

Parametrization of Charged-Pion Photoproduction in Terms of Complex Regge Poles*

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A fit of forward charged-pion photoproduction data is attempted by parametrizing Regge pole-cut combinations in the region $t < 0$ in terms of complex conjugate pairs of Regge poles. In particular, the pion-Pomeranchukon cut is taken into account by this parametrization; in addition to the pion, exchange of the A_2 meson is considered. Predictions are made for the differential cross section and the asymmetry parameters for pion production by linearly polarized photons and polarized target nucleons. The results are found to be in good agreement with pion photoproduction data for laboratory photon energies ranging from 3.4 to 18 GeV and t values up to -0.2 (GeV/c)².

I. INTRODUCTION

The considerable number of data which have recently become available on forward charged-pion photoproduction have initiated numerous attempts to explain them in the frame of Regge-pole phenomenology. It has become clear, however, that the original simple model of Ball *et al.*¹ and Henyey² in terms of a pion parity doublet does not explain polarization data despite the reasonable fit it provides for the differential cross section at small momentum transfers. Attempts have, therefore, been made to incorporate, in addition to poles, absorptive effects in the form of Regge cuts. Several models have been developed for calculating the contributions of these cuts, the most popular being the absorption (or Reggeized Gottfried-Jackson) model – and some of the most successful fits for differential cross-section and polarization data are those of Jackson and Quigg,³ the Michigan group,⁴ Blackmon *et al.*,⁵ and Frøylund and Gordon.⁶

The conspiracy problem which one encounters in charged-pion photoproduction has been one of the vexing aspects of high-energy Regge-pole phenomenology. Conspiracy of some type (i.e., pole or cut) is now firmly established as the only way to explain the sharp forward peaks in charged-pion photoproduction and n - p charge-exchange scattering. The explanation, therefore, requires the existence of related terms with the same energy dependence at $t=0$ in both the amplitude containing pion exchange and the amplitude of opposite parity. It is known that a conspirator pole may lead not only to interpretational difficulties (regarding its physical manifestation), but also, as LeBellac⁷ pointed out, to a violation of the factorization theorem for residues. These difficulties are avoided in cut models as, e.g., those of Jackson and Quigg and the Michigan group, by attribut-

ing the role of the conspirator to the Regge cuts, so that the poles themselves may be evasive. This is possible since cut contributions are not associated with definite parity and hence may conspire.

Recently Ball *et al.*⁸ suggested that Regge pole-cut combinations should, in the region of negative momentum transfers and until extremely high energies are reached, be parametrized simply by a complex conjugate pair of Regge poles. Support for the possible complexity of Regge trajectories has recently been found by Ball and Marchesini⁹ and Chew and Snider¹⁰ in various approaches to the multiperipheral model. Ball *et al.*⁸ show that at moderately high energies, i.e., those presently available, the effects of cuts are well represented by pairs of complex conjugate poles in much the same way as branch cuts in energy may be parametrized by a Breit-Wigner formula in terms of nearby second-sheet poles. Using this formalism Desai *et al.*¹¹ obtained excellent fits to the πN charge-exchange differential cross-section and polarization data.

The aim of this note is to present a fit employing complex conjugate pairs of Regge poles for the data of charged-pion photoproduction. We recall that the pion-Pomeranchukon cut contributes to both natural- and unnatural-parity amplitudes which at $t=0$ are related by the so-called conspiracy relation. The work of Ball *et al.*⁸ shows that the unnatural-parity cut contribution and the pion pole which we will assume to be evasive can be represented by a complex pair of Regge poles. This will lead naturally to a zero in the residue function for this combination as the original pion residue is proportional to t , while the cut has a nonvanishing portion for $t=0$. Since the conspiracy relation *must* hold for the pion-Pomeranchukon cut, the natural-parity part of the cut must also be represented by a pair of complex poles at $t=0$,

and we will assume such a representation to be reasonable in the small- t region. The trajectory and residues of these natural-parity Regge poles are, of course, independent of the pion trajectory except at $t=0$. In this picture the role of the conspirator is thus played by the natural-parity poles which are not on the physical sheet of the J plane, and hence need not have any manifestation as a resonance in the region of positive t .

II. MODEL AMPLITUDES

We use s - and t -channel amplitudes as given by Jackson and Quigg.³ The s -channel helicity amplitudes are $g_{0\lambda';\lambda\gamma\lambda} = g_j(s, t)$, where $j=1, 2, 3, 4$ stand for the photon, initial nucleon, and final nucleon helicities $(\lambda_\gamma, \lambda_N, \lambda_{N'}) = (-1, -\frac{1}{2}, \frac{1}{2}), (1, -\frac{1}{2}, \frac{1}{2}), (1, \frac{1}{2}, \frac{1}{2}), (1, -\frac{1}{2}, -\frac{1}{2})$, respectively. At high energies and small momentum transfers these amplitudes are related to the parity-conserving t -channel helicity amplitudes F_i by

$$g = (\nu/\sqrt{2})XF, \quad (1)$$

where $\nu = (s - m^2)/2m$ is the photon energy in the laboratory, m the nucleon mass, and X the crossing matrix given by

$$X = \begin{pmatrix} t/2m & -2m/(\mu^2 - t) & 1 & 0 \\ (-t)^{1/2} & 0 & (-t)^{1/2}/2m & 2m(-t)^{1/2} \\ (-t)^{1/2} & 0 & (-t)^{1/2}/2m & -2m(-t)^{1/2} \\ t/2m & 2m/(\mu^2 - t) & 1 & 0 \end{pmatrix} + O\left(\frac{t}{4m^2}\right). \quad (2)$$

(Note that the F_i as defined by Jackson and Quigg³ differ by some factors from those of Ball and Jacob.¹²) The F_i , of course, have an index $(\pm, 0)$ corresponding to the exchange of particles of well-defined isotopic spin in the t channel.¹³

In constructing our amplitudes we use the same t -channel exchanges as Jackson and Quigg³ — i.e., π and A_2 , so that $F_4=0$. We make this choice because a study of the over-all effective trajectory indicates that the A_2 is relatively important over a large range of t . Moreover, the classical Born approximation indicates dominance of π exchange in the region of small t . In the region of $(-t)^{1/2} > 0.6$ GeV/ c , other exchanges become important — particularly ρ exchange (in the isoscalar-photon amplitudes) as is indicated by the dropoff of the ratio $d\sigma(\pi^-)/d\sigma(\pi^+)$ in this region. Here, however, we neglect ρ exchange; we give our justification below. With due regard to the kinematical factors characteristic of the evasive pion,¹² we may write in standard Reggeized form (incorporating the electric Born approximation)

$$F_2^{(-)} = -\frac{eg\pi t}{2m\nu} \frac{1 + e^{-i\pi\alpha_\pi(t)}}{[\sin\pi\alpha_\pi(t)]\Gamma(\alpha_\pi(t))} \left(\frac{\nu}{\nu_0}\right)^{\alpha_\pi(t)}, \quad (3)$$

where e, g have their usual meaning as coupling constants at the γ, π vertices, respectively ($e^2 = 1/137, g^2/4\pi = 14$). This expression is valid in the $t \approx \mu^2$ region. We now assume¹⁴ that near $t=0$ the collision of the dominant, i.e., pion pole with the pion-Pomeranchukon cut can be represented by a pair of complex conjugate poles (on the physical sheet of the J plane). In the region $t < 0$ the amplitude may therefore be rewritten

$$F_2^{(-)} = (t + \lambda) \frac{2\mu^2\beta(t)}{\nu \sin\pi\alpha_\pi(t)} \left[\alpha_+(t) \left(\frac{\nu}{\nu_0}\right)^{\alpha_+(t)} e^{-i\pi\alpha_+(t)/2} + \alpha_-(t) \left(\frac{\nu}{\nu_0}\right)^{\alpha_-(t)} e^{-i\pi\alpha_-(t)/2} \right] \quad (4)$$

with $\alpha_\pm = \alpha_\pi \pm i\alpha_I$. Here a Γ function and part of the signature factor have been absorbed in the function $\beta(t)$, and λ is a constant proportional to α_I (more generally, some singularity-free t dependence of λ would be allowed, but the data require λ to remain nonzero at $t=0$). We assume that the real part of the complex trajectories behaves much the same as in the $t > 0$ region. Clearly the part proportional to t represents the evasive pion pole-cut contribution in the region $t < 0$, and the part proportional to λ is the conspiring portion of the unnatural-parity pion-Pomeranchukon cut. In the following we leave the A_2 contribution unchanged, the assumption being that the A_2 meson leads to nondominant cut effects, so that the imaginary part of its trajectory may be neglected in the $t < 0$ region. The conspiracy relation

$$F_2^{(-)}(0)/F_3^{(-)}(0) = -\mu^2/2m$$

must now be satisfied by the natural- and unnatural-parity parts of the pion-Pomeranchukon cut. Hence the natural-parity part belonging to $F_3^{(-)}$ must also be represented by a complex conjugate pair of Regge poles at $t=0$. We assume that such a representation is reasonable at least in the region of small $|t|$. The trajectories and residues of these poles, of course, are related to the pion trajectory and residue only at $t=0$. These poles, therefore, play the role of a "conspirator" but they do not lie on the physical sheet of the J plane. Then adding the relevant A_2 contribution, we may write

$$F_3^{(-)} = -\lambda \frac{4m}{\nu} \bar{\beta}(t) \left[\alpha_+'(t) \left(\frac{\nu}{\nu_0} \right)^{\alpha_+'(t)} e^{-i\pi\alpha_+'(t)} + \alpha_-'(t) \left(\frac{\nu}{\nu_0} \right)^{\alpha_-'(t)} e^{-i\pi\alpha_-'(t)} \right] \\ - \frac{eg t \xi_2}{4m^2 \nu} \left(1 - \frac{t}{4m^2} \right) \Gamma(1 - \alpha_A(t)) (1 + e^{-i\pi\alpha_A(t)}) \left(\frac{\nu}{\nu_0} \right)^{\alpha_A(t)}, \quad (5)$$

where ξ_2 is the corresponding singularity-free residue in $F_3^{(-)}$, and $\bar{\beta}(t)$ is a singularity-free function of t such that

$$\bar{\beta}(0) = \frac{\beta(0)}{\sin\pi\alpha_\pi(0)}.$$

For convenience we choose $\alpha_\pm(t) = \alpha_\pm'(t)$. The amplitude $F_1^{(-)}$, of course, contains only the A_2 contribution and is given by

$$F_1^{(-)} = -\frac{1}{2} \frac{eg}{m\nu} \xi_1 \Gamma(1 - \alpha_A(t)) (1 + e^{-i\pi\alpha_A(t)}) \left(\frac{\nu}{\nu_0} \right)^{\alpha_A(t)}, \quad (6)$$

where ξ_1 is the corresponding singularity-free residue.

We summarize further essential formulas. The cross section for unpolarized photons is given by

$$\frac{d\sigma}{dt} = \frac{1}{128\pi m^2} \left[|F_1|^2(-t) \left(1 - \frac{t}{4m^2} \right) + \frac{4m^2}{(\mu^2 - t)^2} |F_2|^2 + \left(1 - \frac{t}{4m^2} \right) |F_3|^2 \right] \text{ for } F_4 = 0. \quad (7)$$

For photons polarized linearly in the production plane, the “polarized-photon asymmetry parameter” Σ is given by

$$\Sigma = \frac{|F_1|^2(-t)(1 - t/4m^2) - [4m^2/(\mu^2 - t)^2] |F_2|^2 + (1 + t/4m^2) |F_3|^2}{|F_1|^2(-t)(1 - t/4m^2) + [4m^2/(\mu^2 - t)^2] |F_2|^2 + (1 - t/4m^2) |F_3|^2} \text{ for } F_4 = 0. \quad (8)$$

A different combination of amplitudes from that measured in the polarized-photon experiments is measured by photoproduction from a polarized target (see, e.g., Jackson and Quigg³). The production cross section of pions from polarized protons can be expressed as

$$\frac{d\sigma}{dt} \Big|_{\text{polarized}} = \frac{d\sigma}{dt} \Big|_{\text{unpolarized}} [1 + A(t) \vec{\mathbf{P}}_T \cdot \hat{\mathbf{n}}],$$

where $\vec{\mathbf{P}}_T$ is the target polarization vector and $\hat{\mathbf{n}}$ the normal to the production plane (positive in the direction of $\vec{\mathbf{k}} \times \vec{\mathbf{q}}$). The parameter $A(t)$ is then a measure of the asymmetry in the produced pions between two states of proton polarization; it is therefore called the “left-right asymmetry parameter” by Jackson and Quigg,³ and is given by

$$A(t) = \frac{2(-t)^{1/2}(1 - t/4m^2) \text{Im}(F_3 F_1^*)}{|F_1|^2(-t)(1 - t/4m^2) + [4m^2/(\mu^2 - t)^2] |F_2|^2 + (1 - t/4m^2) |F_3|^2} \text{ for } F_4 = 0. \quad (9)$$

III. FIT TO DATA

The essential aspects of the model have been formulated in Sec. II. We now set for computational convenience

TABLE I. Results of two least-square fits.

Trajectories and parameters (for $s_0 = 1 \text{ GeV}^2$)	$\alpha_\pi(t)$	$\alpha_A(t)$	A (GeV^{-2})	B (GeV^{-1})	C (GeV^{-2})	D (GeV^{-2})	α_I	λ (GeV^2)	Total χ^2 (for 47 data points)	eg (exact 1.2)
First fit	$-0.025 + 1.25t$	$0.56 + t$	15.3	50.8	-12.2	-1.6	1.04	0.08	62	0.6
Second fit	$-0.02 + t$	$0.25 + t$	41.3	51.1	-11.6	-0.96	1.03	0.08	60	0.5

$$F_1^{(-)} = -A\Gamma(1 - \alpha_A(t))(1 + e^{-i\pi\alpha_A(t)})\left(\frac{s}{s_0}\right)^{\alpha_A(t)-1}, \quad (10)$$

$$A = eg\xi_1/(s_0 - m^2),$$

and

$$F_2^{(-)} = \frac{\mu^4}{2m\lambda} B e^{Ct} \frac{t+\lambda}{t-\mu^2} \left[\alpha_+(t) \left(\frac{s}{s_0}\right)^{\alpha_+(t)-1} e^{-i\pi\alpha_+(t)/2} + \alpha_-(t) \left(\frac{s}{s_0}\right)^{\alpha_-(t)-1} e^{-i\pi\alpha_-(t)/2} \right]. \quad (11)$$

The constant B may be related to the coupling constants e and g by extrapolation of $F_2^{(-)}$ to the well-known electric Born approximation at the pion pole, i.e., at $t = \mu^2$. Then

$$F_2^{(-)}(\mu^2) = 2eg\mu^2/(s - m^2),$$

so that

$$eg \simeq \frac{\mu^4}{2m\lambda} B e^{C\mu^2} s_0 \alpha'_\pi \left(1 + \frac{\lambda}{\mu^2}\right). \quad (12)$$

In $F_3^{(-)}$ we neglect the A_2 contribution, partly because of the factor t and partly because calculation shows this term to be negligible compared with the pion-Pomeranchukon-cut contribution. Since the latter does not possess the pion pole, we parametrize it as

$$F_3^{(-)} = B e^{Dt} \left[\alpha_+(t) \left(\frac{s}{s_0}\right)^{\alpha_+(t)-1} e^{-i\pi\alpha_+(t)/2} + \alpha_-(t) \left(\frac{s}{s_0}\right)^{\alpha_-(t)-1} e^{-i\pi\alpha_-(t)/2} \right]. \quad (13)$$

The constant B , of course, was chosen so that F_2, F_3 satisfy the conspiracy constraint.

In Table I and Figs. 1 to 3 we summarize the results of the least-squares fits. The total χ^2 per data point is approximately 1.2, the number of parameters being six. For the differential cross section we took the data points of SLAC (Boyarski *et al.*¹⁵) for photon energies from 5 to 18 GeV, and for the asymmetry parameter the data points of DESY (Geweniger *et al.*¹⁶) for photon energies of 3.4 GeV. The polarization data of the recoil nucleon are those of the Berkeley-SLAC-Argonne collaboration¹⁷ for photon energies of 5 and 16 GeV.

We observe that in the region $|t| < \mu^2$, the fits reproduce the Born approximation fairly well – as expected, since the success of the electric Born approximation in this region is well known and has been discussed by many authors (e.g., Harari¹⁸). It is also well known that the Born approximation of the differential cross section exhibits a strong upward trend beyond $t = \mu^2$ in violent disagreement with observation. The width of the forward peak and the effective Regge trajectory α_{eff} which is

known¹⁹ to lie somewhere between 0.2 and -0.2 are clear evidence for pion dominance in this region. The A_2 meson plays only a secondary role. This is borne out by our calculations which showed that the cross section and polarized-photon asymmetry vary little if the intercept of the A_2 trajectory is varied from 0.56 to 0.25, whereas the polarized-nucleon asymmetry shifted slightly within the experimental error bars (for this reason we give

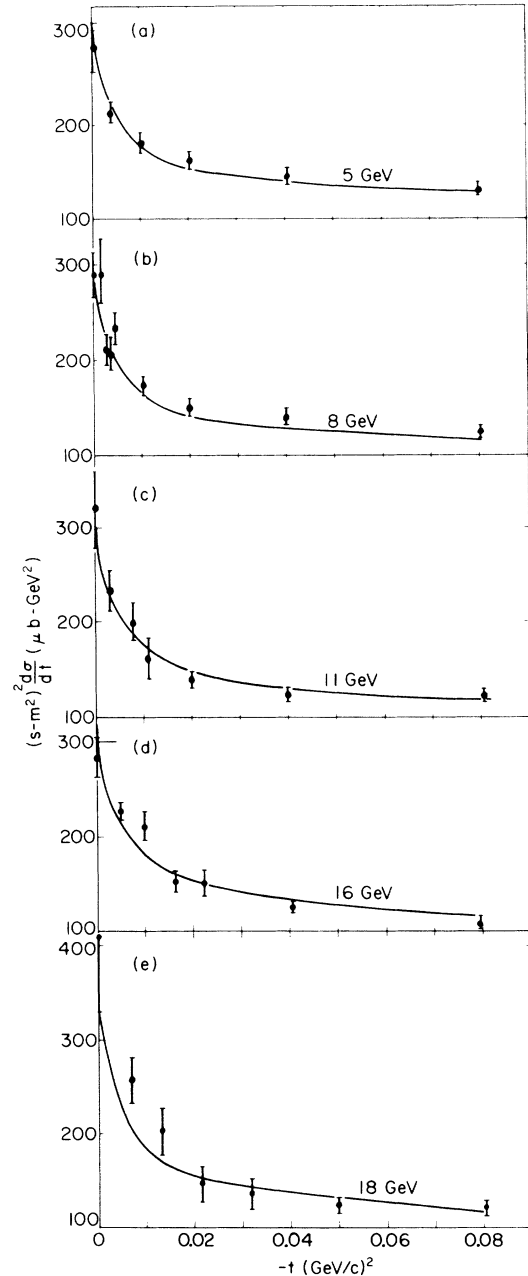


FIG. 1. Predictions for the differential cross section in $\gamma p \rightarrow \pi^+ n$. Data are from Ref. 15 (SLAC).

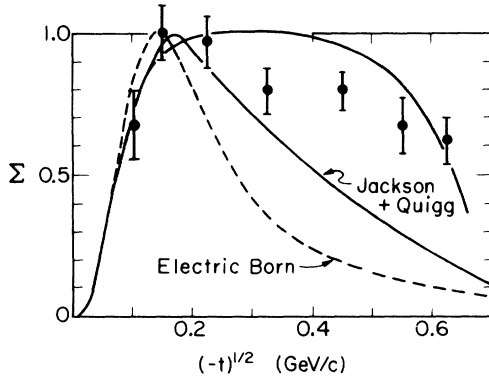


FIG. 2. Model (solid line) and gauge-invariant perturbation-theory (broken line) predictions for the polarized-photon asymmetry parameter Σ . Data are from Ref. 16 (DESY).

only the curves of the "second fit"). The results were found to be insensitive to multiplication of A by a factor e^{Bt} . With increasing $|t|$ the effects of A_2 and ρ exchange become more pronounced because of their higher intercepts. A clear indication of the growing importance of ρ exchange in the $|t| > \mu^2$ region is given by the ratio of π^- to π^+ production cross sections,^{20,21} the corresponding reactions being related by s - u crossing (line reversal) at fixed t .²² This difference between the cross sections away from the forward region (where it is ~ 1) is due to the isoscalar amplitude $F_4^{(0)}$ which we have neglected altogether. If the photon had a well-defined isotopic spin, the ratio would always be one. In view of its high trajectory the ρ meson is expected to yield the dominant contributions to the isoscalar amplitudes and, in fact, this is verified by the π - ρ cut model of Frøylund and Gordon.⁶ An interesting, though probably model-dependent estimate of the relative magnitudes of the dominant isoscalar, isovector amplitudes is quoted by Blackmon *et al.*⁵ They estimate $|T^0|^2/|T^{(-)}|^2 \leq 0.3$ for $(-t)^{1/2} \leq 0.6$ GeV/c and $E_\gamma \leq 16$ GeV with further increase of importance of the ρ meson at still higher energies. The predictions for the polarized-photon asymmetry are in reasonable qualitative agreement with experimental data. The agreement in the region of small $|t|$ is, of course, ensured by the zero in the $F_2^{(-)}$ amplitude as provided by the pion-Pomeranchukon cut. In particular, the results exhibit a flattening of the asymmetry beyond $(-t)^{1/2} = 0.2$ GeV/c and thus indicate a trend which is different from that predicted by either the electric Born term or the pseudomodel of Jackson and Quigg.³ An analogous over-all agreement is found for the left-right asymmetry parameter. In either case one would not expect quantitative agreement beyond $-t \approx 0.4$ (GeV/c)² in view of the growing

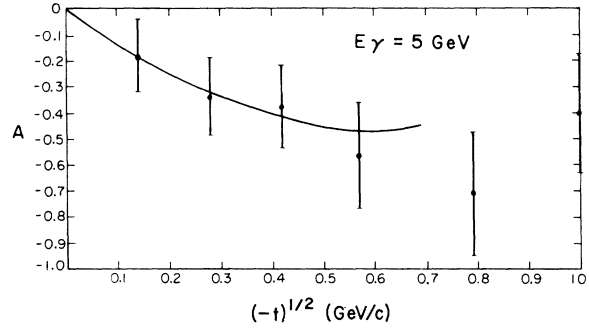


FIG. 3. Left-right asymmetry A from a polarized target predicted by the model. Data are from Ref. 17. Model agreement with 16-GeV data is similar to that for 5 GeV.

importance of isoscalar contributions.

One may ask how the calculations of our model would have to be altered if measurements of the π^-/π^+ ratio were to be included in the data which we fit. The calculations of Jackson and Quigg³ on the one hand and those of Frøylund and Gordon⁶ on the other show that the ρ and A_2 interfere very differently with the pion, but, as pointed out above, contributions of both ρ and A_2 are essential for a fit of all data presently available (i.e., including the π^-/π^+ ratio). The A_2 and ρ contribute, respectively, to isovector and isoscalar parts of both F_1 and F_3 . The ρ -pole contributions contain an additional factor t in the numerator⁶ (compared with the A_2 contributions) and so have little effect in changing our fits of Σ and A in the region of small $|t|$. The ρ is vital, however, for the fit of the π^-/π^+ ratio. A reasonable fit of this ratio as well as the other three quantities may, therefore, be expected by including A_2 -pole as well as ρ -pole contributions in both F_2 and F_3 , necessitating thereby the introduction of three more parameters. Since the π -Pomeranchukon cut is well known to be essential to reproduce the sharp forward peak of the cross section, other cut contributions such as those of ρ -Pomeranchukon double exchange do not seem to be essential to fit data presently available. Additional contributions necessary to fit the π^-/π^+ ratio are, therefore, not characteristic of our model and would only obscure its power through a larger number of parameters. We, therefore, present fits which ignore ρ contributions in analogy to the model of Jackson and Quigg.³

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Aspects of the Scalar-Dominance Hypothesis*

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The scalar-dominance hypothesis, incorporating $\epsilon(700)$ - $\epsilon'(1060)$ mixing, is applied to a large class of elastic, single-particle matrix elements of the energy-momentum trace operator.

I. INTRODUCTION

Among the diverse applications of scaling to particle physics is the concept of a scale-invariant world. This symmetry limit corresponds to the vanishing of all matrix elements of the energy-momentum trace operator $\theta \equiv \theta_\mu{}^\mu$. A recent study¹ of renormalizable field theories has displayed the existence of a scale current D^μ defined in terms of the symmetric energy-momentum tensor $\theta^{\mu\nu}$,

$$D^\mu(x) = x_\nu \theta^{\mu\nu}(x), \quad (1)$$

with divergence

$$\partial_\mu D^\mu(x) = \theta(x) \quad (2)$$

and charge

$$D(x^0) = \int x_\nu \theta^{0\nu}(x) d^3x. \quad (3)$$

Thus, scale invariance can be described in terms of the vanishing of a current divergence. The idea of associating a symmetry with θ is clearly of interest. Taken together with studies of chiral invariance, it constitutes an important probe of the