akci and M. B. Halpern, *ibid.* 183, 1456 (1969).

[12]M. A. Virasoro, Ref. 9.

[13]This procedure is necessary, and is crucial in obtaining the correct threshold behavior discussed by Bloom and Gilman in Ref. 4.

[14]Here we make the usual smoothed-average assumption for the Veneziano formula in the $+\infty$ direction.

[15]The more precise form of $\ln(1-1/\omega)^{-1}$ dependence is

$$\frac{\ln[1/(1-1/\omega)]}{a\ln[1/(1-1/\omega)]+b\ln|q^2|}\,.$$

See Appendix C.

[16]S. D. Drell and T. M. Yan, Phys. Rev. Letters 24, 181 (1970); G. West, *ibid.* 24, 1206 (1970).

[17]We should mention that because of the $i\epsilon$ prescription in Eq. (3), the factor $\ln|q^2|$ should become $(\ln^2|q^2|+\pi^2)^{1/2}$.

[18]I. S. Gerstein, K. Gottfried, and K. Huang, Phys. Rev. Letters 24, 294 (1970).

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Predictions of the Parton Dual-Resonance Model for the Total Cross Section of Electron-Positron Annihilation into Hadrons*

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On the assumption that a heavy virtual photon behaves like a parton-antiparton pair when participating in the strong interaction, we predict that the total cross section for $e^+e^-\to$ hadrons, as $q^2\to\infty$, behaves like $Cq^{-4}(|q^2|\ln|q^2|)^{-1}$ for spin-0 partons, and like $C'q^{-4}(\ln|q^2|)^{-1}$ for spin-$\frac{1}{2}$ partons, where q^2 is the virtual photon's mass squared. This provides a simple, yet important test of the parton dual-resonance model for deep-inelastic lepton-hadronic scattering, proposed in a previous paper.

Because the final-state interaction among partons has not been taken into account, all previous parton models for the scatterings $e^-+N\to e^-+$hadrons and $e^++e^-\to N+$hadrons consider a parton successively coupled (pointlike) to a pair of heavy virtual photons; and to obtain the structure functions W_1, νW_2, an imaginary-part cut across this parton is necessary. On doing this, one immediately faces the puzzle: What is a parton? Why is the parton not observed experimentally? Motivated by this, we have proposed[1] a six-point-function dual-resonance model for the virtual forward-Compton-scattering amplitude, in order to take the final-state interaction into account and hence to resolve the puzzle.

It turns out[1] that the final-state interaction among the partons is of the diffractive type (Pomeranchukon exchange in the parton-parton channel), and it breaks the scaling law by a factor of $(a+b\ln|q^2|)^{-1}$. Two crucial assumptions[1] in constructing that model are: that the parton is pointlike coupled to the heavy virtual photon, and that the final-state interaction effect does not allow the particular parton, which absorbs the virtual photon, to be observed experimentally. Because of these two assumptions, a heavy virtual photon is naturally pictured as a parton-antiparton pair when participating in the strong interaction. In this note, we suggest a simple experimental test of this interesting idea.

We consider the measurement of the total cross section for the process $e^+e^-\to$hadrons. Within the

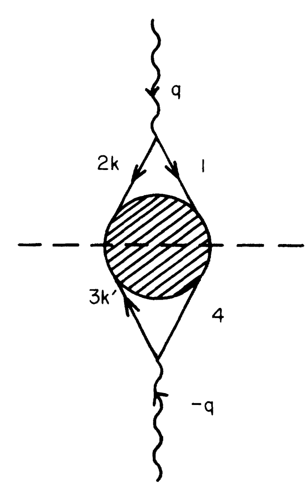

FIG. 1. The four-legs-off-shell parton dual-resonance model for the total cross section $\sigma(e^+e^-\to$hadrons). The dashed line indicates the imaginary part in q^2.

framework of the parton dual-resonance model discussed in Ref. 1, we, therefore, consider a four-parton-leg Veneziano formula (with two loop integrations) for the total cross section $\sigma_{e\bar{e}}$, shown in Fig. 1, in which the dashed line indicates the imaginary part in the virtual-photon mass variable q^2.

We formulate the model for the total cross section as

$$\sigma_{e\bar{e}}^{(i)}(q^2)=\frac{4\pi^2\alpha}{q^4}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)^{-1}\operatorname{Im}_{q^2}T_{\mu\nu}^{(i)}, \tag{1}$$

with

$$T_{\mu\nu}^{(i)}=\int d^4k_2d^4k_3\,\frac{K_{\mu\nu}^{(i)}\overline{B}_4(q-k_2,k_2,k_3,-q-k_3)}{[(k_2-q)^2-m^2](k_2{}^2-m^2)(k_3{}^2-m^2)[(k_3+q)^2-m^2]}, \tag{2}$$

where m is the common mass of the four partons,

$$K_{\mu\nu}^{(i)}=\begin{cases}-(2k_2-q)_\mu(2k_3+q)_\nu & \text{for spin-0 partons}\quad (i=1),\\ -(2k_2-q)_\mu(2k_3+q)_\nu+q^2\left(g_{\mu\nu}-\dfrac{q_\mu q_\nu}{q^2}\right) & \text{for spin-}\tfrac{1}{2}\text{ partons}\quad (i=2),\end{cases} \tag{3}$$

and

$$\overline{B}_4=\int_0^1 dx\,x^{-\alpha_{12}(q)-1}(1-x)^{-\alpha_{23}(k_2+k_3)-1}. \tag{4}$$

As in Ref. 1, and again in Eq. (4), the q^2 variable belonging to legs 1 and 2 must be analytically continued in the opposite direction to that belonging to legs 3 and 4 in crossing the q^2-plane cut. We indicate this by a "bar" over the ordinary four-point Veneziano formula.

Substituting Eqs. (3) and (4) into Eq. (2), and using the identity

$$\frac{1}{k_j{}^2-m^2+i\epsilon}=-\int_0^\infty da_j\exp[a_j(k_j{}^2-m^2)+i\epsilon],\quad j=1,2,3,4 \tag{5}$$

we can perform the double loop integrations over k_2 and k_3, and obtain[2]

$$T_{\mu\nu}^{(i)}=\int_0^\infty da_1da_2da_3da_4\int_0^\infty d[\ln(1/x)]\,x^{-\alpha_{12}}(1-x)^{-\alpha_{23}-1}K_{\mu\nu}^{(i)}\frac{\pi^4}{|C|^2}\exp\{q^2[\ln(1/x)+D/C]\}\,e^{-J}, \tag{6}$$

where

$$\begin{aligned}C&=(a_1+a_2+a_3+a_4)\ln(1-x)^{-1}+(a_1+a_2)(a_3+a_4),\\ D&=(a_1+a_4)C-\{2a_1a_2\ln(1-x)^{-1}+a_4{}^2[a_1+a_2+\ln(1-x)^{-1}]+a_1{}^2[a_3+a_4+\ln(1-x)^{-1}]\},\end{aligned} \tag{7}$$

and

$$J=m^2(a_1+a_2+a_3+a_4)+i\epsilon.$$

We now take the limit $q^2\to-\infty$, where the Veneziano formula converges nicely. From Eqs. (6) and (7), we see that the important region is when $\ln(1/x)$, a_1, a_4 are small. We thus make the scale transformation[3]

$$a_1=\rho\beta_1,\quad a_4=\rho\beta_2,\quad \ln\frac{1}{x}=\rho(1-\beta_1-\beta_2). \tag{8}$$

Make the approximations $(1-x)\approx\rho(1-\beta_1-\beta_2)$, $\ln(1-x)^{-1}\approx\ln q^2$, and put $\rho\approx 0$ everywhere else except in the coefficient of q^2 in the exponent of Eq. (6). We then do the ρ integration and set[4] $\alpha_{23}\equiv 1$, as suggested in Ref. 1. We find

$$T_{\mu\nu}^{(i)}\underset{q^2\to-\infty;\,\alpha_{23}\equiv1}{\sim}\pi^4\int_0^\infty da_2da_3\tilde{K}_{\mu\nu}^{(i)}\frac{\exp[-m^2(a_2+a_3)]}{|a_2a_3+(a_2+a_3)\ln q^2|^2}\int_0^1 d\beta_1\int_0^{1-\beta_1}d\beta_2\frac{1}{q^2(1-\beta_1-\beta_2)^2+i\epsilon}, \tag{9}$$

where

$$\tilde{K}_{\mu\nu}^{(i)}\approx\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\times\begin{cases}1-\dfrac{2}{a_2+a_3+i\epsilon}, & i=1\\ q^2, & i=2.\end{cases} \tag{10}$$

The $i\epsilon$ prescription in Eqs. (9) and (10) is from the Feynman field-propagator prescription for the four parton legs. It is crucial to have it here, otherwise the imaginary part of q^2 vanishes. Because the β_1, β_2

integrals are divergent, a further enhancement in the q^2 asymptotic behavior comes from the region $\beta_1 \approx 0$, $\beta_2 \approx 1$. We make the second scale transformation

$$\beta_1 = \rho'\alpha_1, \quad 1-\beta_2 = \rho'(1-\alpha_1). \tag{11}$$

We then integrate ρ' over a small region, i.e.,

$$\int_0^1 d\beta_1 \int_0^{1-\beta_1} d\beta_2 \frac{1}{q^2(1-\beta_1-\beta_2)^2 + i\epsilon} \approx \int_0^1 d\alpha_1 \int_0^{\epsilon'} d\rho' \frac{1}{q^2\rho' + i\epsilon}$$
$$= \ln\left(\frac{\epsilon' q^2 + i\epsilon}{i\epsilon}\right) \approx \ln q^2[1 + O(\ln^{-1} q^2)]. \tag{12}$$

The double integrals over a_2 and a_3, in Eq. (9), can also be reduced to a single integral over $x \equiv a_2 + a_3$, as has been shown in Appendix C of Ref. 1. We thus find,[5] from Eq. (9),

$$T^{(i)}_{\mu\nu} \underset{q^2\to-\infty;\alpha_{23}\equiv 1}{\sim} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \pi^4 \int_0^\infty dx \frac{e^{-m^2 x} K^{(i)}}{|\frac{1}{4}x + \ln q^2|} \left(\frac{\ln q^2}{q^2}\right), \tag{13}$$

with

$$K^{(i)} = \begin{cases} 1 - \dfrac{2}{x+i\epsilon}, & i=1 \\ q^2, & i=2. \end{cases} \tag{14}$$

Now we analytically continue[6] Eq. (13) to $q^2 \to +\infty \pm i\epsilon$ and take its imaginary part in q^2. Remember that the q^2 variable belonging to legs 1 and 2 must be analytically continued opposite to the q^2 variable belonging to legs 3 and 4. This is reflected in the fact that the $\ln q^2$ factor is changed into $\ln|q^2| \pm i\pi$. On the other hand, the denominator in Eq. (13), which contributes only the residues of the resonance poles in q^2, must remain real and positive definite when it is analytically continued to $q^2 \to +\infty \pm i\epsilon$. This is due to the optical theorem for the cross section. Therefore, the denominator factor is changed into $\frac{1}{4}x + (\ln^2|q^2| + \pi^2)^{1/2}$. We thus take the imaginary part in q^2 from Eq. (13), resulting in

$$\operatorname{Im} T^{(i)}_{\mu\nu} \underset{q^2\to+\infty}{\sim} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \pi^5 \int_0^\infty dx \frac{e^{-m^2 x}}{\frac{1}{4}x + (\ln^2|q^2| + \pi^2)^{1/2}} \times \begin{cases} \dfrac{C}{|q^2|}, & i=1 \\ 1, & i=2. \end{cases} \tag{15}$$

We thus predict, from Eqs. (15) and (1), that

$$\sigma^{(i)}_{e\bar{e}}(q^2) \underset{q^2\to+\infty;\ln|q^2|\to\infty}{\sim} \frac{1}{q^4 \ln|q^2|} \times \begin{cases} \dfrac{C}{|q^2|} & \text{for spin-0 partons} \quad (i=1), \\ C' & \text{for spin-}\tfrac{1}{2}\text{ partons} \quad (i=2). \end{cases} \tag{16}$$

C and C' are two constants. This prediction shows that the total cross section for $e^+e^- \to$ hadrons is of the same order of magnitude as the pointlike interaction.

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[1]L.-p. Yu, preceding paper, Phys. Rev. D 4, 2775 (1971).

[2]The determinant factor C must be real and positive definite since we are making a model for the cross section.

[3]Contrary to Ref. 1, here we do not need to consider the extra pieces of the imaginary part cutting across the two partons, since taking the imaginary part along the line cutting through the two partons is equivalent to putting the two partons on the mass shells, which then vanishes by the energy-momentum conservation at the $\gamma p\bar{p}$ vertex.

[4]This crucial step was used in Ref. 1 to obtain a result consistent with the scaling law. We have interpreted this by saying that the final-state interaction among partons is diffractive in nature. (See Ref. 1.)

[5]The exact $\ln|q^2|$ dependence, as shown in Appendix C of Ref. 1, is

$$\frac{1}{|\frac{1}{4}x^2 + x\ln|q^2||^{1/2}} \ln\left[\frac{\frac{1}{2}x + (\frac{1}{4}x^2 + x\ln|q^2|)^{1/2}}{\frac{1}{2}x - (\frac{1}{4}x^2 + x\ln|q^2|)^{1/2}}\right].$$

We have approximated the complicated ln[ ] factor by assuming $\ln|q^2|$ large.

[6]We make the usual smoothed-average assumption in the $+\infty$ direction of the Veneziano amplitude.