with q and s the relative momentum and effective mass squared of the final π-π system.

Though all the distributions were fitted, the one most useful in determining the phase shifts was the $\cos\theta$ distribution, where θ is the scattering angle in the π-π center-of-mass system. At each π-π effective mass, the distribution was expanded in a power series in $\cos\theta$,

$$\frac{dN}{d\cos\theta}=\sum_{n=0}^{4} C_n \cos^n\theta. \tag{2}$$

In Fig. 1 we show the experimental values of the expansion coefficients as well as the best fits with and without a D-wave contribution. Whereas the previous analysis of these data[3,4] had difficulty in obtaining a good fit to the magnitude of the forward-backward asymmetry, the inclusion of the D wave reduces the discrepancy. We likewise present, in Fig. 2, the results for the fit to the distribution in the Treiman-Yang angle ϕ. Comparisons with other distributions are presented in Ref. 5.

The best fit to the data was attained with an S wave $I=0$ dominated by a broad resonance centered at 820 MeV with a width $\Gamma=600$ MeV. This is consistent with the findings of other groups.[6] To recapitulate, the results we obtain for the π-π amplitudes are (the quantities in square brackets were not varied)

$$\begin{aligned}
&\frac{q}{\sqrt{s}}\cot\delta_{00}(s)=\frac{{m_S}^2-s}{m_S\Gamma_S}\frac{q_S}{m_S},\\
&q\cot\delta_{20}(s)=-12.5\mu+0.25q^2/\mu,\\
&\frac{q^3}{\sqrt{s}}\cot\delta_{11}(s)=\frac{{m_P}^2-s}{m_P\Gamma_P}\frac{{q_P}^3}{m_P}\frac{1+0.193\mu^{-2}q^2}{1+0.193\mu^{-2}{q_P}^2},\\
&\frac{q^5}{\sqrt{s}}\cot\delta_{02}(s)=\frac{{m_D}^2-s}{m_D\Gamma_D}\frac{{q_D}^5}{m_D},
\end{aligned} \tag{3}$$

where μ is pion mass in MeV, $m_S=820$ MeV, $\Gamma_S=600$ MeV, $[m_P]=765$ MeV, $[\Gamma_P]=125$ MeV, $[m_D]=1264$ MeV, $[\Gamma_D]=151$ MeV, and q_l for $l=S,P,D$ is q at the resonance in lth partial wave.

A graphical presentation of these results is given in Fig. 3 together with the results of Ref. 1.

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T, P, and C Symmetries in $K_{L,S}\to\gamma\gamma$

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The Bernstein-Michel analysis of $\pi^0\to\gamma\gamma$ is applied to the decays $K_{L,S}\to\gamma\gamma$. It is shown that, in contrast to the situation in $\pi^0\to\gamma\gamma$, a hypothetical violation of CP invariance in $K_{L,S}\to\gamma\gamma$ can, in principle, be detected by a measurement of the circular polarization of *one* of the two photons.

I. INTRODUCTION

In a paper entitled "T, P, and C Symmetries in $\pi^0\to\gamma\gamma$," Bernstein and Michel[1] obtained the necessary and sufficient conditions for the two-photon state created in this decay to be an eigenstate of P, T, and PT (or, equivalently, of CP, T, and CPT, since the state necessarily has $C=+1$). Their conclusions were as follows: If the polarization wave function of the photons is

$$\psi=\alpha\vec{\epsilon}\cdot\vec{\epsilon}\,'+\beta\vec{\epsilon}\times\vec{\epsilon}\,'\cdot\hat{k}, \tag{1}$$

then the two-photon state is an eigenstate

$$\text{of } P, \text{ if and only if } b \equiv \frac{|\alpha|^2 - |\beta|^2}{|\alpha|^2 + |\beta|^2} = \pm 1,$$

$$\text{of } T, \text{ if and only if } c \equiv \frac{2\,\mathrm{Re}\alpha^*\beta}{|\alpha|^2 + |\beta|^2} = 0, \qquad (2)$$

$$\text{of } PT, \text{ if and only if } d \equiv \frac{2\,\mathrm{Im}\alpha^*\beta}{|\alpha|^2 + |\beta|^2} = 0,$$

the condition $b=+1$ (-1) corresponding to even (odd) parity.

The significance of the parameters b, c, and d is that they completely characterize the density matrix of the 2γ system and can be experimentally measured. Two types of experiment are needed for their determination. The parameter d may be found from an experiment which measures the circular polarization of *one* of the photons produced in $\pi^0 \to \gamma\gamma$, leaving the other undetected. Such an experiment was performed by Garwin *et al.*,[2] who measured the Compton scattering of the photon at 90° from a target of magnetized iron, and compared the rates when the magnetization was parallel and antiparallel to the incident photon. The difference in rates is given by

$$\frac{R_1 - R_2}{R_1 + R_2} = 2Ad, \qquad (3)$$

where A is the "analyzing power" of the polarimeter. The results of this experiment were consistent with $d=0$, showing that the 2γ state was an eigenstate of CPT. For the determination of b and c, it is necessary to measure the polarization of *both* photons in coincidence. Thus, if ω is the angle between the planes of polarization of the two photons, the coincidence rate is

$$R = \text{const}[1 + u^2(b\cos\omega + c\sin\omega)], \qquad (4)$$

where u is the analyzing power of each detector. In practice, the planes of polarization of the photons are determined by the planes of the $e^+ - e^-$ pairs produced by external or internal conversion. An experiment to find b and c for the decay $\pi^0 \to \gamma\gamma$ was carried out by Samios *et al.*,[3] who measured the correlation of the internally converted pairs. Using for u^2 the value predicted by quantum electrodynamics, they found results consistent with $b=-1$, $c=0$. Thus, the complete set of parameters describing the 2γ state in $\pi^0 \to \gamma\gamma$ was found, within experimental errors, to be $(b,c,d) = (-1, 0, 0)$. This is, of course, what one expects if the π^0 is pseudoscalar and its decay conserves parity.

It is of some interest to ask which parameters would describe the 2γ state resulting from the decays $K_S \to \gamma\gamma$ or $K_L \to \gamma\gamma$. The question assumes importance if one admits the possibility of *large* CP violation in these processes. Such a violation could arise, for instance, from a violation of C invariance in the electromagnetic interaction, or from a CP violation peculiar to weak-electromagnetic decays. Even if such large violations are absent, a certain degree of CP violation is always to be expected, since the K_S and K_L states have been demonstrated not to be pure CP eigenstates. In this paper, we shall not concern ourselves with small effects of the latter kind, as these would not be measurable for a long time. We shall, instead, ask the following question. Assuming K_S and K_L to be identical with the CP eigenstates K_1 and K_2, also assuming CPT invariance, but allowing for a substantial CP violation in the decay amplitudes of $K_1 \to \gamma\gamma$ and $K_2 \to \gamma\gamma$, what can one say about the parameters of the emerging two-photon state?

II. ANALYSIS OF $K^0, \bar{K}^0 \to \gamma\gamma$

Consider first the situation when CP is conserved. The decay $K_1 \to \gamma\gamma$ will produce photons in a state of even parity, with parameters $b=+1$, $c=0$, $d=0$, while the decay $K_2 \to \gamma\gamma$ will produce a state of odd parity characterized by $b=-1$, $c=0$, $d=0$. Let us now allow for a violation of CP invariance. Following the notation of Sehgal and Wolfenstein,[4] we consider the CP eigenstates of the two-photon system,[5]

$$|X_e\rangle = (|RR\rangle + |LL\rangle)/\sqrt{2} \quad (CP \text{ even}),$$
$$|X_o\rangle = (|RR\rangle - |LL\rangle)/\sqrt{2} \quad (CP \text{ odd}). \qquad (5)$$

Assuming CPT invariance, the decay amplitudes of $|K_1\rangle$ and $|K_2\rangle$ to these states may be written as

$$\langle X_e|T|K_1\rangle = c_e e^{i\rho_e},$$
$$\langle X_o|T|K_1\rangle = ic_o e^{i\rho_o},$$
$$\langle X_e|T|K_2\rangle = -id_e e^{i\mu_e}, \qquad (6)$$
$$\langle X_o|T|K_2\rangle = d_o e^{i\mu_o}.$$

The amplitudes (6) are so defined that c_e, c_o, d_e, d_o are real quantities, while ρ_e, ρ_o, μ_e, μ_o are "unitarity" phases, which are equal to zero if the decay amplitudes have no absorptive parts. In general, both $K_1 \to \gamma\gamma$ and $K_2 \to \gamma\gamma$ amplitudes do have absorptive parts, since, as was emphasized by Dreitlein and Primakoff,[6] real intermediate states are possible in both cases, as a result of which the T matrix (or "effective Hamiltonian") is not Hermitian. The polarization wave function of the photons resulting from $K_{1,2} \to \gamma\gamma$ will be

$$\psi_{1,2} = \alpha_{1,2}\vec{\epsilon}\cdot\vec{\epsilon}\,' + \beta_{1,2}\vec{\epsilon}\times\vec{\epsilon}\,'\cdot\hat{k}, \qquad (7)$$

where

$$\alpha_1 = c_e e^{i\rho_e}, \quad \beta_1 = c_o e^{i\rho_o},$$
$$\alpha_2 = -id_e e^{i\mu_e}, \quad \beta_2 = -id_o e^{i\mu_o}. \qquad (8)$$

Therefore the parameters $(b,\ c,\ d)$ of the 2γ state are [using subscripts 1 (2) for K_1 (K_2)$\rightarrow\gamma\gamma$]:

$$b_1=\frac{c_e^2-c_o^2}{c_e^2+c_o^2},$$

$$c_1=\frac{2c_ec_o}{c_e^2+c_o^2}\cos(\rho_e-\rho_o), \tag{9a}$$

$$d_1=\frac{-2c_ec_o}{c_e^2+c_o^2}\sin(\rho_e-\rho_o);$$

$$b_2=\frac{d_e^2-d_o^2}{d_e^2+d_o^2},$$

$$c_2=\frac{2d_ed_o}{d_e^2+d_o^2}\cos(\mu_e-\mu_o), \tag{9b}$$

$$d_2=\frac{-2d_ed_o}{d_e^2+d_o^2}\sin(\mu_e-\mu_o).$$

The interesting feature of Eqs. (9) is the appearance of the factors $\sin(\rho_e-\rho_o)$ and $\sin(\mu_e-\mu_o)$ in the parameters d_1 and d_2. Since both $K_1\rightarrow\gamma\gamma$ and $K_2\rightarrow\gamma\gamma$ admit of intermediate states whose mass is lower than the K-meson mass, one must expect, *a priori*, that the phase differences $(\rho_e-\rho_o)$ and $(\mu_e-\mu_o)$ will be different from zero. *It follows that if CP invariance is violated (that is, if $c_o, d_e\neq 0$), not only will the 2γ state emerging from K_1 or K_2 not be an eigenstate of CP or of T, but it may, by virtue of $d_{1,2}\neq 0$, not be an eigenstate of CPT.* This possibility occurs even though K_1 and K_2 have been taken to be eigenstates of CPT, and CPT invariance has been assumed. The fact that d_1 and d_2 can be nonzero in the presence of CP violation implies that such a violation in the decays $K_{1,2}\rightarrow\gamma\gamma$ can, in principle, be detected in a one-photon experiment of the type performed by Garwin *et al.* This is in contrast to the situation with $\pi^0\rightarrow\gamma\gamma$, where the relevant unitarity phases are necessarily zero to a high degree of accuracy. (For example, the amplitude for $\pi^0\rightarrow\gamma\gamma$ could acquire an absorptive part from the final-state interaction of the photons, but the resulting unitarity phase would be of order α^2 only.) A hypothetical violation of CP invariance in $\pi^0\rightarrow\gamma\gamma$, therefore, necessarily requires for its detection a two-photon experiment of the type performed by Samios *et al.*

The unitarity phases ρ_e and μ_o depend on the mechanisms underlying the CP-conserving transitions. Assuming that the amplitude $c_e e^{i\rho_e}$ of the transition $K_1\rightarrow 2\gamma$ ($CP=+1$) is the result of the sequence $K_1\rightarrow 2\pi\rightarrow 2\gamma$, Martin, de Rafael, and Smith[7] have obtained, using perturbation theory, a number proportional to $(1.25-2.21i)$, which implies $\rho_e=-61$ degrees (or 119 degrees). A possible model for the amplitude $d_o e^{i\mu_o}$ of the transition $K_2\rightarrow 2\gamma$ ($CP=-1$) is to assume that it occurs via $K_2\rightarrow 3\pi\rightarrow 2\gamma$. In this case the phase μ_o is difficult to calculate. To the extent, however, that the phase space available for the real decay $K_2\rightarrow 3\pi$ is small, the phase μ_o is unlikely to be large. As regards the phases ρ_o and μ_e, these will be determined by the mechanisms underlying the CP-violating transitions. In order that these phases be different from zero, it is essential that these transitions be mediated by some intermediate states of mass lower than the K mass. Such states are limited to 2π, 3π, $\pi\pi\gamma$, etc., all of which have zero strangeness. In some models of CP violation, such low-mass intermediate states may not be possible.[8] In such cases, the phases ρ_o and μ_e will be zero.

Finally, it may be remarked that an independent way of establishing CP violation in the decays $K_{S,L}\rightarrow\gamma\gamma$ is to look for an interference term in the time dependence of the two-photon decay rate of a K_S-K_L mixture.[4] Analysis shows that such an effect is likely to be measurable only if $\Gamma(K_S\rightarrow\gamma\gamma)$ is much larger than $\Gamma(K_L\rightarrow\gamma\gamma)$. Since theoretical indications are that the two rates are comparable,[9] the polarization measurements discussed above might be the only way of detecting CP violation in these processes.

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