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[12] We ignore η-η' mixing. Taking mixing into account, we would have $F_0 g_0 = m_{\eta'}^2 + m_\eta^2 - \frac{4}{3} m_K^2 + \frac{1}{3} m_\pi^2$ in place of Eq. (39).

[13] Symmetry breaking of tri-octet couplings such as Eq. (46b) was first studied by S. Glashow and M. Muraskin, Phys. Rev. 132, 482 (1963).

[14] For convenience, we have defined $\bar{g}_0 = \frac{1}{4}\sqrt{2}\, g_0$, $\bar{g}_8 = 5^{-1/2} g_8$, $Y_8' = 5^{-1/2} Y_1$, $X_{27}' = \frac{1}{40} X_{27}$, $X_{DD}' = \frac{1}{10} X_{DD}$, and $X_1' = \frac{1}{8} X_1$.

[15] Particle Data Group, Phys. Letters 33B, 1 (1970).

[16] For instance, see Refs. 4 and 11.

[17] H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[18] For an example of this, see H. Pagels, Phys. Rev. 179, 1337 (1969).

[19] For instance, see E. Golowich, Phys. Rev. 177, 2295 (1968), and references cited therein.

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Sum Rule for the Σ Term from the Bjorken Limit and Mass Dispersion Relations*

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We present a sum rule for the Σ term derived from the assumptions of Bjorken limit, precocious asymptopia, and finite-mass dispersion relations. We use the commutation relations of the gluon model as input for a numerical evaluation.

I. INTRODUCTION

The successes of current algebra and partial conservation of axial-vector current (PCAC)[1] have led to the introduction of an underlying SU(3)×SU(3) symmetry[2] for the strong interactions. Mainly because the pion mass is so anomalously small, it has been postulated that the SU(2)×SU(2) subgroup is a much better symmetry than the subgroup SU(3). In view of certain difficulties encountered in this "strong PCAC" approach, Brandt and Preparata recently proposed a "weak PCAC" scheme[3] in which SU(3) is a good symmetry, while the pion-pole domination of matrix elements of the axial-vector current divergence (PCAC) is merely a dynamical accident.

Thus arises the question of the relative accuracy of SU(2) and SU(3). In a $(3,3^*)+(3^*,3)$ theory[4] of broken SU(3)×SU(3), such a question is decided by the value of a single parameter c, which ranges from $-\sqrt{2}$ for perfect SU(2)×SU(2) to 0 for perfect SU(3). In this framework, the so-called "Σ term" encountered in the Ward identity for pion-nucleon scattering is proportional to $\sqrt{2}+c$, and should be very small (of the order of 10 MeV) if strong PCAC is indeed correct. The actual value of the Σ term thus provides a sensitive test for distinguishing between the two possibilities.

Earlier determinations[5] of this quantity were inconclusive. Recently Cheng and Dashen[6] performed an accurate evaluation of the Σ term, and obtained the result $|\Sigma| = 110$ MeV. However, their derivation[7] makes use of an expansion in ϵ_2, the SU(2)×SU(2)-breaking parameter, and this approximation might not be good if ϵ_2 is large, as we now believe.[8]

It has been suggested by the SLAC electroproduction experiments[9] that asymptotic behavior sets in quite early (around 2.5 GeV^2) for large values of the photon mass and energy (the phenomenon of precocious asymptopia). This motivates the use of *finite* dispersion relations[10] in treating current correlation functions. Tests of Callan-Gross sum rules[11] in these experiments favor the gluon model of hadrons, and a formalism of Reggeized symmetry breaking[12] has been developed which extracts from the gluon model excellent numerical predictions of experimental results.

In this paper we use these techniques to derive and numerically evaluate a sum rule that relates the Σ term to the asymptotic behavior of the current correlation function, which is given by certain commutators via the Bjorken limit.[13] We abstract the commutators from the gluon model.

The sum rule is derived in Sec. II, numerically evaluated in Sec. III, and discussed in Sec. IV. The Appendix is devoted to some results from the gluon model.

II. DERIVATION OF THE SUM RULE

The so-called Σ term,

$$i\,\delta^4(x)\Sigma_{\alpha\beta}(x) \equiv \delta(x_0)\left[\partial_\mu A^\mu_\alpha(x), A^0_\beta(0)\right], \qquad (1)$$

appears in the Ward identity

$$
\begin{aligned}
i\int d^4x\, e^{iqx}\langle p|T(D_\alpha(x)D_\beta(0))|p\rangle
&=\int d^4x\, e^{iqx}\delta^4(x)\langle p|\Sigma_{\alpha\beta}(x)|p\rangle \\
&\quad +iq_\mu q_\nu\int d^4x\, e^{iqx}\langle p|T(A^\mu_\alpha(x)A^\nu_\beta(0))|p\rangle \\
&\quad +iq_\nu\epsilon_{\alpha\beta\gamma}\int d^4x\, e^{iqx}\delta^4(x)\langle p|V^\nu_\gamma(x)|p\rangle ,
\end{aligned}
\tag{2}
$$

where $A^\mu_\alpha(x)$ is the axial-vector current, $D_\alpha\equiv\partial_\mu A^\mu_\alpha$, $|p\rangle$ is a one-nucleon state of momentum p, and $V^\nu_\gamma(x)$ is the vector current. Denoting the left-hand side by $F(\kappa,\nu)$, where $\kappa=q^2$ and $\nu=p\cdot q$, we shall adopt the usual decomposition $F=A+\not{q}B$, which nucleon-spin averaging transforms into $F=A+(\nu/m_p)B$ (m_p is the nucleon mass). A symmetrization with respect to isospin indices yields

$$
F^{(+)}(0,0)=\Sigma+iq_\mu q_\nu\int d^4x\, e^{iqx}\langle p|T(A^\mu_\alpha(x)A^\nu_\beta(0)^+|p\rangle . \tag{3}
$$

Separating off the Born-term contributions to both sides, we can let $\kappa\to 0$, $\nu\to 0$ with impunity and obtain

$$
\tilde{F}^{(+)}(0,0)=\Sigma+m_p g_A^{\,2}, \tag{4}
$$

when $\tilde{F}$ indicates the truncated amplitude, and g_A is the axial-vector-current coupling constant, experimentally determined to be 1.24.

We intend to evaluate $\tilde{F}^{(+)}(0,0)$ by using finite-mass dispersion relations,[10] substituting for $\tilde{F}^{(+)}(\kappa,0)$, on the contour of finite radius $\Lambda(=2.5\ \mathrm{GeV}^2)$, its Bjorken limit

$$
F^{(+)}(\kappa,\nu)\xrightarrow[q_0\to i\infty]{}\frac{-i}{q_0^{\,2}}c^{(+)}, \tag{5}
$$

where

$$
c^{(+)}=\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\langle p|[\dot{D}_\alpha(0,\vec{x}),D_\beta(0)]^{(+)}|p\rangle . \tag{6}
$$

The substitution is perfectly reasonable provided that the limit (5) sets in precociously, as is actually the case. Proof of this statement can be given by considering the finite-energy dispersion relation

$$
F^{(+)}(\kappa,\nu)=\frac{1}{\pi}\int_{\nu_0}^{\Lambda}d\nu'\frac{\mathrm{abs}_{\nu'}F^{(+)}(\kappa,\nu')}{\nu'-\nu}+\frac{1}{2\pi i}\oint_{c_\Lambda}d\nu'\frac{F^{(+)}(\kappa,\nu')}{\nu'-\nu} \tag{7}
$$

and recalling an old argument by Bjorken,[14] according to which, in the Bjorken limit, the relevant domain of integration is restricted to the scaling region. This makes Eq. (7), in that limit, independent of ν. To reach the desired conclusion it is

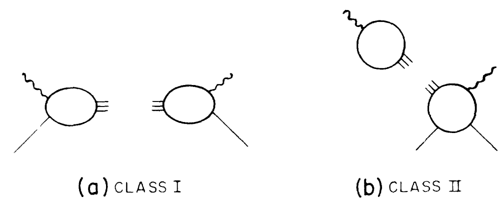

FIG. 1. Classification of singularities in $F^{(+)}(\kappa,0)$, imposed by unitarity requirements, arising from cutting the diagram as in (a) Class I and (b) Class II.

then sufficient to observe that the interpretation of SLAC data on the deep-inelastic region of electron-proton scattering in the light-cone framework[15] strongly suggests universal scaling behavior and universal precocity.

We shall assume, in the following, maximal analyticity in the variable κ, that is, the only singularities will be those imposed by unitarity requirements. These can be classified into two types,[16] as illustrated in Figs. 1(a) and 1(b). Class I contains a nucleon pole at $\kappa=0$, a cut starting at $\kappa_1=\mu^2+2\mu M$, etc., and Class II contains single and double pion poles at $\kappa=\mu^2$, a cut starting $\kappa_2=9\mu^2$, etc. The analytic structure is summarized in Fig. 2.

We can now proceed to introduce the announced finite-mass dispersion relation

$$
F^{(+)}(0,\nu)=\frac{1}{\pi}\int_{\kappa_0}^{\Lambda}d\kappa'\frac{\mathrm{abs}_{\kappa'}F^{(+)}(\kappa',\nu)}{\kappa'}+\frac{1}{2\pi i}\oint_{c_\Lambda}d\kappa'\frac{F^{(+)}(\kappa',\nu)}{\kappa'} \tag{8}
$$

together with the superconvergence condition

$$
0=\frac{1}{\pi}\int_{\kappa_0}^{\Lambda}d\kappa'\,\mathrm{abs}_{\kappa'}F^{(+)}(\kappa',\nu)+\frac{1}{2\pi i}\oint_{c_\Lambda}d\kappa' F^{(+)}(\kappa',\nu). \tag{9}
$$

Exhibiting the structure of $c^{(+)}$, defined in (5), $c^{(+)}=ap_0^{\,2}+b$, we obtain, from Eqs. (8) and (9),

$$
F^{(+)}(0,\nu)\simeq\frac{1}{\pi}\int_{\kappa_0}^{\Lambda}d\kappa'\frac{\mathrm{abs}_{\kappa'}F^{(+)}(\kappa',\nu)}{\kappa'}-\frac{i}{2\pi i}\oint_{c_\Lambda}\frac{d\kappa'}{\kappa'}\left(\frac{b}{\kappa'}+\frac{a\nu^2}{\kappa'^3}\right) \tag{10}
$$

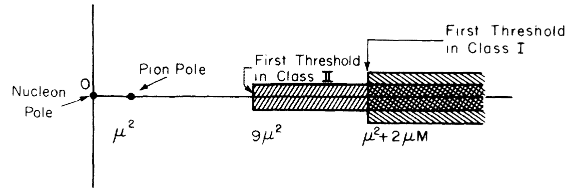

FIG. 2. Analytic singularity structure of $F^{(+)}(\kappa,0)$.

and

$$0 \simeq \frac{1}{\pi}\int_{\kappa_0}^{\Lambda} d\kappa' \,\mathrm{abs}_{\kappa'} F^{(+)}(\kappa',\nu) - \frac{i}{2\pi i}\oint_{c_\Lambda} d\kappa' \left(\frac{b}{\kappa'} + \frac{a\nu^2}{\kappa'^3}\right). \tag{11}$$

Finally, setting $\nu = 0$,

$$i\Sigma \simeq m_p g_A{}^2 + \frac{1}{\pi}\int_{\kappa_0}^{\Lambda} d\kappa' \frac{\mathrm{abs}_{\kappa'} \tilde{F}^{(+)}(\kappa',0)}{\kappa'} \tag{12}$$

and

$$0 \simeq \frac{1}{\pi}\int_{\kappa_0}^{\Lambda} d\kappa' \,\mathrm{abs}_{\kappa'} F^{(+)}(\kappa',0) - ib. \tag{13}$$

Next we explicitly take the low-lying pion poles in $\tilde{F}^{(+)}(\kappa,0)$ into account:

$$F^{(+)}(\kappa,0) = i^2 \frac{\mu^4 T_{\pi N}^{(+)}(\nu=0)}{(2f_\pi)^2(\kappa-\mu^2)^2} + 2i\frac{\mu^2\langle p,\pi_\alpha|\bar{D}_\beta|p\rangle^{(+)}}{2f_\pi(\kappa-\mu^2)} + (\text{terms regular at } \kappa=\mu^2), \tag{14}$$

where f_π is the pion-decay constant, $T_{\pi N}$ is the forward πN scattering amplitude, and a bar denotes subtraction of the pion poles. Since, according to Ref. 3, corrections to pion-pole dominance of matrix elements of D between physical states are universal, we can replace $\langle p,\pi_\alpha|\bar{D}_\beta|p\rangle$ by

$$\langle p,\pi|\bar{D}_\beta|p\rangle^{(+)} \simeq +\delta\frac{i}{2f_\pi} T_{\pi N}^{(+)}(\nu=0), \tag{15}$$

where

$$\delta = 1 - \frac{2m_p g_A f_\pi}{G} \simeq 0.093. \tag{16}$$

By substituting Eqs. (14) and (15), Eqs. (12) and (13) become, respectively,

$$\Sigma = m_p g_A{}^2 - (1-2\delta)\frac{A_{\pi N}^{(+)}(0)}{(2f_\pi)^2} + \frac{1}{\pi}\int_{\text{rest}}^{\Lambda} d\kappa' \frac{\mathrm{abs}_{\kappa'}\tilde{F}^{(+)}(\kappa',0)}{\kappa'} \tag{17}$$

and

$$0 = +2\delta\frac{\mu^2 A_{\pi N}^{(+)}(0)}{(2f_\pi)^2} + \frac{1}{\pi}\int_{\text{rest}}^{\Delta} d\kappa' \,\mathrm{abs}_{\kappa'}\tilde{F}^{(+)}(\kappa',0) - ib. \tag{18}$$

III. NUMERICAL EVALUATION

To evaluate Σ numerically, first we have to use some model to fix the unknown constant b. The gluon model has already been used in a nonperturbative manner to correlate and predict experimental results with great success.[12] The various commutators we shall need are exhibited in the appendix. Needless to say, utilization of these commutation relations by no means commits us to the actual existence of quarks.

The last ingredient of our calculation is a connection between the integrals which occur in Eqs. (17) and (18). We first notice that the analytic structure of the integrand is somewhat different from those which have occurred in the literature.[16] The crucial difference lies in the fact that, in our case, the discontinuities of Classes I and II do not, in general, come in with the same sign. We introduce the notation

$$\frac{1}{\pi}\mathrm{abs}_{\kappa'}\tilde{F}^{(+)}(\kappa',0) = I_1 + I_2, \tag{19}$$

where the subscripts denote the type of the discontinuity, and we propose to consider them separately. Clearly, I_1 has a definite sign, since unitarity relates it to a sum of total cross sections. It is also reasonable to assume that I_1 is dominated by the 3-3 resonance. There is little direct information on I_2, yet we can say definitely that there are no prominent resonances in the current channel up to $\kappa \approx 2.5\ \text{GeV}^2$. It is then clear that I_1 and I_2 can be separately treated by well-known techniques in finite-mass dispersion relations.

Equations (17 and (18) can be written as

$$\Sigma = \Pi + \int^{\Lambda} d\kappa' \frac{1}{\kappa'}(I_1 + I_2), \tag{20}$$

$$0 = \int^{\Lambda} d\kappa' (I_1 + I_2) + (f_1\Sigma + f_2), \tag{21}$$

where

$$\Pi = m_p g_A{}^2 - (1-2\delta)\frac{A_{\pi N}^{(+)}(0)}{(2f_\pi)^2},$$
$$f_1 = \tfrac{1}{3}\epsilon_2{}^2, \quad f_2 = \frac{-1}{3\sqrt{3}}\epsilon_2{}^2 m_p{}^2\tilde{E} + 2\delta\frac{\mu^2 A_{\pi N}^{(+)}(0)}{(2f_\pi)^2}, \tag{22}$$

and use has been made of Eqs. (A4) and (A8).

To relate the integrals in Eqs. (20) and (21), we need the next moment condition:

$$0 = \int^{\Lambda} d\kappa' \,\kappa' (I_1 + I_2) + (\alpha\Sigma + \beta), \tag{23}$$

where α, β are constants which should be computable from the dynamics of the system. Invoking precocious asymptopia, we simply demand that

$$\frac{\alpha}{\Lambda} \ll f_1, \quad \frac{\beta}{\Lambda} \ll f_2. \tag{24}$$

Now we observe that in the short integration range 0 to Λ, it is reasonable to apply the mean-value theorem to I_1 and I_2 *separately*, and write

$$\Sigma = \Pi + \frac{1}{\kappa_1}\bar{I}_1 + \frac{1}{\kappa_2}\bar{I}_2, \tag{25}$$

$$0 = \bar{I}_1 + \bar{I}_2 + (f_1\Sigma + f_2), \tag{26}$$

$$0 = k_1\bar{I}_1 + k_2\bar{I}_2 + (\alpha\Sigma + \beta), \tag{27}$$

where κ_1, κ_2, κ_1, and κ_2 are mean-value masses, and

$$\bar{I}_n = \int d\kappa' \, I_n, \quad n = 1,2. \tag{28}$$

The system of algebraic equations (25)–(27) can be solved easily to yield

$$\Sigma = A/B, \tag{29}$$

where

$$A = \Pi - \frac{1}{k_2 - k_1}\left(\frac{1}{\kappa_1}(k_2 f_2 - \beta) - \frac{1}{\kappa_2}(k_1 f_2 - \beta)\right),$$
$$B = 1 + \frac{1}{k_2 - k_1}\left(\frac{1}{\kappa_1}(k_2 f_1 - \alpha) - \frac{1}{\kappa_2}(k_1 f_1 - \alpha)\right). \tag{30}$$

Since I_1 is dominated by the sharply peaked 3-3 resonance, we assume that

$$\kappa_1 = k_1 = 0.65 \text{ GeV}^2, \tag{31}$$

which corresponds to the position of the resonance. For the Class-II integral, we assume that it is smoothly varying, and it is reasonable to put

$$\kappa_2 = 1.5 \text{ GeV}^2, \quad k_2 = 2.0 \text{ GeV}^2. \tag{32}$$

The value of Σ obtained is not very sensitive to these choices of the mean-value masses. We shall take the approximation of ignoring α, β relative to $k_1 f_1$, $k_2 f_1$; $k_1 f_2$, $k_2 f_2$. As to the other constants, ϵ_2^2 has been determined by Brandt and Preparata[8] to be 0.36 GeV^2, $\tilde{E}$ was given by the same authors as 0.85,[12] and $A_{\pi N}^{(+)}(0)$ has been evaluated by Adler,[17] among others, to be 26.0 in units where $\mu = 1$.

Numerically,

$$\Pi = 0.09 \text{ GeV}, \tag{33}$$

$$\frac{1}{k_2 - k_1}\left(\frac{k_2}{\kappa_1} - \frac{k_1}{\kappa_2}\right) = 2.0 \quad \text{GeV}^{-1}. \tag{34}$$

Then we finally obtain

$$\Sigma = 0.22 \text{ GeV}. \tag{35}$$

Theoretical uncertainty arises mainly from our *ignorance* about the parameters α, β, and the uncertainty in the various mean-value masses. It is thus difficult to arrive at a precise estimate of the errors involved. The question of the possible effects of nonleading singularities certainly deserves further investigation.

IV. COMMENTS

Since our numerical result differs from previous determinations, it is of some interest to elucidate the apparent discrepancy. Our result, Eqs. (29) and (30), can be expanded in a formal power series in ϵ_2:

$$\Sigma = \frac{\Pi + \lambda_1 \epsilon_2^2}{1 + \lambda_2 \epsilon_2^2}$$
$$= \Pi + (\lambda_1 - \lambda_2 \Pi)\epsilon_2^2 + \cdots. \tag{36}$$

The determination of Ref. 6 ignores terms of order ϵ_2^2 and higher. The first term, Π, in our framework agrees numerically with the value of the Σ term obtained. Assuming a reasonable dynamical model for the symmetry breaking, we have simply succeeded in estimating the effect of higher-order terms in the case where ϵ_2^2 is not particularly small.

Another determination of the Σ term has been performed by Brandt and Preparata,[18] who obtained $|\Sigma| \approx 450 \pm 150$ MeV. Our result is not incompatible with theirs.

ACKNOWLEDGMENTS

We are deeply indebted to Richard Brandt and Giuliano Preparata for suggesting the method and for many helpful discussions.

APPENDIX: RESULTS FROM THE GLUON MODEL

The gluon model as used in Ref. 12 is defined by the Hamiltonian density

$$\mathcal{H} = \psi^\dagger(-i\vec{\alpha}\cdot\vec{\nabla} + \beta M + g\beta\gamma_\mu B^\mu)\psi + \mathcal{H}_B, \tag{A1}$$

with

$\psi \equiv$ quark field,

$B^\mu \equiv$ gluon field,

$\mathcal{H}_B \equiv$ free Hamiltonian for the gluon,

$M \equiv \alpha_0 \lambda_0 + \alpha_8 \lambda_8$, the mass term.

We define various currents by

$$A_\alpha^\mu \equiv \bar{\psi}\tfrac{1}{2}(\lambda_\alpha)\gamma_5\gamma^\mu\psi,$$
$$P_\alpha \equiv -i\bar{\psi}\tfrac{1}{2}(\lambda_\alpha)\gamma_5\psi, \quad S_\alpha \equiv \bar{\psi}\tfrac{1}{2}(\lambda_\alpha)\psi, \tag{A2}$$

and a trivial computation yields

$$D_\alpha = -(2/\sqrt{3})\epsilon_2 P_\alpha, \quad \text{where} \quad \epsilon_2 \equiv \sqrt{2}\,\alpha_0 + \alpha_8, \tag{A3}$$

$$\Sigma_{\alpha\beta} = \delta_{\alpha\beta}\tfrac{2}{3}\epsilon_2 \tilde{S}, \tag{A4}$$

$$\delta(x_0)[\dot{D}_\alpha(x), D_\beta(0)]$$
$$= i\delta_{\alpha\beta}\delta^4(x)\tfrac{8}{3}\epsilon_2^2[\tfrac{1}{3}\epsilon_2\tilde{S} + (1/\sqrt{3})\bar{\psi}\tfrac{1}{2}(\tilde{\lambda})\vec{\Delta}\cdot\vec{\gamma}\psi], \tag{A5}$$

where

$$A = \sqrt{2}\,A_0 + A_8, \quad \vec{\Delta} = -i\vec{\nabla} + g\vec{B}. \tag{A6}$$

Using the technique of Ref. 12, the matrix element of (A5) between nucleon states is simplified to

$$c^{(+)} = i\tfrac{4}{3}\epsilon_2{}^2[\tfrac{1}{6}\epsilon_2\tilde{S} - (1/4\sqrt{3})m_p{}^2\tilde{E} + (1/\sqrt{3})p_0{}^2\tilde{E}] \tag{A7}$$

so that b, as defined in the text, is given by

$$b = i\tfrac{4}{3}\epsilon_2{}^2[\tfrac{1}{6}\epsilon_2\tilde{S} - (1/4\sqrt{3})m_p{}^2\tilde{E}]. \tag{A8}$$

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Kaon Production and Multiperipheral Dynamics

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Using experimental data on the pion momentum distribution, together with the dynamics of the multiperipheral model, we show that the asymptotic energy dependence of the total nonstrange cross section and of the total cross section differ only slightly. The difference is determined by the energy dependence of the kaon production rate.

INTRODUCTION

Only a small fraction of the particles produced in high-energy reactions of hadrons have strangeness. Accelerator experiments[1] with p_{lab} between 10 and 70 GeV/c give the ratio of K mesons to π mesons produced as less than 0.1 (essentially only kaons and pions are produced), while cosmic-ray data[2] indicate a somewhat higher value for the ratio of nonpions to pions, perhaps $\frac{1}{4}$. This does not mean, however, that the cross section for strange-particle production is necessarily a small part of the total cross section. Waters *et al.*[3] have inferred from their own and other experimental data the total cross section for strange-particle production in $\pi^- p$ reactions. They find that the fraction of events having strange-particle production is increasing with energy and is approximately $\frac{1}{6}$ at $p_{\text{lab}} = 25$ GeV/c. By subtracting the cross section for strange-particle production from the total cross section, we obtain the "total nonstrange cross section" of interest to the multiperipheral model, since it corresponds more closely to the "total cross section" calculated in a model which assumes no strange-particle production.

It is well known that the multiperipheral model leads to a power law s^α for total cross sections (i.e., Regge behavior). We shall present first a simple ansatz and later a more detailed model to explore the changes in this power law due to the