Veneziano Model for $K^- p \to K^{*-}\pi^+ n$ and $K^- p \to \bar{K}^{*0} n$ Taking Some Account of Spin

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(Received 23 February 1971)

A Veneziano amplitude for $K^- p \to K^{*-}\pi^+ n$, which takes the spin of the external particles into account, is constructed. From this amplitude an amplitude for $K^- p \to \bar{K}^{*0} n$ is obtained. Comparisons with the data for both processes are made.

INTRODUCTION

In an earlier paper[1] we applied a simple generalized Veneziano model to $K^- p \to K^{*-}\pi^+ n$. In this model we followed previous authors in essentially ignoring the spins of the external particles and regarding our amplitude as describing only the spin-average process. In this paper we propose a prescription in which we take into account the spins of the external particles. Although less elegant than the supermultiplet model which we used to describe the process $K^- p \to \pi^-\pi^+\Lambda$ in a recent paper,[2] this prescription provides a simpler method for constructing amplitudes. We show that our five-point amplitude may be consistently reduced to obtain an amplitude for $K^- p \to \bar{K}^{*0} n$, and compare the results for this amplitude with differential cross section data.

I. THE AMPLITUDE FOR $K^- p \to K^{*-}\pi^+ n$

We write the five-point amplitude as

$$A = K \sum_{i=1}^{12} B_5^i\,(x_{12}, x_{23}, x_{34}, x_{45}, x_{51}),$$

where $x_{ij} = J - \alpha_{ij}$, and J is the spin of the lowest resonance on the linear trajectory α_{ij} coupling to external particles i and j. The sum is over the twelve noncyclic permutations of the external particles.

Because there are no trajectories coupling to the $K^- n$, $K^{*-} p$, or $K^-\pi^+$ channels, only two permutations contribute to the amplitude. These are shown in Fig. 1 together with the trajectories we have used for each channel. The choice of trajectories has already been discussed in Ref. 1.

Our amplitude is written as

$$A = K[B_5(1-\alpha_\rho, 1-\alpha_{K^*}, \tfrac{3}{2}-\alpha_\Delta, -\alpha_\pi, \tfrac{3}{2}-\alpha_\Sigma) + B_5(1-\alpha_\rho, \tfrac{3}{2}-\alpha_\Sigma, \tfrac{3}{2}-\alpha_\Delta, \tfrac{3}{2}-\alpha_N, \tfrac{3}{2}-\alpha_\Sigma)].$$

We note that in the second term we have shifted the argument of the N_α in order to obtain the correct asymptotic behavior of the amplitude.

As before, we add an imaginary part to the trajectories above threshold in order to give the resonances a finite width.

Our trajectories are thus

$$\alpha_{K^*}(s) = 0.18 + 0.9s + i0.10(s - s_0), \qquad s_0 = (m_{K^*} + m_\pi)^2,$$

$$\alpha_\rho(t) = 0.48 + 0.9t,$$

$$\alpha_\Sigma(s) = -0.22 + 0.9s + i0.15(s - s_0), \qquad s_0 = (m_p + m_K)^2,$$

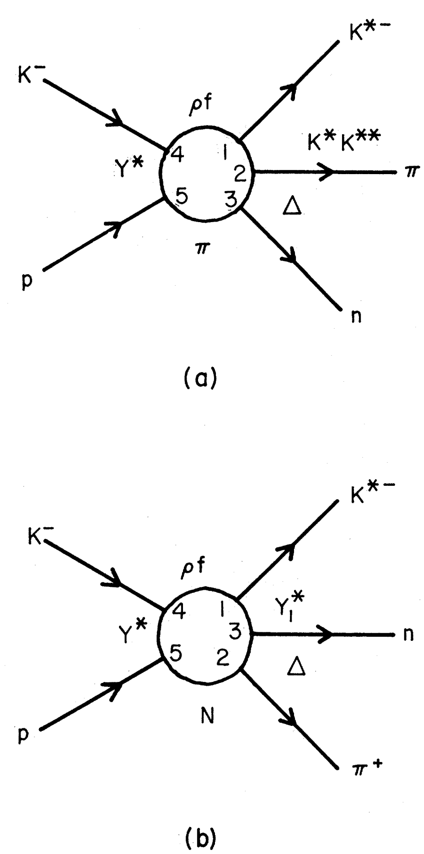

FIG. 1. The two permutations contributing to the amplitude.

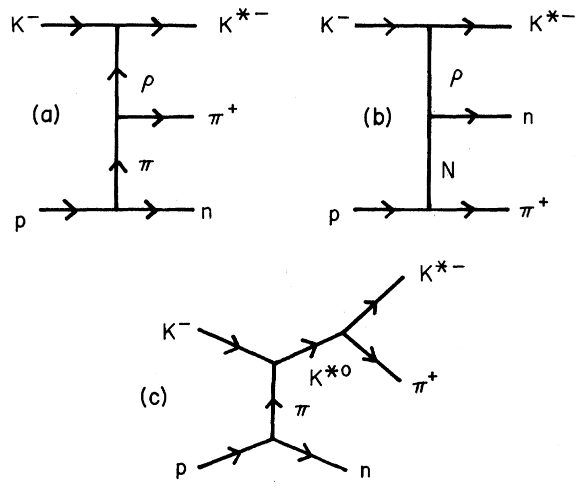

FIG. 2. Peripheral diagrams corresponding to Fig. 1 and the diagram for K^* production.

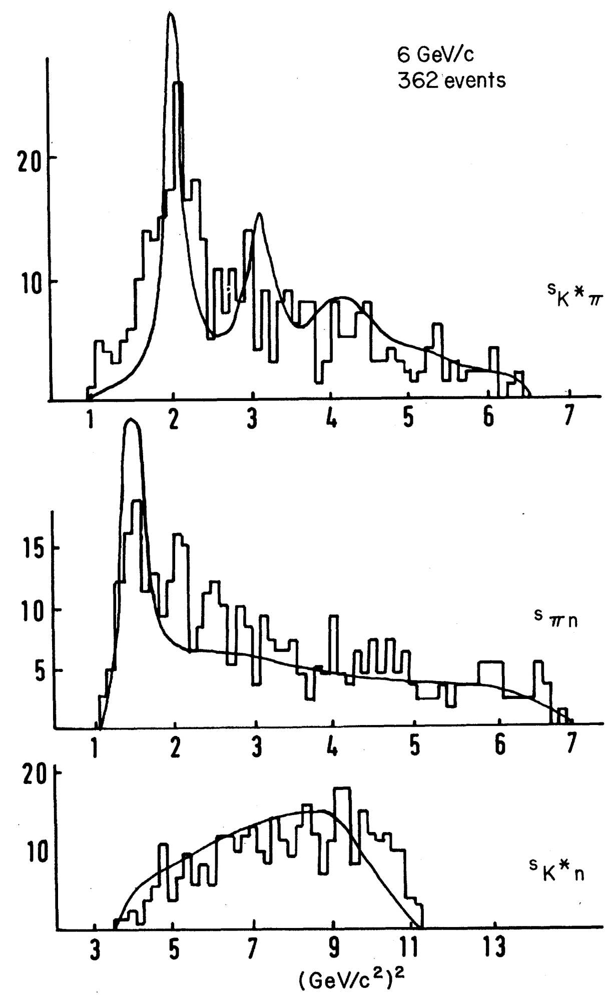

FIG. 3. Mass-squared distributions at 6.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

$$\alpha_N(t) = -0.3 + 0.9t,$$

$$\alpha_\pi(t) = -0.0175 + 0.9s,$$

$$\alpha_\Delta(s) = 0.12 + 0.9s + i0.25(s - s_0),$$

$$s_0 = (m_n + m_\pi)^2.$$

Up to now we have essentially followed Ref. 1. We now come to the problem of evaluating the kinematic factor K in a way which takes account of the spins of the external particles and ensures correct forward asymptotic behavior of the amplitude. Because the process is peripheral, we expect the first resonances on each exchange trajectory to be most important. In Fig. 2 we show the peripheral diagrams corresponding to the blob diagrams of Fig. 1. We expect Fig. 2(a) to be dominant since Fig. 2(b) involves baryon exchange. Hence we will base our choice of K on consideration of Fig. 2(a). This diagram may be evaluated in a single-particle-exchange model using the covariant vertex functions and propagators of Scadron.[3]

The vertices are given by

$$C_{\alpha\beta}(K, K^*, \rho_{\rm ex}) = \tfrac{1}{2} g_1 \epsilon_{\alpha\beta\gamma\delta}(K^- + \rho_{\rm ex})_\gamma (K^- - \rho_{\rm ex})_\delta,$$

$$C_\alpha(\rho_{\rm ex}, \pi, \pi_{\rm ex}) = \tfrac{1}{2} g_2(\pi^+ + \pi^+_{\rm ex})_\alpha,$$

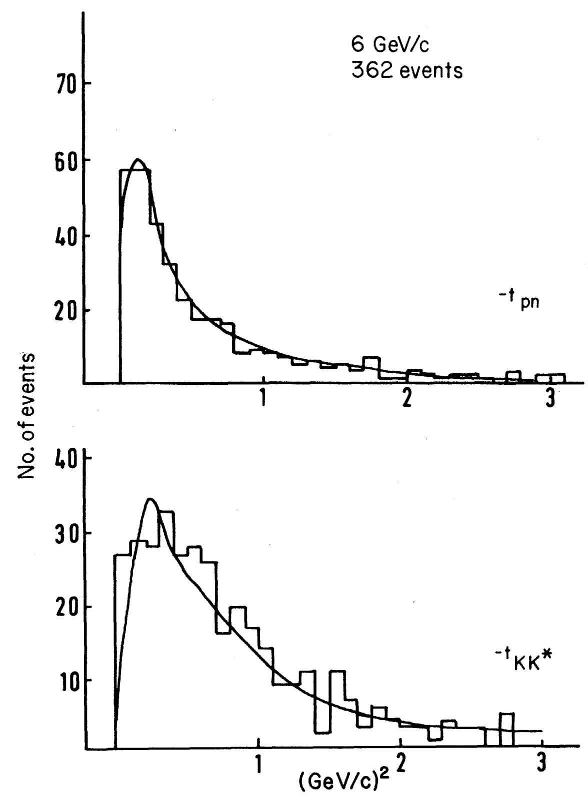

FIG. 4. Momentum-transfer distributions at 6.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

$C(p, \pi^+, n) = g_3\gamma_5$,

where we have denoted the momentum of the K^-, etc., as $(K^-)_\mu$. The subscript "ex" refers to exchanged particles.

The propagators are simply

$$P_{\mu\nu}(\rho_{ex}) = \frac{g_{\mu\nu} - (\rho_{ex})_\mu(\rho_{ex})_\nu/m_\rho^2}{s_{KK^*} - m_\rho^2},$$

$$P(\pi) = \frac{1}{s_{pn} - m_\pi^2}.$$

The pole graph then gives

$$g_3\epsilon_\mu(\lambda)C^{\mu\delta}(K, K^*, \rho_{ex}) \times P_{\delta\alpha}(\rho_{ex})C^\alpha(\rho_{ex}, \pi, \pi_{ex})P(\pi_{ex})\bar{u}\gamma_5 u \propto \frac{\epsilon_\mu(\lambda)\epsilon^{\mu\nu\sigma\tau}K_\nu^- K_\sigma^{*-}\pi_\tau^+\bar{u}\gamma_5 u}{(s_{KK^*} - m_\rho^2)(s_{pn} - m_\pi^2)}. \quad (1)$$

Now consider the first term of our five-point amplitude at the π and ρ poles:

$$A \simeq \frac{K}{-\alpha_\pi(1-\alpha_\rho)} = \frac{K}{\alpha'^2(s_{pn} - m_\pi^2)(s_{KK^*} - m_\rho^2)}. \quad (2)$$

Comparing (1) and (2), we may write

$$K = c\epsilon_\mu(\lambda)\epsilon^{\mu\nu\sigma\tau}K_\nu^- K_\sigma^*\pi_\tau^+\bar{u}\gamma_5 u,$$

where c is used as an over-all normalization parameter.

In the double Regge limit corresponding to Fig. 2(a), $|K|^2$ behaves as $s_{K^*\pi}^2$ and the total amplitude will have correct asymptotic behavior in the forward direction. We also obtain correct asymptotic behavior for all other double Regge limits.

In Figs. 3–6 we compare the results obtained for our five-point amplitude with the data[4] at 6 and 10 GeV/c.

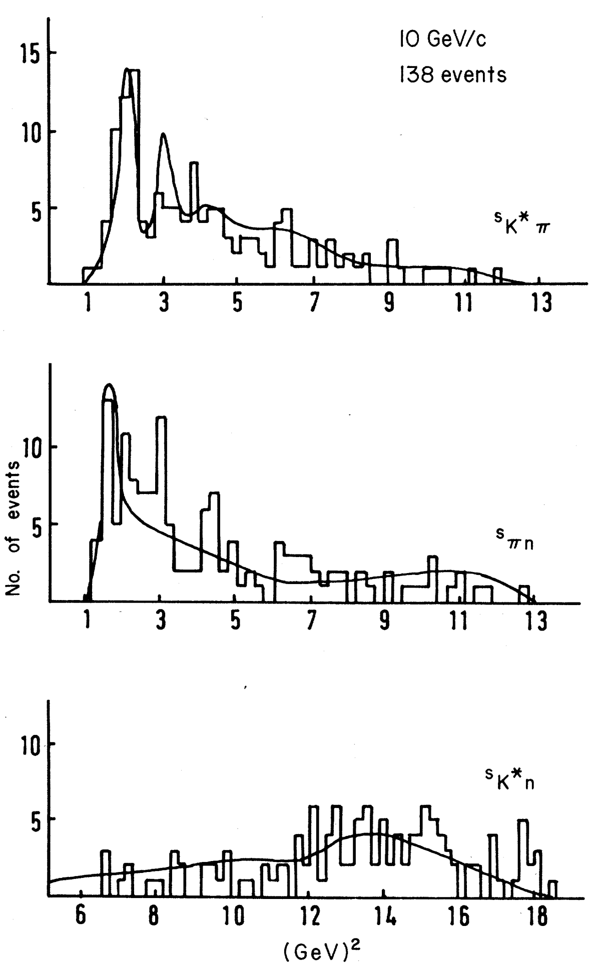

FIG. 5. Mass-squared distributions at 10.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

II. THE FOUR-POINT AMPLITUDE FOR $K^-p \to \bar{K}^{*0}n$

To obtain the amplitude for $K^-p \to \bar{K}^{*0}n$, we consider our five-point amplitude at the K^* pole in the $K^*\pi$ channel. This pole only occurs in the first

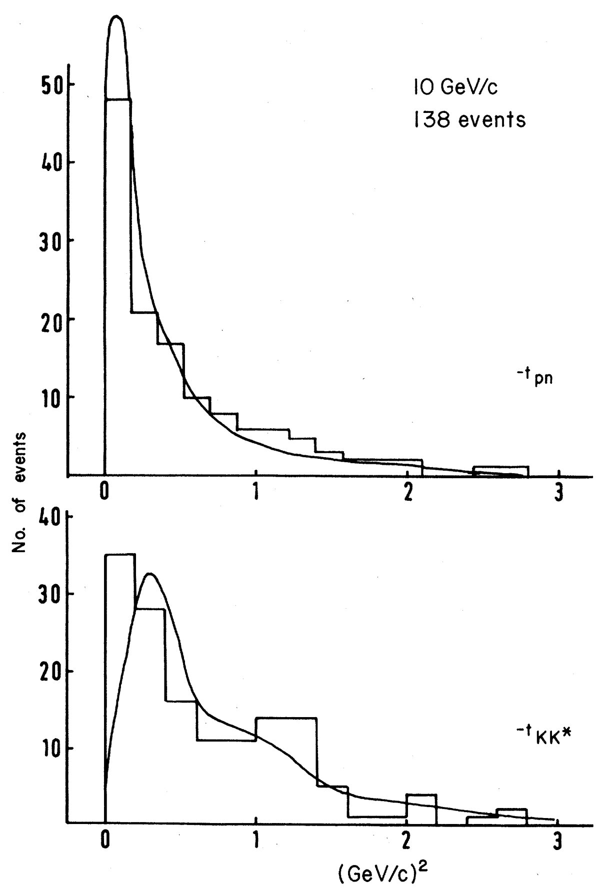

FIG. 6. Momentum-transfer distributions at 10.0 GeV/c for $K^-p \to K^{*-}\pi^+n$.

term of the amplitude. At the pole we obtain

$$A_{\text{pole}} = \frac{K}{1-\alpha_{K^*}} B(\tfrac{3}{2}-\alpha_\Sigma, -\alpha_\pi).$$

The K^* production corresponds to the diagram of Fig. 2(c). To find out how the factor K should factorize at the pole, we evaluate this diagram as before. We obtain

$$\begin{aligned} A' = {} & \tfrac{1}{4} g_1' g_2' g_3 \bar{u}\gamma_5 u \frac{1}{s_{pn}-m_\pi^2}(K^- - \pi_{\text{ex}})_\alpha \\ & \times \frac{g_{\alpha\beta} - (K^*_{\text{ex}})_\alpha (K^*_{\text{ex}})_\beta / m_{K^*}^2}{s_{K^*\pi} - m_{K^*}^2} \\ & \times \epsilon^{\beta\sigma\nu\tau}(K^*+\pi)_\sigma (K^*-\pi)_\nu \epsilon_\tau(\lambda). \end{aligned} \qquad (3)$$

However, this reduces to

$$A' \propto \frac{\bar{u}\gamma_5 u \epsilon^{\alpha\beta\gamma\tau}(K^*)_\alpha (K^-)_\beta (\pi^+)_\gamma \epsilon_\tau(\lambda)}{(s_{pn}-m_\pi^2)(s_{K^*\pi}-m_{K^*}^2)}.$$

The numerator of this expression is just the factor K.

From Eq. (3) we can easily see how to factorize K. We do this simply by replacing the K^* propagator numerator by spin-1 projection operators, i.e.,

$$\begin{aligned} A_{\text{pole}} &= \frac{K}{1-\alpha_{K^*}} B(\tfrac{3}{2}-\alpha_\Sigma, -\alpha_\pi) \\ &= \frac{c}{4\alpha'} \bar{u}\gamma_5 u (K^- - \pi_{\text{ex}})^\alpha \frac{\epsilon_\alpha(\lambda'_{\text{ex}}) \sum_{\lambda'} \epsilon_\beta(\lambda'_{\text{ex}})}{s - m_{K^*}^2} \\ & \quad \times \epsilon^{\beta\sigma\nu\tau}(K^*+\pi)_\sigma (K^*-\pi)_\nu \epsilon_\tau(\lambda) B(\tfrac{3}{2}-\alpha_\Sigma, -\alpha_\pi). \end{aligned}$$

Thus we obtain our four-point amplitude

$$\begin{aligned} A_{Kp\to K^*n} &= \frac{c}{2\alpha' g_{K^*K^*\pi}} \bar{u}\gamma_5 u (K-\pi_{\text{ex}})^\alpha \\ & \quad \times \epsilon_\alpha(\lambda') B(\tfrac{3}{2}-\alpha_\Sigma, -\alpha_\pi) \\ &= c' \bar{u}\gamma_5 u (2K - K^*)^\alpha \epsilon_\alpha(\lambda') B(\tfrac{3}{2}-\alpha_\Sigma, -\alpha_\pi), \end{aligned}$$

where

$$c' = \frac{c}{2\alpha' g_{K^*K^*\pi}}.$$

This amplitude has correct forward asymptotic behavior. The backward asymptotic behavior for the crossed process ($K^*p \to K^+n$) is also correct.

In Fig. 7 we compare the results obtained for this amplitude with cross-section data[5] at 4–10 GeV/c. In doing this we use c' as a parameter, obtaining best fits for $c'^2 = 2.3\times10^3$ GeV^{-5}.

CONCLUSIONS AND RESULTS

The results obtained with the five-point amplitude are a considerable improvement on those obtained in the previous paper. The π^+n mass distribution is especially improved. However, because we have not included a term with a $N(1470)$ trajectory, we have no resonance corresponding to this particle. As a consequence of this, the Δ peak is too high. In the $K^*\pi^+$ channel there is a resonance corresponding to the 3^- recurrence of the $K^*(890)$, which is not seen in the data.

For $K^-p \to \bar{K}^{*0}n$ we have good agreement with the differential cross-section data. The predictions for the density matrices are

$$\rho_{00}(t) = 1, \quad \text{Re}\rho_{10}(t) = \rho_{1\,-1}(t) = 0.$$

These are the values expected for simple pion exchange. However, we would expect the density matrices to vary from these values as there are also exchange contributions from higher-spin particles.

Our four-point amplitude has been obtained by

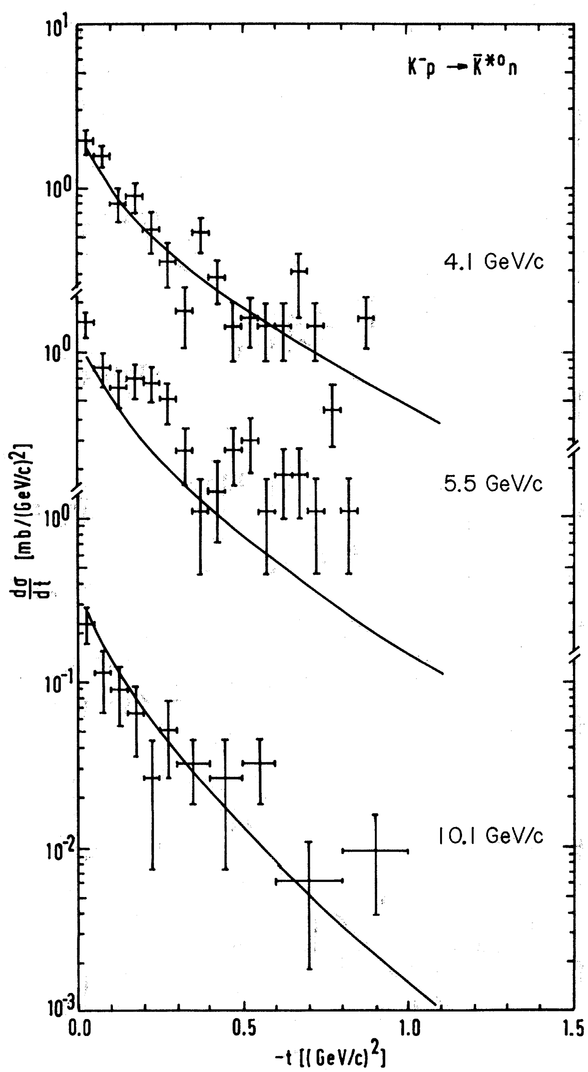

FIG. 7. Differential cross sections for $K^-p \to K^{*0}n$.

reduction of a five-point amplitude which, for computational reasons, is restricted to one Veneziano term for each permutation. As a result, the four-point amplitude only involves one Veneziano term. As we would expect, this is not sufficient to provide a good fit to the density-matrix data. As well as satellite terms, it would also be necessary to include terms with ρ and A_2 exchange and absorptive corrections[6] in a more realistic model for $K^-p \to \bar{K}^{*0}n$. However, these remarks apply equally to all applications of the Veneziano model to three-particle production in which the five-point amplitude is used to obtain amplitudes for various subprocesses. The fault lies mainly in the inadequacy of present-day computers to cope with satellite terms in the five-point model and the poor statistics of the data, which prevent detailed tests of the model.

Accepting these difficulties, we feel that the technique suggested here for the construction of kinematic factors for five-point Veneziano models provides a considerable improvement on the previous rather arbitrary choices.

It is especially pleasing to be able to include pion exchange.[7] The technique also takes account of the spins of the external particles and correct forward asymptotic behavior is always ensured.

It should be noted, however, that our amplitude is constructed by reference to the dominant peripheral diagram for a given process. In general, it will not be the leading term for other processes obtained by crossing the external lines. If we started by considering crossed processes having baryon exchange as the dominant mechanism, we would obtain different kinematic factors.

ACKNOWLEDGMENTS

The authors are indebted to Professor P. T. Matthews for encouragement in this work and for a critical reading of the manuscript. One of us (S.A.A.) wishes to thank the University of Ghana for a postgraduate scholarship, and the others wish to thank the Science Research Council for financial support.

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[6]If we include absorptive corrections [see S. A. Adjei, P. A. Collins, B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Imperial College Report No. ICTP/70/19 (unpublished)], the polarization results are improved. However, since we have not included absorption corrections in the five-point amplitude, we do not feel justified in doing so just for the four-point case.

[7]For another treatment of pion exchange see J. Bartsch *et al.* (Aachen-Berlin-CERN-London-Vienna collaboration), Nucl. Phys. B23, 1 (1970).

PHYSICAL REVIEW D VOLUME 4, NUMBER 1 1 JULY 1971

Regge Cuts in the Van Hove Model: Pion-Nucleon Forward Charge-Exchange Scattering*

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A Regge pole-cut model based on the Durand–Van Hove model is presented for forward pion-nucleon charge exchange. Moving cuts are introduced through the j dependence of the mass of the exchanged spin-j object. The model fits the differential cross sections quite well. The polarization predictions are quite distinct from the Reggeized absorption model, and polarization measurements at $t = -0.5$ GeV^2 will be decisive between these two models.

In this paper we present a Regge pole-cut model for forward processes, in particular for π^-p forward charge exchange. Our model is based on the Durand–Van Hove model,[1] with Regge-cut contributions introduced by the j dependence of the effective mass of the exchanged spin-j object. The model amplitudes exhibit moving Regge cuts whose discontinuities vanish at the branch point, in agree-