 0$ for the case of $U_3 \times U_3$ symmetry considered in the first paper of Ref. 5.) A simple analysis now gives the allowed domains of Fig. 1. It is seen that on the whole very little is changed when the symmetry of $\hat{\Theta}_{00}$ is reduced from $U_3 \times U_3$ to $SU_3 \times SU_3$. However, stronger results may be obtained if one has more information on the breaking of $U_3 \times U_3$, as expressed in Eq. (1.4).

From Eq. (2.6) we see that the positivity conditions place restrictions on the three variables a, b and $\zeta \equiv a'b'$. Hence, the allowed domains for our generalized problem are certain volumes in a three-dimensional space having Cartesian coordinates (a, b, ζ). When $\zeta = 0$ (as happens when $\epsilon_9 = 0$), Eqs. (2.6) coincide with the $U_3 \times U_3$ case and the (a, b) domains are the same as given in the first paper of Ref. 5.

Before considering the general case we analyze the simple case $b=0$. (Note that b vanishes for the case of an SU_3-symmetric vacuum.) When $b=0$, Eqs. (2.6) reduce to

$$\begin{aligned} I_{00} &= \gamma(1+\zeta), \\ I_{08} &= \sqrt{2}\,\gamma a, \\ I_{88} &= \gamma(1-a), \\ I_{33} &= \gamma(1+a), \\ I_{44} &= \gamma(1-\tfrac{1}{2}a), \\ K_{44} &= 0, \end{aligned} \qquad (3.1)$$

and Eq. (2.9) may be written as

$$\zeta \geqslant \frac{2a^2+a-1}{1-a}. \qquad (3.2)$$

The equations $I_{88} \geqslant 0$, $I_{33} \geqslant 0$, and $I_{44} \geqslant 0$ permit a to lie in the domain $-1 \leqslant a \leqslant 1$ when $\gamma > 0$. For $\gamma < 0$ there is no solution. (This result is independent of any information about F_0^5.) $I_{00} \geqslant 0$ then requires $\zeta \geqslant -1$. But a sharper result follows from (3.2) as shown in Fig. 2.

The same domain shape is found for $a=0$, $b \neq 0$ because of the basic symmetry of Eqs. (2.6) under interchange of a and b. This corresponds, in general, to $\epsilon_8 = 0$ and hence spontaneous breakdown of SU_3 is typical of this case.

The foregoing cases $(a, b, \zeta) = (a, b, 0)$, $(a, 0, \zeta)$, $(0, b, \zeta)$ must be intersections of the three-dimensional domain boundaries with the planes through the appropriate coordinate axes.

To analyze the general case, we first note that the values of (a, b) shown in Fig. 1 were obtained independently of ζ. Hence *all* possible values must lie inside right cylinders having the allowed domains of Fig. 1 as cross sections. Since $I_{88} > 0$ (except on the hyperbola $1/a + 1/b = 1$), $I_{00} \geqslant {I_{08}}^2/I_{88}$ is positive because of the stronger condition (2.9). Hence ζ is constrained by Eq. (2.10) which we rewrite as

$$\begin{aligned} &-\zeta \leqslant F(a, b), \\ &F(a, b) = \frac{(1-2a)(1+a)(1-2b)(1+b)}{1+3ab-a-b}. \end{aligned} \qquad (3.3)$$

Hence $-\zeta$ must lie below the surface $\zeta_0 = F(a, b)$.

Instead of finding the range of ζ allowed for a given a and b, we can fix ζ_0 and find the allowed domain of a and b. This amounts to drawing contours on the surface $\zeta_0 = F(a, b)$. In Fig. 3 we exhibit a contour map of the function $F(a, b)$. For $\zeta = 0$ the allowed (a, b) lie in the original $(U_3 \times U_3)$ domain of Okubo and Mathur. In general, the

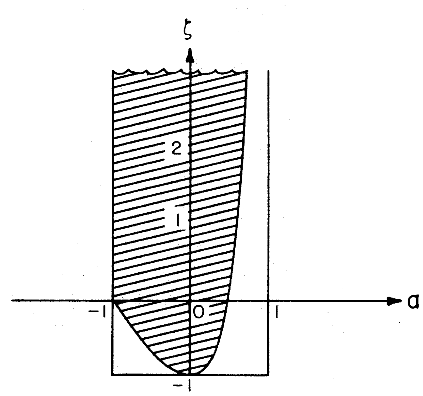

FIG. 2. When $b = 0$, as in the case of an SU_3-symmetric vacuum, the allowed values of a and ζ are indicated by the shaded area. If nothing is known about the mechanism of breakdown of $U_3 \times U_3$ to $SU_3 \times SU_3$, all that can be said is that ζ lies above the value -1 in the range $-1 \leq a \leq 1$.

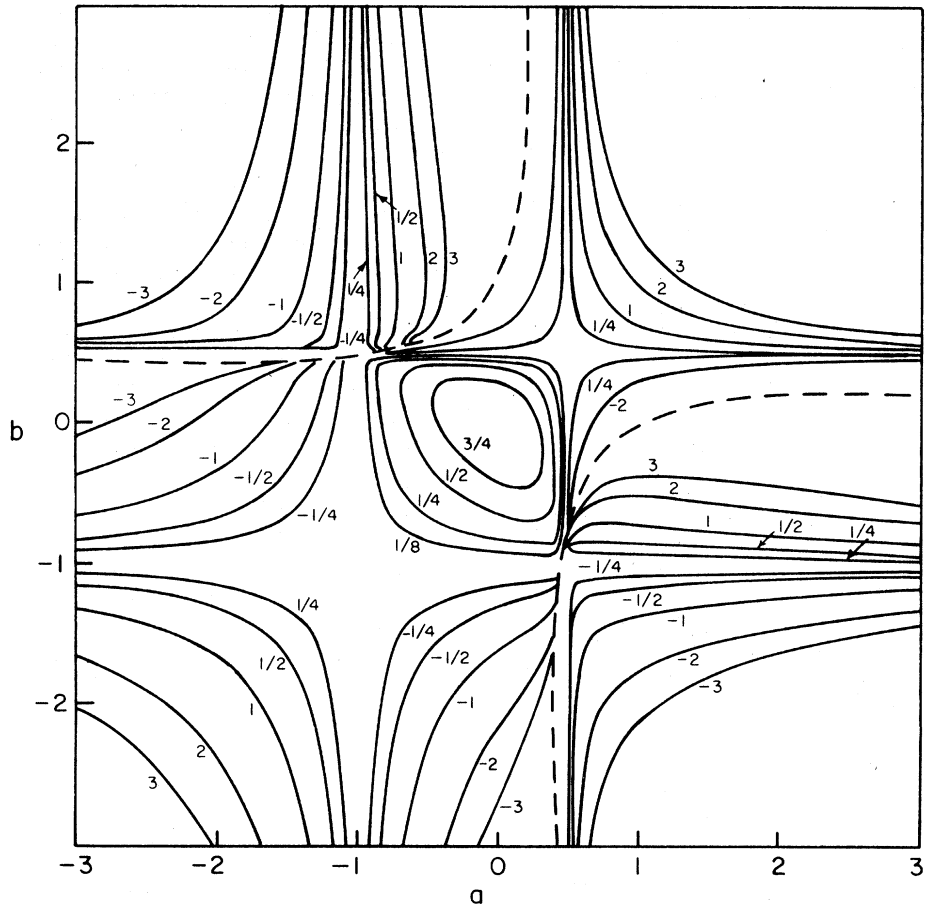

FIG. 3. The contours of the function $F(a,b)$ defined in Eq. (3.3) are shown for various values. The hyperbola is the locus of a singularity of F. The function vanishes along the lines $a=\frac{1}{2}$, $a=-1$, $b=\frac{1}{2}$, $b=-1$. The parameter $-\zeta$, defined to be $-a'b'$ [see Eq. (2.5)], must lie below the surface.

allowed volumes are those in the vertical tubes lying above the $SU_3\times SU_3$ (a,b) domains of Fig. 1 which satisfy $-\zeta\leqslant F(a,b)$.

In a separate paper[6] we shall elaborate upon the physical significance of our results in the context of simple Lagrangian models. It is interesting to note that most estimates of the parameters ϵ_0, ϵ_8, $\langle u_0\rangle$, and $\langle u_8\rangle$ place the allowed solutions under the convex surface lying in the interval $-1\leqslant a$, $b\leqslant 0$. Most authors agree that b is small (approximate SU_3 invariance) but different values of a have been obtained.[2,12] Recalling that the quantity c of Ref. 2 is the same as our $\sqrt{2}\,a$, the authors of Ref. 2 and Ref. 12, respectively, recommend $a=-0.89$ and $a=-0.017$. Setting $b=0$ we find from Eq. (3.2) the restrictions $\zeta\geqslant -0.16$ and $\zeta\geqslant -1.0$. From Eq. (2.5) we see that these cases permit rather different values for the quantity $\epsilon_9\langle u_9\rangle$.

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T-Matrix Perturbation Theory for the Faddeev Equations

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A T-matrix perturbation theory for the Faddeev equations is developed in two different ways giving slightly differing results for the first-order correction to the ground-state energy of the three-body system, and both expressions differ from the result recently obtained by Fuda. The application of the theory to the system of three identical spinless particles is discussed.

I. INTRODUCTION

When solving the Faddeev equations,[1] a separable two-body T matrix is often used as input, since this reduces the problem (after angular momentum decomposition) to one of solving one-dimensional (possibly coupled) integral equations instead of the more general two-dimensional equations. In many applications the potentials one wants to use are not separable, and consequently the corresponding two-body T matrices will not be separable, and thus some separable approximation, T_s, to the actual T matrix must be used. The difference between these two, $T_p=T-T_s$, the perturbing part of the T matrix, may not be negligibly "small," and it is often useful to be able to calculate the first-order correction to the ground-state energy of the three-particle system.

Fuda[2] has written down such a perturbation expansion, but he uses a separable T matrix that comes from a separable potential, while we take advantage of the considerable freedom in the particular breakup of the T matrix into separable and nonseparable parts. Let $V=V_1+V_2$ with V_1 separable. Then

$$T=T_1+T_2+T_{12},$$

where T_1 is the solution of the Lippmann-Schwinger (LS) equation for V_1, T_2 is the solution for V_2, and T_{12} contains both V_1 and V_2. Since V_1 is separable, so is T_1, and it is this that Fuda calls T_s. Thus, for Fuda, $T_p=T_2+T_{12}$. However, T_1+T_{12} is also separable,[3-5] and thus it is possible to identify T_2 as T_p. The later breakup is advantageous, since T_s includes some of the effects of V_2, and T_p may be "small" even when V_2 is not. Yaes[5] has also developed a perturbation expression, but he emphasizes expansion in terms of the potential.

In Sec. II we derive two relations for the first-order correction to the energy, both of which differ from Fuda's, and in Sec. III we discuss how the formulas may be applied in order to find the first-order Coulomb correction to the ground-state energy for a system consisting of three identical, spinless, charged particles.

II. THEORY

We first write the homogeneous Faddeev equations in the form

$$(E-H_0)|\psi\rangle=T(E)|\psi\rangle, \tag{2.1}$$

where

$$|\psi\rangle=\begin{pmatrix}|\psi^1\rangle\\|\psi^2\rangle\\|\psi^3\rangle\end{pmatrix} \tag{2.2}$$

and

$$T=\begin{pmatrix}0 & T_1 & T_1\\ T_2 & 0 & T_2\\ T_3 & T_3 & 0\end{pmatrix}, \tag{2.3}$$

with T_i given by

$$T_i=V_i+V_iG_0T_i. \tag{2.4}$$

In the above equation V_i is the potential between particles j and k [throughout this paper $(ijk)=(123)$ or some cyclic permutation thereof], and G_0 is the free-particle Green's function given by