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Comments and Addenda

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Deuteron Spin Alignment as a Test for Double-Scattering Contributions in the Glauber Theory*

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It is shown that in the single-scattering approximation to the Glauber theory of high-energy deuteron scattering there exists a relation between the two nonvanishing parameters describing the deuteron spin alignment. The failure of this relation can be used as a test for the expected importance of double-scattering terms at larger angles in elastic π-d or p-d scattering.

The break in the differential cross section in high-energy π-d or p-d elastic scattering has been interpreted as a change from a predominant single-scattering term to a predominant double-scattering term in the Glauber theoretical analysis.[1] Furthermore, the fact that there is no dip in the cross section where this change takes place has been explained as a result of contributions from the deuteron D state.[2] As a consequence of the D-state contribution, it has been pointed out[3,4] that elastic π-d or p-d scattering should be a method of producing and analyzing spin alignment of the deuteron. This alignment has been observed in a recent experiment.[5] Here we wish to point out that the relation between the two nonvanishing alignment parameters in the theory provides a test for the presence of the double-scattering term.

For an experiment in which unpolarized deuterons are successively scattered from two unpolarized targets at some given pair of angles (or, alternatively, polarized[6] deuterons are scattered once) the differential cross section is given by[7]

$$I = I_0\left[1 + a_1 b_1 + a_2 b_2 \cos 2\varphi + (a_3 b_3 - a_4 b_4)\cos\varphi\right], \tag{1}$$

where I_0 is the cross section without polarization effects, φ is the azimuthal angle of the second scattering, the a_i define the deuteron polarization produced in the first scattering (or the polarization of initially polarized deuterons), and the b_i define the analyzing power of the second scattering. For elastic scattering at a given angle it follows from time-reversal invariance that $a_i = b_i$. In the usual theoretical analysis[3,4] the elementary scattering amplitude (nucleon-nucleon or pion-nucleon) is assumed to be spin-independent and the small-angle approximation is used so that the momentum-transfer direction $\hat{\Delta}$ is at right angles to the incident direction $\hat{k}$. The coefficients of $\cos\varphi$ then vanish, and there remain two alignment parameters[8]; in terms of the alignment produced in the first scattering these are

$$a_1 = (\tfrac{1}{2})^{1/2}(3\langle S_k^2\rangle - 2) = (\tfrac{1}{2})^{1/2} A_k = -(\tfrac{1}{2})^{1/2}(A_n + A_\Delta),$$

$$a_2 = (\tfrac{3}{2})^{1/2}(\langle S_n^2\rangle - \langle S_\Delta^2\rangle) = (A_n - A_\Delta)(\tfrac{1}{6})^{1/2}$$
$$= (2A_n + A_k)(\tfrac{1}{6})^{1/2},$$

where S_i is a deuteron spin operator, $\hat{n}$ is the normal to the scattering plane, and the A_i correspond to the notation of Harrington.[4] The two alignment parameters are constrained by the inequality

$$\sqrt{6}\,|a_2| - 2 \leq \sqrt{2}\,a_1 \leq 1.$$

When the alignment effect is a maximum $a_2^2 = \frac{3}{2}$ and a_1^2 is constrained to equal $\frac{1}{2}$.

If the theoretical analysis contains only a single-scattering term, then the momentum-transfer axis $\hat{\Delta}$ is an axis of symmetry, since the elementary scattering amplitude at fixed energy is only a function of momentum transfer. This symmetry implies

$$A_n = A_k, \tag{2a}$$

$$a_2 = \sqrt{3}\, a_1. \tag{2b}$$

This symmetry would also follow from the assumption that the p-d or π-d elastic scattering considered as an elementary particle process is governed by the exchange of a spinless particle. Similar tests are known in discussions of the density matrices of vector mesons.[9]

Beyond the break in the cross section, however, the double-scattering term is supposed to dominate and the theory predicts $A_n \neq A_k$, as can be seen in Fig. 1 of Ref. 4. Thus an experimental determination that Eq. (2b) is violated would serve as qualitative evidence for the presence of a double-scattering term. The converse is not true, since Eq. (2) remains valid even in the presence of the double-scattering term, provided one can neglect the D-state contribution (F_{22} in Ref. 4).

When the spin dependence of the elementary scattering amplitude is included, Eq. (2) no longer follows for the single-scattering term. However, if the spin dependence is not large, so that we need consider only terms linear in the spin-dependent part of the elementary amplitude, then the only consequence of including spin dependence is that a_3 and a_4 (or b_3 and b_4) become nonzero, while the alignment parameters a_1 and a_2 are unaffected. For the case of pion scattering there is only one spin-dependent elementary amplitude, and it is easy to show that its effect on the alignment parameters in the single-scattering approximation is to increase A_n and decrease A_k. Thus we obtain the more general result for π-d scattering

$$a_2 - \sqrt{3}\, a_1 \geq 0. \tag{3}$$

In this case a violation of Eq. (3) would serve as qualitative evidence for the double-scattering term without any approximation. A test of Eq. (3) requires the determination of the signs as well as the magnitudes of a_2 and a_1.

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[1]V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966).

[2]D. R. Harrington, Phys. Rev. Letters 21, 1496 (1968).

[3]V. Franco and R. J. Glauber, Phys. Rev. Letters 22, 370 (1969).

[4]D. R. Harrington, Phys. Letters 29B, 188 (1969).

[5]G. M. Bunce, Ph.D. thesis, University of Michigan, 1971 (unpublished).

[6]We use the word "polarized" to describe any departure from the unpolarized condition, which may be any combination of vector- and tensor-type polarizations. The tensor-type polarization described by a_1 and a_2 is commonly referred to as alignment.

[7]W. Lakin, Phys. Rev. 98, 139 (1955); for a recent discussion, see V. König *et al.*, Nucl. Phys. A148, 380 (1970).

[8]It should be emphasized that the various experiments with a polarized deuterium target discussed in Ref. 3 may all be described in terms of the parameters a_1 and a_2. Thus, the double-scattering experiment, proposed by Harrington and discussed here (Ref. 4), is equivalent to the polarized-target experiment as a test of the theory, provided both the $\cos 2\varphi$ asymmetry and the change in the differential cross section are measured. It is also possible in both experiments to check the vanishing of the $\cos\varphi$ term. In the notation of Ref. 7, the parameters a_1 and a_2 are $\langle T_{20}\rangle$ and $\sqrt{2}\langle T_{22}\rangle$, respectively.

[9]K. Gottfried and D. Jackson, Nuovo Cimento 33, 309 (1964); 34, 735 (1964). In the notation of these papers, Eq. (2a) is $\rho_{1,-1} = 0$, where $\rho_{mm'}$ is the deuteron density matrix with the z axis defined by the momentum-transfer direction.