PHYSICAL REVIEW D VOLUME 4, NUMBER 9 1 NOVEMBER 1971

Consequences of Relativistic Quark Models for the Interactions of Hadrons

J. Dias de Deus*
University College London, Gower Street, London WC1, England
(Received 17 August 1970; revised manuscript received 9 April 1971)

Hadron interactions are formulated in terms of relativistic quark models. The constraints of vector-meson dominance and partial conservation of axial-vector current are used to select convenient quark-quark-meson vertex functions. These vertex functions correspond to Llewellyn Smith's model I. Interactions of hadrons are assumed to take place through quark interactions, and coupling constants are evaluated using quark graphs. The results are consistent with vector-meson-dominance and the ϕ-ω current-mixing model. Deviations from SU(3) and the nonrelativistic additivity quark model are predicted.

I. INTRODUCTION

The nonrelativistic quark model has been a very successful model in predicting decay widths and coupling constants in hadron interactions.[1-3] However, when decays involving quark-antiquark annihilation are considered, the model runs into trouble, requiring the inclusion of SU(3)-noninvariant space wave functions.[3]

Discussing the problem of the structure of the meson wave functions from a relativistic point of view, Llewellyn Smith[4] found that the wave functions that give good predictions in annihilation processes and preserve SU(3) are not the ones that correspond to the weak-binding limit of the nonrelativistic quark model. We show, in simple realizations of Llewellyn Smith's models, that his best solution is the only one compatible with vector-meson dominance and partial conservation of axial-vector current (PCAC).

Working with wave functions with a relativistic structure, we keep, however, some of the fundamental assumptions of the nonrelativistic quark models, in particular, the idea that quarks move with small space momenta within hadrons, which require a deep flat-bottomed potential with a long-range force and massive quarks[5,6] ($M \gtrsim$ GeV).

Our basic assumptions will be that the hadronic interactions take place through the quarks – an assumption whose validity is still an open question[7] – and that the quark currents (or their divergences) are meson-pole-dominated.

It has been shown[8] that in the Bethe-Salpeter formalism applied to quarks, vector-meson dominance and the Goldberger-Treiman relation appear as a result of the saturation of the quark-antiquark propagator by meson bound-state contributions. These are expected to dominate when the meson is near its mass shell. Thus, from the point of view of quarks, the electromagnetic decay of vector mesons (or the weak axial-vector two-body pseudoscalar-meson decay) will be described by the graph of Fig. 1(a).[4] Neglecting three-body interactions, the three-meson vertex will be predominantly given by a triangle (renormalized) graph [Fig. 1(b)], which may be justifiably evaluated as a Feynman integral.[9] But in the four-legged process the graph of Fig. 1(c) cannot be treated as a Feynman graph because there will be large contributions from poles in some channels. Only some kind of duality interpretation is adequate. Baryon interactions will similarly be decomposed into their quark constituents.

In Sec. II we discuss vertex functions which are realizations of Llewellyn Smith's models and show that only model I is consistent with vector-meson dominance and the Goldberger-Treiman relation at quark level. The basic $QQ\pi$ and weak quark axial-vector coupling constants are derived and the results agree with additivity. In Sec. III we evaluate coupling constants using the Bethe-Salpeter formalism. As the hadron interactions take place through the same basic quark-level couplings, the mass factors present in the quark vertex functions determine SU(3)-breaking factors in the various coupling constants. In general, the results improve the SU(3)-symmetric limit and nonrelativistic quark model.

II. MODELS FOR THE QQV AND QQP VERTEX FUNCTIONS

Llewellyn Smith[4] discusses Bethe-Salpeter wave functions for $Q\overline{Q}$ pseudoscalar- and vector-meson bound states. These wave functions have the structure corresponding to the outer product of a spinor and a conjugate spinor and can be described in terms of four two-by-two matrices: χ^{++}, χ^{--}, χ^{+-}, χ^{-+}, the first corresponding to the positive-energy–positive-energy component, and so on. What distinguishes Llewellyn Smith's various models is the relation between the χ^{++} and χ^{--} components of the wave function in the limit of vanishing me-

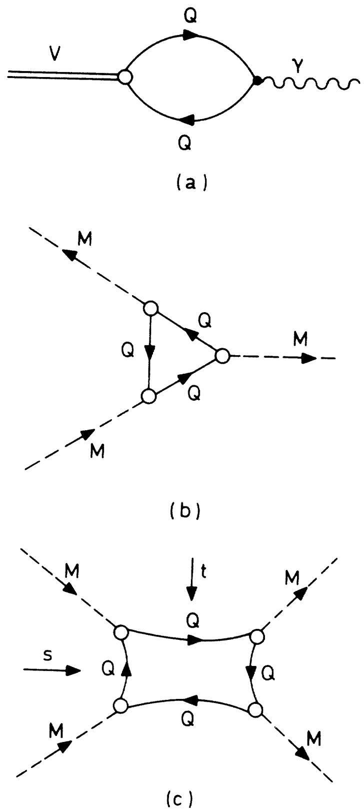


FIG. 1. Quark graphs for meson interactions: (a) Annihilation graph; (b) Triangle graph for three-meson couplings; (c) Graph for a four-legged process. The blob ○ represents the quark-quark-meson (QQM) vertex.

son masses. When these relations are imposed in the Bethe-Salpeter normalization, the various models become perfectly well defined, giving very different predictions in the evaluation of hadron coupling constants. A general discussion of these models is contained in Ref. 4. Model I is the model that satisfies SU(3) for the wave functions and that can be made consistent with approximate SU(6). Model III is the one corresponding to a weak-binding limit and to a nonrelativistic structure of the wave function.

In the present paper we use quark-quark-pseudoscalar-meson (QQP) and quark-quark-vector-meson (QQV) vertex functions which are simple realizations of Llewellyn Smith's models I, II, and III. (See Table I.) They have the structure of solutions of the $Q\overline{Q}$ homogeneous Bethe-Salpeter equation with a separable potential[4] and this simple model is occasionally used for orders of magnitude estimates. Both the QQP and QQV vertices include derivative and nonderivative couplings, pseudovector and pseudoscalar couplings for the QQP interaction and Dirac and Pauli-like couplings for QQV. We write the vertex functions as follows:

QQP:

$$\Gamma_P(p,q) = W(p,q)\gamma^5 G\left[\left(\frac{m_P}{2M}\right)^s R + \left(\frac{m_P}{2M}\right)^v \frac{\not{q}}{2M}\right], \qquad (1)$$

QQV:

$$\Gamma_V^\mu(p,q) = W(p,q)\epsilon_\alpha^\mu F\left[\left(\frac{m_V}{2M}\right)^e \gamma_\alpha - iR'\left(\frac{m_V}{2M}\right)^m \sigma_{\alpha\nu}\frac{q^\nu}{m_V}\right], \qquad (2)$$

where m_P (m_V) is the meson mass and q its momentum; M, $p\pm\frac{1}{2}q$ are the quark mass and momenta; G, R, F, and R' are model-dependent constants; s, v, e, and m are parameters which char-

TABLE I. Characteristics of Llewellyn Smith's models and model parameters in QQV and QQP vertex functions. [See Eqs. (1) and (2) in the text.] χ^{+-} and χ^{-+} are small in all three models.

		Vertex-function parameters			
		QQP		QQV	
Model	Characteristics	s	v	e	m
I	$\chi^{++}\simeq\chi^{--}$ $\frac{\chi^{++}-\chi^{--}}{m_P}=\text{const}$	0	0	1	0
II	$\chi^{++}\simeq-\chi^{--}$ $\chi^{++}-\chi^{--}=\text{const}$	1	−1	0	1
III	$\chi^{++}\gg\chi^{--}$ $\frac{\chi^{++}}{\sqrt{m_P}}=\text{const}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

acterize the models; ϵ_α^μ is the vector-meson polarization ($q_\alpha\epsilon_\alpha^\mu = 0$ when $q^2 = m_V^2$, and $\epsilon_\alpha^\mu\epsilon_\beta^\nu = g_{\alpha\beta} - q_\alpha q_\beta/m_V^2$); and $W(p,q)$ is a form factor required for the convergence of some integrals and satisfies the quark on-mass-shell condition:

$$W(p,q) = 1 \text{ when } (p+\tfrac{1}{2}q)^2 = (p-\tfrac{1}{2}q)^2 = M^2.$$

The quantities G and R (F and R') can, in principle, be determined by solving the Bethe-Salpeter and normalization equations. Table I shows the characteristics of the different models and the values of the parameters s, v, e, and m.

We now compute for the various models,[4] the neutral vector-meson electromagnetic coupling constants f_V, using the graph of Fig. 1(a), and similarly the pseudoscalar-meson annihilation parameter F_P. The result, easily obtained with the vertex functions (1) and (2), can be written in the form

$$\frac{1}{f_V} = \left[\sum g_i^V e_i \Big/ \left(\frac{m_V}{2M}\right)^k\right] I \tag{3}$$

and

$$F_P = \left[2Mg_A \Big/ \left(\frac{m_P}{2M}\right)^l\right] I', \tag{4}$$

where g_A is the ratio of axial-vector to vector coupling for quarks; e_i is the charge of quark Q_i ($i = \mathcal{P}, \mathfrak{N}, \lambda$) in units of electron charge; g_i^V is the Clebsch-Gordan coefficient giving the $Q_i\overline{Q}_i$ contribution to the vector meson V; k, l are numbers depending on the chosen model; and I, I' are the loop integrals which in the approximation

$$m_P/M, m_V/M \ll 1$$

are independent of the meson masses. The quark axial-vector coupling renormalization constant and quark and nucleon Cabibbo angles have been absorbed in I'. The loop integrals I and I' have not necessarily the same values for the various models.

In Table II are shown the parameters k and l in the different cases as well as the mass dependence of f_V and F_P. The experimental ratios of f_V's or F_P's for different vector or pseudoscalar mesons strongly favor model I.[4]

Let us now take the electromagnetic interaction of quarks in the limit $q^2 \to 0$ and consider first the Dirac coupling. In that limit this coupling gives the quark electric charge. As $q^2 = 0$ is not far from the vector-meson poles, it is reasonable to assume that the quark electromagnetic current is dominated by vector mesons. Graphically and making use of Fig. 1(a), we have Fig. 2. The corresponding equations are, from (2) and (3),

$$\sum g_i^V e_i = F\left(\frac{m_V}{2M}\right)^e \frac{1}{f_V} \tag{5a}$$

$$= \left(\sum g_i^V e_i\right)\left(\frac{m_V}{2M}\right)^{e-k} FI. \tag{5b}$$

To avoid contradiction between the left- and the right-hand sides of (5b) in the sense that no dependence on vector-meson masses should occur on the right-hand side, we require that

$$e = k, \tag{6}$$

and this constraint is only satisfied by model I. To satisfy Eq. (5b) one still needs to ensure

$$FI = 1, \tag{7}$$

but this is only a constraint of the form factor for the QQV vertex. For model I the Dirac coupling is then, in the limit $q^2 \to 0$, consistent with a vector-meson-dominance description of the quark electromagnetic interaction.

We treat in a similar way and in the same limit the divergence of the weak axial-vector interaction of quarks (Fig. 3) with the resulting equations:

$$2Mg_A = \sqrt{2}\, g_Q F_\pi \tag{8a}$$

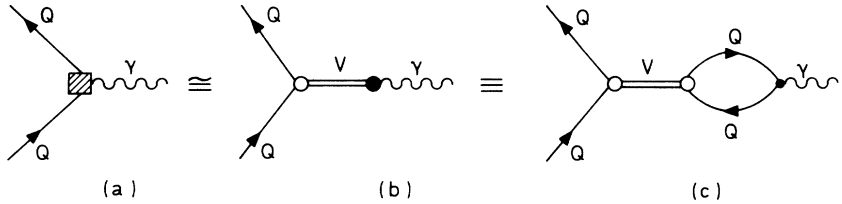
$$= (2Mg_A)\left(\frac{m_P}{2M}\right)^{-l} \sqrt{2}\, g_Q I', \tag{8b}$$

where

TABLE II. Annihilations in Llewellyn Smith's models. Annihilation parameters and mass dependences. [See Eqs. (3) and (4) in the text.]

	Annihilations			
	Pseudoscalar mesons		Vector mesons	
Model	Parameter l	m_P dependence of F_P	Parameter k	m_V dependence of f_V
I	0	const	1	m_V
II	1	$1/m_P$	2	m_V^2
III	$\frac{1}{2}$	$1/m_P^{1/2}$	$\frac{3}{2}$	$m_V^{3/2}$

FIG. 2. Electromagnetic coupling of quarks.



$$g_Q=\frac{1}{\sqrt{2}}G\left(\frac{m_P}{2M}\right)^v\left[\left(\frac{m_P}{2M}\right)^{s-v}R+1\right] \tag{9}$$

is the over-all $QQ\pi^0$ coupling constant. If we assume negligible mass dependence of the last factor in (9), which is strictly true in model I and is justified in the other models if R is not too large, then the consistency of (8b) requires that

$$v=l, \tag{10}$$

a condition that is satisfied only in model I. The additional constraint is

$$\sqrt{2}\,g_Q I'=1. \tag{11}$$

Equation (8a) contains nothing but a Goldberger-Treiman relation for quarks.

It is remarkable that only model I satisfies vector-meson dominance and PCAC for quarks.[10] It is also the only model that predicts a Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin- (KSRF) type relation[11]:

$$f_\rho F_\pi=cm_\rho, \tag{12}$$

where in this model, from (5a), (8a), (10), and (11),

$$c=\sqrt{2}\,g_A I'/I=g_AF/g_Q.$$

We shall use the experimental determination of c to relate g_A and g_Q.[12] From now on we restrict ourselves to model I: $s=v=0$, $e=1$, $m=0$, $l=0$, $k=1$.

Applying vector-meson dominance to the Pauli coupling, one obtains

$$\sum e_i g_i^V K_i=\frac{FR'}{m_V}\frac{1}{f_V}, \tag{13}$$

where K_i gives the anomalous contribution to the magnetic moment of the quark in units of the quark mass. From relations (13) and with a mixing angle θ close to the ideal mixing angle, we have approximately,

$$K_\mathfrak{N}=2\frac{1-(\frac{1}{3})^{1/2}\sin\theta\,(m_\rho/m_\omega)^2}{1+(\frac{1}{3})^{1/2}\sin\theta\,(m_\rho/m_\omega)^2}K_\mathfrak{P} \tag{14}$$

and

$$K_\lambda=\frac{4}{3}\,\frac{(\frac{3}{2})^{1/2}(m_\rho/m_\phi)^2\cos\theta}{1+(\frac{1}{3})^{1/2}\sin\theta\,(m_\rho/m_\omega)^2}K_\mathfrak{P}. \tag{15}$$

For ideal mixing of octet and singlet SU(3) states ($\phi\equiv\lambda\bar{\lambda}$) and with $m_\rho{}^2\approx m_\omega{}^2$, (14) and (15) give

$$K_\mathfrak{N}=K_\mathfrak{P},\quad K_\lambda=(m_\rho/m_\phi)^2K_\mathfrak{P}. \tag{16}$$

The first relation is the usual nonrelativistic quark-model assumption; the second differs by the factor $(m_\rho/m_\phi)^2$.

As we mentioned before, G and R (F and R') can be determined by solving the Bethe-Salpeter and normalization equations, but to obtain such a solution a knowledge of the interacting $Q\bar{Q}$ potential and form factor $W(p,q)$ is required. As we do not know these we use general arguments to relate G to F and R to R'.

Llewellyn Smith showed that in the case of a spin-independent potential, the pseudoscalar and vector relativistic wave functions for vanishing (m_P/M) and (m_V/M) are related by SU(6). This implies, in our model,

$$RG=R'F, \tag{17}$$

a result which we take as more fundamental than the form of the potential. On the other hand, the normalization equation[13] written in the rest frame of the meson gives

$$\mathrm{Tr}\left\{\int\frac{d^4k}{(2\pi)^3}\left[\bar{\Gamma}(k,-q)\frac{i}{\not{k}+\frac{1}{2}\not{q}-M}\gamma^0\frac{i}{\not{k}+\frac{1}{2}\not{q}-M}\Gamma(k,-q)\frac{i}{\not{k}-\frac{1}{2}\not{q}-M}\right.\right.$$
$$\left.\left.-\bar{\Gamma}(k,-q)\frac{i}{\not{k}-\frac{1}{2}\not{q}-M}\Gamma(k,-q)\frac{i}{\not{k}-\frac{1}{2}\not{q}-M}\gamma^0\frac{i}{\not{k}-\frac{1}{2}\not{q}-M}\right]\right\}=4q^0, \tag{18}$$

where Γ stands for Γ_P or Γ_V^μ. The potential contributions have been neglected as in Ref. 4. Working out the trace calculations for P and V mesons under the assumption $\langle\vec{k}^2\rangle<M^2$ and comparing the results,

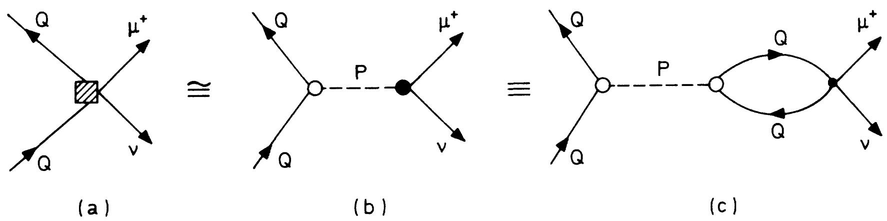


FIG. 3. Divergence of the weak axial-vector coupling of quarks.

$$(aR^2+bR)G^2=(aR'^2+bR')F^2 \,, \tag{19}$$

where, in the approximation $m_V^2(m_P^2) \ll M^2$, a and b are integrals independent of the meson masses. Equations (17) and (19) then give

$$G=F \tag{20}$$

and

$$R=R' \,. \tag{21}$$

We note that the separable-potential model, in general, predicts R to be of the order of R', but a precise relation like (21) is model-dependent. It also gives, for a dominantly scalar interaction, $R<1$ and positive.

To find an estimate for R and R', we use the nonrelativistic quark-model result that the magnetic moments of the quark and the proton are equal[5,14]: $K_p = 2.79/2m_{\text{proton}}$. Equation (5a) for the ρ meson becomes

$$F\frac{m_\rho}{2M}=\frac{f_\rho}{\sqrt{2}} \,, \tag{22}$$

and combined with (13) gives

$$R'=\left(\frac{2.79}{2m_{\text{proton}}}m_\rho\right)\frac{m_\rho}{2M}\simeq\frac{m_\rho}{2M} \,, \tag{23}$$

where the last expression is an approximate numerical result. It is worth recalling that R' does not depend on m_V, but (23) shows that R' (and R) is small of order (m_ρ/M). With (23) the QQV coupling satisfies broken $SU(6)_W$ universality.[10]

Neglecting R, compared to 1, we derive from (8a), (9), (12), (20), (21), and (22) an expression for the basic quark-level weak axial-vector coupling constant g_A and pseudovector $QQ\pi$ coupling constant (g_Q/M) in terms of known quantities:

$$g_A=c/\sqrt{2} \tag{24}$$

and

$$g_Q/M=f_\rho/m_\rho \,. \tag{25}$$

Equation (25) is an additivity quark-model result. In (24) with the experimental value of c, $c\approx 1$, g_A agrees with the additivity result for axial-vector weak decays for baryons, $g_A \simeq 0.70$.[14]

We have shown that Llewellyn Smith's model I contains PCAC and vector-meson dominance consistently built in. In general, we may say that in this model the interactions of hadrons take place through the quark currents and the interactions of quarks are mediated by hadron fields. The model is then dynamically consistent with a description of the meson interactions through the splitting of the mesons into the quarks and the rebuilding of the mesons from the quarks.

III. HADRON COUPLING CONSTANTS

By reversing the argument of Sec. II we see alternatively that in model I the vertex functions (1) and (2) for the basic QQP and QQV couplings and the constraints of vector-meson dominance and pole dominance of the divergence of the weak axial-vector current impose on f_V and F_P the meson mass dependences of Eqs. (3) and (4) without the need to use the graph of Fig. 1(a). For the interesting cases we write their predictions for the f_V's and F_P's:

$$F_\pi=F_K \tag{26}$$

and

$$f_\rho^{-2}:f_\phi^{-2}:f_\omega^{-2}=\frac{1}{m_\rho^2}:\frac{\cos^2\theta}{3m_\phi^2}:\frac{\sin^2\theta}{3m_\omega^2} \,. \tag{27}$$

Neither relation assumes mass degeneracy, but none the less (26) remains the exact SU(3) relation. Equations (27), on the other hand, are broken-SU(3) relations. They satisfy the first Weinberg sum rule[15] and are typical of a current-mixing model[16] of ω-ϕ mixing. We refer to Ref. 4 for further discussion of Eqs. (26) and (27).

We will now show how the mass factors appearing in (1) and (2) affect other couplings and start with PPV interactions [graph of Fig. 1(b)]. We illustrate for the $\pi^+\pi^-\rho$ $[\rho\equiv(\frac{1}{2})^{1/2}(\mathcal{P}\overline{\mathcal{P}}-\mathfrak{N}\overline{\mathfrak{N}})]$ vertex,

$$\langle\pi^+(p_1)\,|\,V_\rho^\mu(q)\,|\,\pi^-(p_2)\rangle=(p_1+p_2)^\mu f_{\rho\pi\pi}(q^2) \,. \tag{28}$$

In the limit $q^2\to 0$ only the quark Dirac coupling, $f_\rho/\sqrt{2}$, acts and if the $QQ\rho$-vertex $W(p,q)$ form factor does not drastically modify the triangle-graph integration, the evaluation of (28) from the triangle graph of Fig 1(b) is then equivalent to the evaluation of the Bethe-Salpeter normalization equation, (18). We thus obtain the vector-meson-dominance result,

$$f_{\rho\pi\pi}(0)=f_\rho \,. \tag{29}$$

In each triangle graph, because of the symmetry of the Bethe-Salpeter wave function,[4] only terms odd in m_P survive so that up to terms of order $(m_P/M)^2$, the coupling constants $f_{PPV}(0)$ for $q^2=0$ are proportional to the vector-meson mass [from (2)] and are independent of the pseudoscalar-meson masses. With the approximation $f_{PPV}(0) \approx f_{PPV}(m_V{}^2)$ and in the ideal mixing limit, we compare in Table III the ratios $\Gamma(V\to PP)/\Gamma(\rho\to\pi\pi)$ predicted in this model with the SU(3)-symmetry limit and experimental values.[17]

It should be noticed that the mass dependence, $f_{PPV}(0)\sim m_V$, is again compatible with the current-mixing model and that when the results for vector-meson electromagnetic decays and PPV couplings are combined, some general vector-meson-dominance results are rederived. For instance,

$$\frac{f_{KK\omega}(0)}{f_\omega}+\frac{f_{KK\phi}(0)}{f_\phi}=\frac{1}{2}\,, \tag{30}$$

which is usually derived by saturating the hypercharge current form factor of the kaon by vector-meson poles, and also the ω-ϕ mixing relation[16]

$$\frac{f_{KK\phi}}{f_\phi}=\tfrac{1}{2}\cos^2\theta\,. \tag{31}$$

The PVV couplings can be worked out in a similar way. One obtains for the $f_{\rho\omega\pi}$ coupling the relation,

$$\frac{f_{\omega\rho\pi}{}^2 m_\rho{}^2}{[(\frac{2}{3})^{1/2}\cos\theta+(\frac{1}{3})^{1/2}\sin\theta]^2}\simeq 4f_{\rho\pi\pi}{}^2\,, \tag{32}$$

which, for ideal mixing, reproduces the $SU(6)_W$ result.[18] In this limit using vector-meson dominance, we obtain for the $\omega\pi\gamma$ coupling,

$$f_{\omega\pi\gamma}{}^2=4e^2/m_\rho{}^2\,. \tag{33}$$

Quark-model additivity calculations[1,14] successfully predicted $f_{\omega\pi\gamma}{}^2=4e^2\mu_{\text{proton}}{}^2$. In the present model this result comes about from (33) and the numerical coincidence exploited in (23).

We note in a similar manner that the VVP couplings are proportional to $1/m_V{}^2$, and when the experimental angle $\theta\approx 34°$ is used in decays involving the ϕ meson, this factor helps to bring down the $\phi\to\pi\gamma$ and $\phi\to\eta\gamma$ rates as seems to be experimentally required.[19-21] Our results on the VVP couplings are in qualitative agreement with the ones of Ref. 20.

TABLE III. Ratios $\Gamma(V\to PP)/\Gamma(\rho\to\pi\pi)$.

Decay	Present work	SU(3)	Experimental
$\phi\to K^+K^-$	0.0216	0.0122	0.0150 ± 0.0033
$\phi\to K^0\bar{K}^0$	0.0139	0.0078	0.0112 ± 0.0026
$K^*\to K\pi$	0.397	0.290	0.404 ± 0.080

Many of the results obtained so far in this section have been derived by other approaches[20,22] involving either the two Weinberg sum rules or in the nonrelativistic quark model,[23] the use of meson wave functions with large SU(3) symmetry breaking. Some related relativistic calculations are contained in Refs. 4, 8, and 24.

Formally, baryon couplings can be treated in a similar way. The electric coupling to vector mesons is related to the Bethe-Salpeter normalization equation provided, as before, that the potential-term contribution is neglected, and we obtain relations such as

$$g_{\rho NN}(0)\simeq\tfrac{1}{2}f_\rho\,, \tag{34}$$

$$g_{\omega NN}(0)\simeq 3(\tfrac{1}{2}f_\rho)\,, \tag{35}$$

and the relation

$$\frac{g_{\rho pp}(0)}{f_\rho}+\frac{g_{\omega pp}(0)}{f_\omega}=1\,, \tag{36}$$

normally obtained by the saturation of the proton electromagnetic current by ρ and ω poles.

As the vertex integral only reduces to the normalization condition at zero momentum transfer, $q^2=0$, one can make model-independent statements only about the electric couplings. To evaluate the magnetic coupling, the weak axial-vector decay coupling, and the couplings to pseudoscalar mesons, whose matrix elements are linear in q, some further assumptions are required. In a three-body problem this also implies additional complications.

The additivity rules for magnetic moments, as well as for the other couplings referred to above, may be rederived with a specific model[25] by doing the calculations in the Breit frame of the baryon, neglecting the interacting quark-space momentum, $\vec{k}$, and assuming that on the average it behaves as if it were on the mass shell,[26] $\langle k_0{}^2\rangle\approx M^2$. For instance, for the magnetic moment of the proton, we then have

$$\mu_p\approx\mu_{\mathcal{P}}=\frac{1}{2M}+K_{\mathcal{P}}\,. \tag{37}$$

To satisfy (37) the magnetic moment of the quarks has to be mostly anomalous if we are working in the Bethe-Salpeter formalism with massive quarks. In the present model the anomalous magnetic moment of the $\mathfrak{N}$ and λ quarks differs from that of the $\mathcal{P}$ quark. Applying (14) and (15), in the extreme case of quarks with only anomalous moment and in the ideal miximg limit, $\tan\theta=(\frac{1}{2})^{1/2}$, we derive corrections to the additivity calculations. From

TABLE IV. Magnetic moments of octet baryons ($\mu_{\text{proton}}=2.793$).

	Theoretical				
	No correction $K_\lambda=K_\mathfrak{N}=K_\mathfrak{P}$		Correction α and β		
Baryon	μ/μ_{proton}	μ	μ/μ_{proton}	μ	Experimental
N	$-\frac{2}{3}$	-1.86	$-\frac{2}{3}\left(\frac{1+2\beta/3}{1+\beta/9}\right)$	-1.90	-1.913
Λ	$-\frac{1}{3}$	-0.93	$-\frac{1}{3}\left(\frac{1-\alpha}{1+\beta/9}\right)$	-0.53	-0.73 ± 0.16
Σ^+	1	2.793	$\left(\frac{1-\alpha/9}{1+\beta/9}\right)$	2.66	$2.5\ \pm0.5$

(14) and (15) we obtain $\beta=0.03$ and $\alpha=0.43$,[17] where α and β are defined by

$$K_\mathfrak{N}=K_\mathfrak{P}(1+\beta)\,,$$

$$K_\lambda=K_\mathfrak{P}(1-\alpha)\,.$$

The effect of these corrections on baryon moments is shown in Table IV (experimental data from Ref. 27). In all cases the corrections act in the right direction, but in the Λ case seem to be more than required. The corrections α and β depend appreciably on the mixing angle. With an angle $\theta<35°$, as required by experiment,[28] the value of the magnetic moment improves, for instance $\mu_\Lambda=0.57$ for $\theta=33°$. Note that large mixing angles, $\theta>36.5°$, give a negative β and a correction in the wrong direction for the magnetic moment of the neutron.

IV. CONCLUSIONS

We started by using the criterion of consistent $Q\overline{Q}$-channel meson-pole dominance to select from the three models of Llewellyn Smith the preferred one: model I. The corresponding vertex function was then systematically applied to hadron couplings, assuming that hadron interactions take place primarily through quark interactions.

Vector-meson dominance, satisfied at the quark level, is then transmitted to the hadron level by using the Bethe-Salpeter normalization equation. The mass factors in the basic vertex functions are SU(3)-breaking factors that affect all the couplings to vector mesons. In particular, we have the mass dependences,

$$f_{PPV}\sim m_V\,,$$

$$f_{VVP}\sim 1/m_V^2\,.$$

The same mass factors are the origin of corrections to the baryon magnetic moments. Because there is no m_P dependence in the QQP vertex function, axial-vector couplings to pseudoscalar mesons remain unchanged. This implies, for instance, the validity of SU(3) for octet baryon pseudoscalar-meson couplings with $F/(F+D)=\frac{2}{5}$ and of generalized Goldberger-Treiman relations.

It should be mentioned that this model can be tranposed without major modifications to the high-energy region. A process $A+B\to A+B$ is described by a graph of the type of Fig. 1(c) and in the region of large s, $t\simeq 0$ the poles in the t $Q\overline{Q}$ channel will then be dominant. Our model can in this way be interpreted either as an exchange model with universality and factorization[29] or as the usual high-energy additivity quark model.[30] We may write the amplitude either as a sum over the exchanged particles or as a sum over the interacting quarks. This aspect has been exploited in Ref. 31.

ACKNOWLEDGMENTS

I am grateful to Professor L. Castillejo for his continued advice and for many suggestions and helpful critical comments. Thanks are also due to N. Craigie and D. Maharajh for discussions. I should also like to acknowledge a research fellowship from the Calouste Gulbenkian Foundation.

*Supported by a Calouste Gulbenkian Foundation Fellowship.

[1] C. Becchi and G. Morpurgo, Phys. Rev. 140, B687 (1965).

[2] C. Becchi and G. Morpurgo, Phys. Rev. 149, 1284 (1966).

[3] R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50, 617 (1967).

[4] C. H. Llewellyn Smith, Ann. Phys. (N.Y.) 53, 521 (1969).

[5] G. Morpurgo, Physics 2, 95 (1965).

[6]R. H. Dalitz, in *Elementary Particle Physics* (Benjamin, New York, 1967).
[7]E. J. Squires and P. J. S. Watson, Ann. Phys. (N.Y.) 41, 409 (1967).
[8]T. Kitazoe and T. Teshima, Nuovo Cimento 57A, 498 (1968).
[9]See, for instance, T. Gudehus, Phys. Rev. 184, 1788 (1969).
[10]P. G. O. Freund, Phys. Letters 15, 352 (1965); P. G. O. Freund, in *Fundamental Particle Physics* (Benjamin, New York, 1968).
[11]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966).
[12]In Ref. 4 a derivation of the KSFR relation with $c=1$ is given, but such a derivation is questionable as it requires that the quark unrenormalized g_A be identical to the renormalized one and neglects devergent terms in the loop integrals I and I' to make them equal.
[13]See, for instance, D. Lurié, A. J. Macfarlane, and Y. Takahashi, Phys. Rev. 140, B1091 (1965).
[14]J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969) or Ref. 3.
[15]S. Weinberg, Phys. Rev. Letters 18, 507 (1967); J. J. Sakurai, *ibid.* 19, 803 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* 19, 470 (1967).
[16]N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).
[17]Data from: Review of Particle Physics, Phys. Letters 33B, 1 (1970).
[18]B. Sakita and K. C. Wali, Phys. Rev. 139, B507 (1965).
[19]E. Cremer, Nucl. Phys. B14, 52 (1969).
[20]L. H. Chan, L. Clavelli, and R. Torgerson, Phys. Rev. 185, 1754 (1969).
[21]D. F. Greenberg, Carnegie Mellon University report, 1969 (unpublished).
[22]S. Bose and R. Torgerson, Phys. Rev. Letters 19, 1151 (1967).
[23]A. Dar and V. F. Weisskopf, Phys. Letters 26B, 670 (1968).
[24]K. Fukushima, M. Kaiba, and K. Kudo, Progr. Theoret. Phys. (Kyoto) 43, 172 (1970).
[25]J. Dias de Deus, Ph.D. thesis, University of London, 1971 (unpublished).
[26]The same condition, in a different context, was applied in the parton model to ensure gauge invariance at the electromagnetic parton vertex: G. B. West, Phys. Rev. Letters 24, 1206 (1970).
[27]D. Flamm, HEP VII (Vienna), 1969.
[28]E. Lohrmann, in *Proceedings of the Lund International Conference on Elementary Particles, 1969,* edited by G. von Dardel (Berlingska, Lund, Sweden, 1970).
[29]P. G. O. Freund, Nuovo Cimento, 43A, 1171 (1966). N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters 22, 336 (1966).
[30]E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [Soviet Phys. JETP Letters 2, 65 (1965)]. H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).
[31]J. Dias de Deus, Phys. Rev. D (to be published).

PHYSICAL REVIEW D VOLUME 4, NUMBER 9 1 NOVEMBER 1971

Remarks on the Hadron Distributions in Electroproduction*

Rudolph C. Hwa† and C. S. Lam‡
The Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790
(Received 13 July 1971)

A model for electroproduction is discussed. It is an extrapolation from the diffractive model for the hadron-hadron collisions. The principle involved in the extrapolation is the idea that coherent particles can be produced when momentum transfer between the virtual photon and the target proton can be sufficiently small. In the deep-inelastic region there are unavoidably many incoherent products. A comparison of our model with that of Chou and Yang recently proposed for electroproduction is discussed. Multiplicity and momentum distributions of the secondary hadrons are described.

I. INTRODUCTION

Various models[1,2] have been suggested to describe the production processes in electron-proton collisions with varying degree of physical and mathematical details. In this paper we present yet another view of the subject. Our description will be mainly qualitative, stressing more the general picture rather than technical details. We aim to cover the whole inelastic region and offer a unified view of the production processes as one goes from the resonance region to the diffraction region and to the deep-inelastic region. Conceptually, we shall not hesitate to oscillate between the complementary pictures as viewed in the s and t channels. In the s channel one has the more physical picture that the geometrical considerations can provide, while in the t channel one has the mathematical description based on crossing symmetry and Regge behavior. In fact, an objective of our study is an