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Quark-Model Sum Rules for Hyperon-Nucleon Total Cross Sections

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Predictions of hyperon-nucleon total cross sections are made based upon quark-model additivity sum rules. Using various kinematic conditions for performing the quark decomposition, large qualitative differences result in the $\Sigma^- p$ total cross section predicted at high energies. Data taken in proposed hyperon-beam experiments should be able to differentiate between predictions of the various kinematic schemes. Also, Serpukhov data are used to confront old quark-model sum rules for conventional beams; results are consistent with additivity sum rules taken with the various hadron systems compared at fixed relative velocity.

The quark model of hadron structure has had considerable success in predicting the qualitative features of strong interactions.[1] Of particular interest have been the sum rules relating total cross sections in meson-baryon and baryon-baryon scattering.[2] These relations, which follow from the assumption that hadronic forward elastic scattering amplitudes may be decomposed into a sum of quark-quark amplitudes, have received rigorous testing thus far only for reactions involving scattering of $\pi^\pm$, $K^\pm$, p, and $\bar{p}$ on nucleons. In this paper we reexamine the validity of these sum rules using the recent Serpukhov data. Furthermore, we derive sum rules for the total cross sections of $\Sigma^\pm$ and Ξ^- on nucleons in anticipation of proposed experiments using hyperon beams at the National Accelerator Laboratory and other accelerators.[3]

In each case, various kinematical schemes which have been suggested for use in evaluation of the sum rules are considered. The predictions are found to differ to such an extent that even quite crude data on $\Sigma^- N$ or $\Xi^- N$ will be sufficient to rule out some approaches. The hyperon sum rules are especially "clean" in that they follow from quark additivity alone; no assumptions regarding asymptotics or symmetries among the quark-quark amplitudes are required.

Following Kokkedee and Van Hove,[4] we adopt as our statement of additivity of scattering amplitudes

$$A_{AB}(s_{AB}, t) = \sum_{i,j} f_i^A(t) f_j^B(t) A_{ij}(s_{ij}, t), \qquad (1)$$

where A and B refer to the hadrons and the indices i and j run over all quarks and antiquarks composing A and B, respectively; $f_i^A(t)$ is a form factor with the property $f_i^A(0) = 1$ (0) if quark i is (is not) contained in hadron A. The optical theorem then relates $\sigma_{AB}(s_{AB})$ to a sum of the $\sigma_{ij}(s_{ij})$, where i (j) label the constituent quarks of A (B). These relations imply equalities between appropriate combinations of the σ_{AB}. The quark additivity sum rules thus follow from (1) evaluated at $t=0$; this means that details of the quark wave functions are irrelevant (we deal only with spin-averaged quantities). Such sum rules are expected to be valid provided one may neglect double scattering and that one works at energies sufficiently high that the Fermi motions and bindings of the quarks may be neglected. In particular, one should work above the resonance region which extends to a laboratory momentum $P_L \approx 4$ GeV/c in the πN system.

The kinematics under which one should compare the various hadron cross sections appearing in the sum rules has been a subject of controversy. The dynamics of the quark-quark interactions presumably depends upon the kinematic variables in each quark-quark system. This has been input for two of the three common kinematic prescriptions (II and III):

I. The σ_{AB} should be compared at the same laboratory momentum.

II. Meson-baryon cross sections at P_L should be compared to baryon-baryon cross sections at $\frac{3}{2}P_L$,[4] based on the idea that the quarks share the hadron momentum equally.

III. The σ_{AB} should be compared at the same relative velocity (velocity of A in B's rest frame).[5] In this scheme the hadron lab momenta are scaled by the beam masses, $P_{L1}/m_1 = P_{L2}/m_2 \cdots$.[6] If all σ_{AB} eventually approach constants, I, II, and III will give identical asymptotic results.

We first review some of the sum rules previously derived and compare the predictions of prescriptions I–III in the region of the Serpukhov data.

TABLE I. The left- (L) and right-hand (R) sides of the sum rule of Eq. (2) are given for various values of incident nucleon momentum for kinematic prescriptions I–III as described in the text. Cross sections are in mb.

	I		II		III	
P_L (GeV/c)	L	R	L	R	L	R
30	85.9 ± 2.1	73.5 ± 1.3	85.9 ± 2.1	74.1 ± 1.2	85.9 ± 2.1	86.6 ± 0.9
40	83.6 ± 2.2	72.5 ± 1.3	83.6 ± 2.2	73.5 ± 1.1	83.6 ± 2.2	83.2 ± 0.8
50	82.1 ± 2.2	72.6 ± 1.3	82.1 ± 2.2	73.1 ± 0.8	82.1 ± 2.2	81.0 ± 0.9

Equation (1) implies

$$\sigma(\bar{p}p)+\sigma(pn)=2\sigma(\pi^-p)+\sigma(\pi^+p), \qquad (2)$$

and the additional assumptions of charge conjugation and isospin invariance of quark-quark amplitudes lead to

$$\sigma(\bar{p}n)+\sigma(pp)=\sigma(\pi^-p)+2\sigma(\pi^+p) \qquad (3)$$

and

$$\sigma(K\bar{p})-\sigma(K^+p)=\sigma(K^-n)-\sigma(K^+n)+\sigma(\pi^-p)-\sigma(\pi^+p). \qquad (4)$$

The sum rule of Eq. (4) involves differences of the form $\sigma_{\bar{A}B}-\sigma_{AB}$; in view of the experimental uncertainties in those differences, this sum rule does not distinguish between I–III. However, a meaningful test based on (2) and (3) is possible; Tables I and II give the left- (L) and right-hand (R) sides of (2) and (3), respectively, for the three kinematic prescriptions. It is seen that prescription III is in agreement with the data, while I and II disagree by about 15%. This supports the arguments of Ref. (5), although the reader must be cautioned that the pp and pn cross sections were based on curves of as yet unpublished Serpukhov data.

In applying sum rules involving $\sigma_{\bar{A}B}$ where A and B are baryons, some authors[7] have chosen to subtract the portion of $\sigma_{\bar{A}B}$ due to $\bar{A}+B\to$ mesons. This could possibly explain the discrepancies of sum rules (2) and (3) under schemes I and II. Little is known experimentally about the above annihilation cross section; at 5.7 GeV/c this cross section is 22.5 ± 2 mb.[8] One might expect this cross section not to be important at Serpukhov energies, however; such processes involve a baryon exchange and individual contributions to $\bar{A}+B\to$ mesons should fall as p^{-4}, much faster than individual contributions to processes having baryons in the final state. Also, the subtraction procedure is not clear-cut theoretically since crossing relates the baryon-baryon amplitudes to the baryon-antibaryon amplitudes. In addition, while roughly 12 mb of such contributions is needed to reconcile the sum rules with experiment, $\sigma(\bar{p}p)$ only exceeds $\sigma(pp)$ by about 5 mb at 40–50 GeV/c. Clearly experimental data on $\bar{A}+B\to$ mesons at high energies would be welcome.

We turn now to the hyperon beams. The most promising candidates for cross-section measurements are $\Sigma^\pm$ on nucleons. Sum rules for these processes as well as Ξ^- on nucleons will be considered. We observe that each of the following combinations of hadrons has the quark structure of the vacuum:

$$\Sigma^-+p+2\bar{p}-K^--2\pi^+-3\pi^-,$$

$$\Sigma^++\pi^--p-K^-,$$

and

$$\Xi^-+\pi^--\Sigma^--K^-.$$

The "addition" of hadrons is an instruction to sum numerically the various types of constituent quarks. These relations imply the following sum rules for the cross sections of hyperons on nucleons (or on any elementary target):

$$\sigma(\Sigma^-N)=\sigma(K^-N)+2\sigma(\pi^+N)+3\sigma(\pi^-N)-\sigma(pN)-2\sigma(\bar{p}N), \qquad (5)$$

TABLE II. The left- (L) and right-hand (R) sides of the sum rule of Eq. (3) are given for various values of incident nucleon momentum for kinematic prescriptions I–III as described in the text. Cross sections are in mb.

	I		II		III	
P_L (GeV/c)	L	R	L	R	L	R
30	83.4 ± 2.1	72.2 ± 1.5	83.4 ± 2.1	72.0 ± 1.5	83.4 ± 2.1	83.8 ± 0.9
40	81.7 ± 2.1	70.9 ± 1.5	81.7 ± 2.1	72.3 ± 1.3	81.7 ± 2.1	80.9 ± 0.7
50	82.6 ± 2.1	71.4 ± 1.5	82.6 ± 2.1	71.8 ± 0.8	82.6 ± 2.1	78.6 ± 0.9

$$\sigma(\Sigma^+N) = \sigma(pN) + \sigma(K^-N) - \sigma(\pi^-N), \quad (6)$$

$$\sigma(\Xi^-N) = \sigma(\Sigma^-N) + \sigma(K^-N) - \sigma(\pi^-N). \quad (7)$$

A comparison with the data is made below for the case of a proton target. The approximate validity of the Okun-Pomeranchuk theorem suggests that similar results will be found for neutron targets. Our calculations support this, but since the errors are larger for neutron targets these results have not been tabulated.

Before proceeding to calculations of the hyperon-nucleon cross sections from the data, a few points should be emphasized. First, these sum rules follow strictly from additivity without any assumptions concerning the various quark-quark amplitudes. This is in contrast to sum rules (3) and (4) since charge conjugation and isospin invariance are required in these cases. No asymptotic assumptions concerning the σ_{AB} are assumed; only the optical theorem, which follows from unitarity, is needed in addition to additivity. Thus these sum rules should not depend upon ultimate rising, flattening, etc., of hadron cross sections and should provide a very clean test of additivity. An amusing consequence of (5)–(7) is the following relation among baryon-baryon cross sections:

$$\sigma(\Sigma^-p) + \sigma(\Sigma^+p) = \sigma(\Xi^-p) + \sigma(pp).$$

Predictions of hyperon cross sections using prescriptions I–III are limited by the availability of experimental data. Total cross sections for $\bar{p}p$, K^-p, and π^-p have been measured and tabulated by Allaby *et al.*[9] In addition, π^+p may be deduced by isospin invariance from π^-n. Lacking published pp data at energies from 22–50 GeV/c, an extrapolation of existing data[10] between 6 and 22 GeV/c was made using the form

$$\sigma_T(pp) = A + B(P_L)^{-1/2}.$$

The coefficients determined are $A = 36.5$ and $B = 10.3$ (when σ is in mb and P_L in GeV/c). An extrapolation error of $\frac{1}{2}$ mb was included in our error analysis. The extrapolated cross sections were compared to the unpublished Serpukhov results and were found to be consistent within errors; in addition, the asymptotic pp cross section is consistent with recent results from ISR.

We might add that there exist more recent Serpukhov data on negative beams, as well as on the positive beams, which do not agree exactly with the published data. Here, for predictions of the hyperon cross sections, we chose to rely on published data only.

Predictions using kinematic prescription III are limited by the fact that the pion beam energies are scaled to a lab momentum roughly $\frac{1}{8}$ of the Σ^- momentum. The low-energy πp data suggest resonance structure out to about 4 GeV/c.[11] Since collective effects such as resonances will invalidate simple additivity, one is limited to Σ beam momenta of 30 GeV/c and above for testing sum rules using prescription III.

Tables III, IV, and V give predictions for the Σ^-p, Σ^+p, and Ξ^-p cross sections obtained using kinematic prescriptions I, II, and III, respectively. Scale errors have been included in our error estimates because of the fact that we had to compare data obtained in different experiments (see Refs. 9–11).

It is clear from the tables that the predictions of prescription III for Σ^-p and Ξ^-p cross sections differ radically from those of I and II. All three sum rules lead to a flat Σ^+p cross section between 25 and 60 GeV/c on the order of 30–34 mb. Pre-

TABLE III. Predictions of the hyperon-nucleon cross sections at fixed laboratory momentum (prescription I) are given in mb; predictions follow from the sum rules of Eqs. (5)–(7).

P_L (GeV/c)	$\sigma(\Sigma^-p)$	$\sigma(\Sigma^+p)$	$\sigma(\Xi^-p)$
6	2.7 ± 4.4	36.1 ± 1.2	-2.2 ± 5.0
12	4.9 ± 3.7	35.1 ± 1.1	0.6 ± 4.2
20	7.2 ± 7.1	34.6 ± 2.3	3.0 ± 8.4
30	10.9 ± 5.9	34.7 ± 1.9	7.2 ± 6.8
45	12.2 ± 6.1	34.4 ± 1.9	8.5 ± 7.0

TABLE IV. Predictions of the hyperon-nucleon cross sections for baryon laboratory momenta shown are in mb. The meson momenta are scaled to $\frac{2}{3}$ of the baryon momenta in accordance with prescription II.

P_L (GeV/c)	$\sigma(\Sigma^-p)$	$\sigma(\Sigma^+p)$	$\sigma(\Xi^-p)$
12	13.5 ± 3.7	35.5 ± 1.1	9.6 ± 4.2
18	8.4 ± 10.3	34.4 ± 1.1	4.1 ± 10.8
30	11.4 ± 7.1	34.2 ± 2.3	7.2 ± 8.4
45	15.6 ± 6.1	34.4 ± 1.9	11.9 ± 7.0

TABLE V. Predictions in mb for hyperon-nucleon cross sections from sum rules (5)–(7) are presented using the fixed-relative-velocity prescription (III). Cross sections are predicted at momenta scaled from nucleon-nucleon momenta corresponding to Serpukhov data points.

P_L^Σ (GeV/c)	$\sigma(\Sigma^-p)$	$\sigma(\Sigma^+p)$	$\sigma(\Xi^-p)$	P_L^Ξ (GeV/c)
38.1	32.1 ± 5.1	29.9 ± 1.7	23.6 ± 5.8	42
44.5	31.5 ± 6.2	30.9 ± 2.6	24.1 ± 7.8	49
50.8	30.8 ± 5.5	30.8 ± 2.1	23.4 ± 6.6	56
57.2	28.8 ± 5.4	30.8 ± 2.0	21.5 ± 6.4	63
63.5	29.9 ± 5.5	31.1 ± 1.9	23.0 ± 6.4	70

scription III is consistent with a flat $\Sigma^- p$ cross section which is roughly equal to the $\Sigma^+ p$ cross section; the other two prescriptions seem to imply a slowly rising $\Sigma^- p$ cross section roughly $\frac{1}{3}$ in magnitude of the $\Sigma^+ p$ cross section in the same momentum range. The only data consist of a few events at very low energy[12] and do not really confront the sum rules. Flat baryon-baryon cross sections are predicted by exchange-degeneracy models[13]; and perhaps one intuitively feels that isospin would lead to $\Sigma^- p$ and $\Sigma^+ p$ cross sections nearly equal at high energies. But a true test must await the hyperon-beam experiments. It is clear that even rough data on the $\Sigma^- p$ cross section would provide a test of prescription III versus I and II.

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Exact Inequality and Test of Chiral SW(3) Theory in K_{l3} Decay Problem*

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Assuming the chiral SW(3) model, exact inequalities for the K_{l3} scalar form factor $D(t)$ and its derivatives have been studied. We can sharpen our inequalities considerably if we take into account the soft-pion theorem. These inequalities are stringent enough to test the validity of the chiral SW(3) theory.

I. INTRODUCTION AND SUMMARY OF PRINCIPAL RESULTS

The purpose of this paper is to study further applications of the exact inequality which was proved elsewhere[1] for the K_{l3} problem, assuming the validity of the chiral SW(3) model[2] of Gell-Mann, Oakes, and Renner and of Glashow and Weinberg (hereafter referred to as GMORGW). In particular, we estimate upper bounds for higher-order derivatives of the K_{l3} scalar form factor $D(t)$ (see below) and we discuss in some detail consequences of the soft-pion theorem. We find that our inequality is stringent enough to be barely satisfied by the present experimental data, and we suggest a simple form for $D(t)$.

As usual, we define the standard K_{l3} form factor $f_\pm(t)$ by

$$\langle \pi^0(p')|V_\mu^{(4-i5)}(0)|K^+(p)\rangle = -(4p_0p_0'V^2)^{-1/2}(\tfrac{1}{2})^{1/2}[(p+p')_\mu f_+(t) + (p-p')_\mu f_-(t)],\quad t=-(p-p')^2\,. \qquad (1.1)$$

We are especially interested in the scalar form