reduction of a five-point amplitude which, for computational reasons, is restricted to one Veneziano term for each permutation. As a result, the four-point amplitude only involves one Veneziano term. As we would expect, this is not sufficient to provide a good fit to the density-matrix data. As well as satellite terms, it would also be necessary to include terms with ρ and A_2 exchange and absorptive corrections[6] in a more realistic model for $K^-p \to \bar{K}^{*0}n$. However, these remarks apply equally to all applications of the Veneziano model to three-particle production in which the five-point amplitude is used to obtain amplitudes for various subprocesses. The fault lies mainly in the inadequacy of present-day computers to cope with satellite terms in the five-point model and the poor statistics of the data, which prevent detailed tests of the model.

Accepting these difficulties, we feel that the technique suggested here for the construction of kinematic factors for five-point Veneziano models provides a considerable improvement on the previous rather arbitrary choices.

It is especially pleasing to be able to include pion exchange.[7] The technique also takes account of the spins of the external particles and correct forward asymptotic behavior is always ensured.

It should be noted, however, that our amplitude is constructed by reference to the dominant peripheral diagram for a given process. In general, it will not be the leading term for other processes obtained by crossing the external lines. If we started by considering crossed processes having baryon exchange as the dominant mechanism, we would obtain different kinematic factors.

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Regge Cuts in the Van Hove Model: Pion-Nucleon Forward Charge-Exchange Scattering*

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A Regge pole-cut model based on the Durand–Van Hove model is presented for forward pion-nucleon charge exchange. Moving cuts are introduced through the j dependence of the mass of the exchanged spin-j object. The model fits the differential cross sections quite well. The polarization predictions are quite distinct from the Reggeized absorption model, and polarization measurements at $t = -0.5$ GeV^2 will be decisive between these two models.

In this paper we present a Regge pole-cut model for forward processes, in particular for π^-p forward charge exchange. Our model is based on the Durand–Van Hove model,[1] with Regge-cut contributions introduced by the j dependence of the effective mass of the exchanged spin-j object. The model amplitudes exhibit moving Regge cuts whose discontinuities vanish at the branch point, in agreement with the results derived by Bronzan and Jones[2] using partial-wave unitarity. Thus, the model generalizes the Carlitz-Kislinger[3] and Durand-Lipinski[4] models to forward processes. However, unlike the Durand-Lipinski model or the strong-cut Reggeized absorption model,[5] the relative strength of the pole and cut contributions is not a variable parameter in this model. The model provides rather good fits to the forward charge-exchange differential cross sections and polarizations at several energies and over a wide range of t.

Our model for the forward πN amplitude is constructed by summing an infinite set of signatured, natural-parity spin-j exchanges in the crossed (t) channel. There are two couplings of a natural-parity object to the $N\bar{N}$ vertex, which we take as

$$(N\bar{N}\,\text{coupling}) = p_\mu \cdots p_\sigma p_\lambda + ma(j) p_\mu \cdots p_\sigma i\gamma_\lambda\,, \qquad (1)$$

where there are j factors of $p_\mu = \frac{1}{2}(p_1 - p_2)_\mu$ in the first term and $a(j)$ is a j-dependent function measuring the relative contribution of the two types of coupling; the factor m of nucleon mass is inserted for dimensional reasons. Since a spin-zero object only couples to the $N\bar{N}$ vertex through the first term in Eq. (1), $a(j=0)=0$, and $\tilde{a}(j) = a(j)/j$ is regular in j. The signatured t-channel helicity amplitudes, to leading order in $x_t = \cos\theta_t$, are given by

$$-f_{++}^{t\,\tau}(s,t) = \frac{1}{2p}\sum_j (2j+1) g^2(j)[m^2 a(j) - p^2] \times (pq)^j \frac{P_j(-x_t) + \tau P_j(x_t)}{m^2(j) - t}\,, \qquad (2)$$

$$-f_{+-}^{t\,\tau}(s,t) = \frac{m\sqrt{\varphi}}{4p}\sum_j (2j+1) g^2(j)\tilde{a}(j) \times (pq)^{j-1} \frac{P_j'(-x_t) - \tau P_j'(x_t)}{m^2(j) - t}\,, \qquad (3)$$

where $4p^2 = t - 4m^2$, $4q^2 = t - 4\mu^2$, $\sqrt{\varphi} = 2pq(\sin\theta_t)/\sqrt{t}$, $4pqx_t = s - u$, and $g^2(j)$ is the coupling constant for spin j. Note that, in this model, the kinematic constraint at $p^2 = 0$ is automatically satisfied. This is an important factor in constructing the amplitudes for processes like photoproduction, where $t=0$ constraints are important, or $\pi N \to \rho N$, where there is a low-lying pseudothreshold at $t = (m_\rho - m)^2 \approx 1.5\mu^2$.

Motivated by the requirements that $m^2(j) \sim j$ for large j, and that the widths of the resonances along the Regge trajectory be nonzero, we take

$$\alpha_p' m^2(j) = j - \alpha_0 - i\gamma[j - \alpha_c(t)]^{1/2}\,, \qquad (4)$$

where $\alpha_p(t) = \alpha_0 + \alpha_p' t$, $\alpha_c(t) = \alpha_0 + \alpha_c' t = \alpha_p(t) - a^2 t$, and γ determines the strength of the cut contribution. For t in the resonance region, this leads to an effective Regge trajectory

$$j = \alpha_+(t) = \alpha_p(t) - \tfrac{1}{2}\gamma^2 + \tfrac{1}{2} i\gamma(4a^2 t - \gamma^2)^{1/2}\,. \qquad (5)$$

We require $\mathrm{Im}\,\alpha_+(t) = 0$ for $t < 4\mu^2$, which leads to $a^2 = \alpha_p' - \alpha_c' > 0$ and $\gamma = 4a\mu$. In addition, Eq. (4) also represents a Regge cut whose branch point

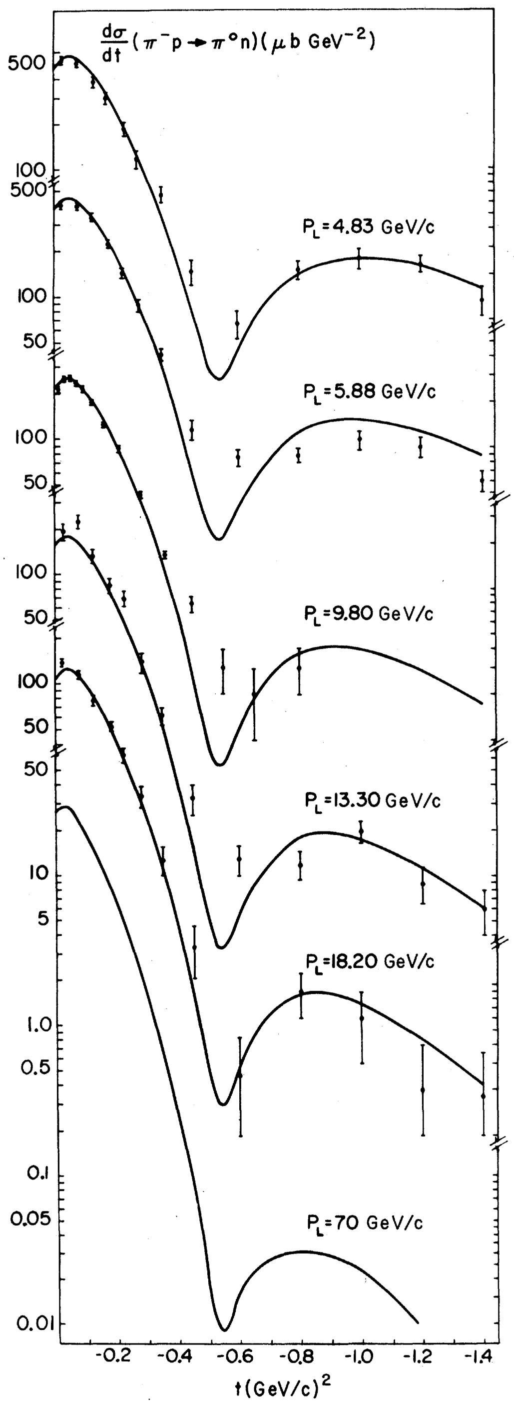

FIG. 1. $\pi^- p \to \pi^0 n$ differential cross sections. The data are taken from Ref. 7.

moves in the j plane according to $j=\alpha_c(t)$. In our model calculations we have set $\alpha_p' = 0.9\ \mathrm{GeV}^{-2} \approx (2m_\rho^2)^{-1}$, as expected phenomenologically. Then $\alpha_0 = 0.51$ is determined by $\mathrm{Re}\,\alpha_+(m_\rho^2)=1$, and $a^2 = 0.22$ is determined by $\mathrm{Im}\,\alpha_+(m_\rho^2) = \alpha_p' m_\rho \Gamma_\rho$ with $\Gamma_\rho = 125$ MeV.

We evaluate the sums in Eqs. (2) and (3) utilizing the Sommerfeld-Watson transformation. The leading contributions at large $-s$, for negative-signature natural-parity exchanges, are given by

$$f^t_{++}(s,t) = i\frac{K(t)}{p}\left\{[m^2\alpha_+(t)\tilde{a}(t)-p^2]\frac{(-is)^{\alpha_+(t)}}{\cos[\frac{1}{2}\pi\alpha_+(t)]\,\Gamma(1+\alpha_+(t))} - \frac{1}{2a(4\mu^2-t)^{1/2}}\left([m^2\alpha_+(t)\tilde{a}(t)-p^2]x_+\,\omega(x_+)\right.\right.$$
$$\left.\left. - [m^2\alpha_-(t)\tilde{a}(t)-p^2]x_-\omega(x_-) - \frac{8ia^2\mu m^2\tilde{a}(t)(4\mu^2-t)^{1/2}}{\sqrt{\pi}}\right)\frac{(-is)^{\alpha_c(t)}}{(\ln s)^{1/2}\cos[\frac{1}{2}\pi\alpha_c(t)]\,\Gamma(1+\alpha_c(t))}\right\}, \tag{6}$$

$$f^t_{+-}(s,t) = \frac{iK(t)}{p}\,\frac{m\tilde{a}(t)\sqrt{\varphi}}{s}\left\{\alpha_+(t)\,\frac{(-is)^{\alpha_+(t)}}{\cos[\frac{1}{2}\pi\alpha_+(t)]\,\Gamma(1+\alpha_+(t))} - \frac{1}{2a(4\mu^2-t)^{1/2}}\left(\alpha_+(t)x_+\,\omega(x_+)\right.\right.$$
$$\left.\left. - \alpha_-(t)x_-\omega(x_-) - \frac{8ia^2\mu(4\mu^2-t)^{1/2}}{\sqrt{\pi}}\right)\frac{(-is)^{\alpha_c(t)}}{(\ln s)^{1/2}\cos[\frac{1}{2}\pi\alpha_c(t)]\,\Gamma(1+\alpha_c(t))}\right\}, \tag{7}$$

where

$$\alpha_\pm(t) = \alpha_p(t) - 8a^2\mu^2 \pm 4a^2\mu(4\mu^2-t)^{1/2}, \quad x_\pm = a(\ln s)^{1/2}[(4\mu^2-t)^{1/2} \mp 2\mu], \quad \omega(x) = e^{-x^2}[1-\mathrm{erf}(-ix)],$$

$\mathrm{erf}(x)$ being the error function.[6] We have taken the scale mass $s_0 = 1\ \mathrm{GeV}^2$. The functions $K(t)$ and $\tilde{a}(t)$ are unknown functions of t, although $K(t=0)$ is determined by the value of the forward differential cross section.

We have used this model to fit the $\pi^- p$ charge-exchange differential cross section[7] and polarization[8] at five energies and over a large region in t in the forward hemisphere. In the fit, we took $K(t) = K(0)e^{b_0 t}$ and $\tilde{a}(t) = \tilde{a}(0)e^{b_1 t}$ as simple parametrizations of these unknown t dependences. Since $K(0)$ is fixed initially by the value of $d\sigma/dt$ at $t=0$ and $P_{\mathrm{Lab}} = 4.83$ GeV/c, we have only three adjustable parameters in the theory. We have not attempted to find *the best* possible fits, but rather some *reasonable* fits to the gross features of the experimental data. The results are shown in Figs. 1 and 2. Quite reasonable fits are obtained for $K(0) = 33.1$, $b_0 = 3.5\ \mathrm{GeV}^{-2}$, $\tilde{a}(0) = -2.65$, $b_1 = -3.70\ \mathrm{GeV}^{-2}$.

The forward dip, the dip at $t \approx -0.6\ \mathrm{GeV}^2$, and the broad secondary maximum are rather well reproduced in our model. The striking dip, at $t = -0.54\ \mathrm{GeV}^2$ in our model, is not associated with the usual nonsense, wrong-signature zero of the helicity-flip amplitude; neither the flip nor the nonflip amplitude has a zero at $\alpha_+(t) = \alpha_\rho(t) = 0$. In our model, the real and imaginary parts of the flip and nonflip amplitudes vanish at different points in the region $+0.4\ \mathrm{GeV}^2 < -t < 0.7\ \mathrm{GeV}^2$; then the dip at $t \approx -0.55\ \mathrm{GeV}^2$ arises from the minimum of the sum of the absolute values squared. We fall consistently below the data throughout the dip region. This may result from the approximations involved in the evaluation of the Sommerfeld-Watson transformation, although the data do not rule out a sharp dip at $t \approx -0.54\ \mathrm{GeV}^2$, rather than a broader dip at $t = -0.6\ \mathrm{GeV}^2$. The strong-cut Reggeized absorption model[5] (SCRAM) also predicts a rather deep dip in this region, although it has some difficulty in fitting the broad secondary maximum at $t \approx -1\ \mathrm{GeV}^2$.

The polarization predictions of the present model differentiate much more strikingly between it and SCRAM (see Fig. 2). In particular, at $P_{\mathrm{Lab}} = 5.9$ GeV/c, SCRAM predicts a large negative polarization ($P \approx -0.75$ at $t = -0.5\ \mathrm{GeV}^2$) while we pre-

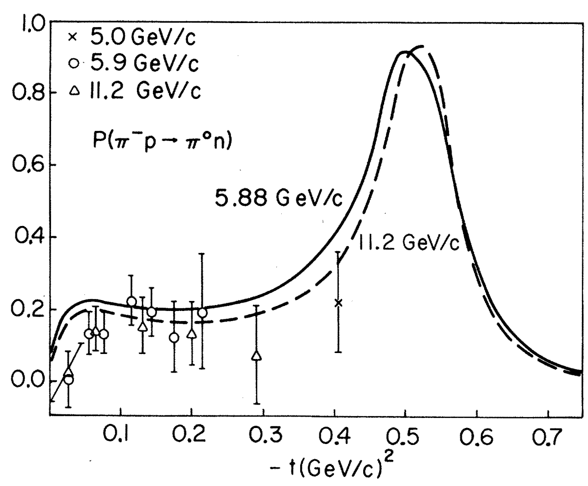

FIG. 2. Charge-exchange polarization. The polarization data are from Ref. 8 (5.9 and 11.2 GeV/c) and Ref. 9 (5.0 GeV/c).

dict $P=+0.90$ at $t=-0.5\ \mathrm{GeV}^2$. A polarization measurement in this region would clearly be decisive. The data[9] at $P_{\mathrm{Lab}}=5.0\ \mathrm{GeV}/c$ indicate a positive polarization at $t=-0.4\ \mathrm{GeV}^2$, but the statistics are somewhat poor.

It should perhaps be emphasized that we have not attempted a χ^2 minimization of all the data, but rather fitted by eye at $P_{\mathrm{Lab}}=4.83\ \mathrm{GeV}/c$. The differential cross sections and polarizations at higher energies are then calculated without further variation of parameters. Somewhat better over-all fits are probably obtainable by χ^2 minimization.

Additional constraints are imposed on the model by $\pi^{\pm}p$ total-cross-section differences as a function of P_{Lab}. Our calculations show a value consistently 30% too large for this difference.[10] This constraint can also be satisfied by introducing additional parameters into the model, e.g., by taking more complex t dependences for the unknown functions $K(t)$ and $\bar{\alpha}(t)$. For example, if one takes $K(t)$ as the difference of two exponentials, one of which is very rapidly decaying in t, one can fit the cross-section difference to within 5% over the range of laboratory energies 10–20 GeV/c. The only change in the fits shown in Figs. 1 and 2 is then a sharpening of the forward dip for $-t<0.05\ \mathrm{GeV}^2$.

In conclusion, we have found quite reasonable fits to the high-energy forward charge-exchange differential cross sections and polarizations with a three-parameter model. Polarization measurements at $t=-0.5\ \mathrm{GeV}^2$ at $P_{\mathrm{Lab}}=5.9\ \mathrm{GeV}/c$ will differentiate between the present model and the strong-cut Reggeized absorption model. As expected, it is difficult to distinguish experimentally the predictions of the present model from those of complex Regge-pole models.[11] We are presently investigating the application of this model to $\pi^{\pm}p$ forward elastic scattering and to pion photoproduction. These results will be presented elsewhere.

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