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Asymptotics in Inelastic Electroproduction and Photoabsorption

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It is shown that the use of normalizable states in quantum field theory strongly suggests that scattering processes at high energies may be determined solely by light-cone contributions. Then configuration-space analysis of electromagnetic current commutators and observed behaviors of photoabsorption and of inelastic electron-proton scattering imply that scaling for deep-inelastic electroproduction *cannot* be related to a diffractive mechanism and that the scaling limit of $\nu W_2(Q^2,\nu)$ and $W_1(Q^2,\nu)$ is broken when $\omega = Q^2/2M\nu$ is very small ($\omega \ll 0.05$). It is shown that our results are consistent with the observed scaling behavior, the parton model, and duality.

I. INTRODUCTION

Inelastic lepton-proton scattering at high energies revealed two remarkable features[1]: (i) a very large cross section and (ii) consistency with scaling behavior.[2] These remarkable properties were related to Regge asymptotics[3-5] and have been used to study the structure of the matrix elements of electromagnetic current commutators near the light cone.[6-12] They are also suggested by the description of the scattering in terms of pointlike constituents of the proton, the partons of Feynman.[13,14] Another important property is that the behavior of electroproduction of resonances is related to that of deep-inelastic electroproduction on protons.[15]

It is the purpose of this paper to investigate the structure of commutators of electromagnetic currents in the light of the scaling behavior of inelastic electron-proton scattering and of the experimental observation which establishes that the total photon-proton cross section is constant at high energy.[16] In Sec. II, it will be proposed in contradistinction with Refs. 7, 11, 12, 17, and 18 that, even in the case where the external masses are small, the scattering processes at high energies are dominated by light-cone singularities and are *not* affected by the behavior of the commutators inside the light cone. This result will be strongly suggested by the usual assumption of quantum theory that any physical system is localized in space. It will be particularly important to notice that, in quantum field theory, we must work with *normalizable* states and be very cautious if we use plane-wave solutions.[19]

In Sec. III, we will show that the leading singularities on the light cone are different for deep-inelastic electroproduction and photoabsorption, if we take seriously the present experimental data.[1,16] Furthermore, we will see that scaling behavior for the deep-inelastic region is decoupled from Pomeranchuk exchange. This will imply that the scaling limit as given by Bjorken is broken when $\omega = Q^2/2M\nu$ is very small ($\omega \ll 0.05$), far away from the range of ω presently accessible.[1] We will obtain

$$\lim_{\mathrm{Bj}} \nu W_2(Q^2, \nu) = F_2(\omega) \quad \text{if} \quad \omega > \omega_0 \; (\omega_0 \ll 0.05), \tag{1}$$

$$\lim_{\mathrm{Bj}} M W_2(Q^2, \nu) = G_2(\omega) \quad \text{if} \quad \omega < \omega_0 . \tag{2}$$

M is the proton mass and $p \cdot q = M\nu$, where ν is the energy of the photon in the rest frame of the proton. $q^2 = -Q^2$, with q^2 the squared mass of the virtual photon. $\lim_{\mathrm{Bj}}$ means $\nu \to \infty$ with ω fixed. As we will see, ω_0 can be arbitrarily small.

In Sec. IV, it will be shown that our results are consistent with the parton model[13,14] and with duality.[15] Finally, the content of the paper will be summarized.

II. CONFIGURATION SPACE PICTURE

We consider the two-body scattering $\alpha + \beta \to \alpha' + \beta'$, where α, β, α', β' represent wave-packet states.[19] α and α' are neutral spin-zero particles. The generalization to particles with spin and charge can be done in the usual manner. Applying the LSZ reduction technique, we can express the S-matrix element of this process as

$$\langle \alpha', \beta'; \mathrm{out} | \alpha, \beta; \mathrm{in} \rangle = \delta_{\alpha', \beta'; \alpha, \beta} + i^2 \int d^4x \int d^4y f^*_{\alpha'}(x) \{(\Box_x + \mu^2)(\Box_y + \mu^2) \langle \beta' | \theta(x_0 - y_0)[\phi(x), \phi(y)] | \beta \rangle \} f_\alpha(y), \tag{3}$$

where the functions $f_\alpha(x)$ are normalized according to the condition

$$i\int d^3x f_\alpha^* \frac{\overleftrightarrow{\partial}}{\partial x_0} f_\beta = \delta_{\alpha\beta}\,.$$

In many applications the wave packet $f_\alpha(x)$, which is spatially localized, is allowed to approach a plane wave:

$$f_\alpha(y) \to f_q(y) = \frac{1}{(2q_0)^{1/2}(2\pi)^{3/2}}\, e^{-iq\cdot y}\,, \tag{4}$$

with $q^2=\mu^2$. But this modification can be made only after the completion of all partial integrations which require that $f_\alpha(x)$ is spatially confined. Hence, any consideration which involves the region where $|\vec{x}|$ is large does *not* allow the replacement of $f_\alpha(x)$ by $f_q(x)$ because the latter is not spatially confined.

Let us forget this difficulty for the time being. Introducing Eq. (4) into Eq. (3) and integrating with respect to y, we get the invariant scattering amplitude M:

$$\frac{1}{(2P_0)^{1/2}(2\pi)^{3/2}}\,\frac{1}{(2P_0')^{1/2}(2\pi)^{3/2}}\,M(\alpha+\beta\to\alpha'+\beta') = -i\int d^4x\, e^{iq'\cdot x}(\Box_x+\mu^2)\theta(x_0)\langle\beta'|[\phi(x),j(0)]|\beta\rangle\,. \tag{5}$$

We have used $(\Box_y+\mu^2)\phi(y)=j(y)$, where $j(y)$ is the source of the field $\phi(y)$; q, q', P, P' are the four-momenta of the states α, α', β, β', respectively, which at this stage in the argument are plane-wave states.

Now, we will consider the forward scattering ($q=q'$; $P=P'$). Taking the target β at rest ($P_0=M$) and choosing q along the z axis, we get, in the limit $P\cdot q \equiv M\nu \to\infty$,

$$q=(\nu,0,0,\nu(1-\tfrac{1}{2}\mu^2/\nu^2))\,. \tag{6}$$

Hence, in Eq. (5),

$$e^{iq\cdot x}\approx e^{i\nu(x_0-x_3)}e^{+i\frac{1}{2}(\mu^2/\nu)x_3}\,. \tag{7}$$

The x_0-x_3 integration in Eq. (5) selects only the region $|x_0-x_3|\lesssim O(1/\nu)$. This follows because, for $|x_0-x_3|\gg 1/\nu$, $\lim_{\nu\to\infty} e^{i\nu(x_0-x_3)}$ oscillates to a zero contribution since the argument of the exponent cannot vanish in the range of integration. A similar statement can be made for the region where $|x_3|\lesssim O(2\nu/\mu^2)$. Otherwise, when $|x_3| \gg 2\nu/\mu^2$, $\exp[i\frac{1}{2}(\mu^2/\nu)x_3]$ oscillates to a zero contribution. Thus, in the limit $\nu\to\infty$, the contributing range of x^2 is given by

$$0\leqslant x^2 \lesssim O\left(\frac{1}{\nu}\,\frac{4\nu}{\mu^2}\right). \tag{7$'$}$$

The lower bound is fixed by microcausality: $(x_0-x_3)(x_0+x_3)\geqslant x_0{}^2-|\vec{x}|^2\geqslant 0$. The upper bound has been obtained, observing that $x_0=x_3+O(1/\nu)$.

As discussed by Gribov, Joffe, and Pomeranchuk,[17,18] there may be two completely different behaviors of $M(\alpha+\beta\to\alpha'+\beta')$ which satisfy inequalities given in (7′) when $\nu\to\infty$:

(i) $|x_0-x_3|\lesssim O(1/\nu)$ and $|x_3|$ or $|\vec{x}|\sim O(2\nu/\mu^2)$; then, $x^2\lesssim O(4/\mu^2)$ and the leading contributions can be *inside* the light cone.

(ii) $|x_0-x_3|\lesssim O(1/\nu)$ and $|x_3|$ or $|\vec{x}|< O(2\nu/\mu^2)$

and the leading contributions are *on* the light cone. The first alternative, (i), which involves the region where $|\vec{x}|\to\infty$, has been obtained because the modification given by (4) – the replacement of a localized wave packet by a plane wave – has been applied *outside the limit of its validity*. Of course the fact that $f_{\alpha'}^*(x)$ and $f_\alpha(x)$ are smearing functions is *not* a sufficient condition to claim that only the regions where $|\vec{x}|$ and $|\vec{y}|$ are finite contribute to Eq. (3), because $j(x)$ and $\phi(x)$ are highly singular objects which are not spatially confined.[19] Nevertheless, we have to be very cautious with alternative (i) in contradistinction to alternative (ii) where the infinite spatial distances are unimportant [at least when $|\vec{x}|\ll O(2\nu/\mu^2)$]. Hence, in strongly interacting processes, we suspect that the light-cone singularities *do* play the dominating role in the asymptotic energy region, the Regge region.

These considerations cast some doubt on the validity of the analysis given in Ref. 17. They can be immediately applied to high-energy Compton scattering on hadron H [$\gamma(q)+H(P)\to\gamma(q')+H(P')$] and to high-energy photoabsorption [$\gamma(q)+H(P)\to$ anything], where the total cross section is proportional to the imaginary part of the forward scattering amplitude of photons on hadron H. And we can conclude that it is very doubtful that, for these last two processes, the leading contributions are inside the light cone.

In Sec. III, we will use these considerations to study the leading space-time singularities which correspond to the high-energy region in inelastic electroproduction and photoabsorption.

III. APPLICATION TO INELASTIC ELECTROPRODUCTION

The hadronic part of electroproduction and photoabsorption is usually defined by

$$W_{\mu\nu} \equiv \frac{1}{M^2}\left(P_\mu - \frac{P\cdot q}{q^2} q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2} q_\nu\right) W_2(Q^2,\nu) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(Q^2,\nu) = \frac{P_0}{M}\int \frac{d^4x}{2\pi} e^{iq\cdot x}\langle P|[J_\mu(x), J_\nu(0)]|P\rangle . \tag{8}$$

This second-rank tensor is proportional to the imaginary part of the forward scattering amplitude of photons of four-momentum q by protons. $J_\mu(x)$ is the electromagnetic current, P is the proton four-momentum, and a spin averaging is understood in Eq. (8).

In the scaling limit, Bjorken has suggested[2] that

$$\lim_{\mathrm{Bj}} \nu W_2(Q^2,\nu) = F_2(\omega), \tag{9a}$$

$$\lim_{\mathrm{Bj}} M W_1(Q^2,\nu) = F_1(\omega), \tag{9b}$$

where $F_i(\omega)$ $(i=1,2)$ are smooth functions for $(1>\omega\geq 0)$.

Experimentally,[1,15] if we choose the following kinematic conditions: $W=[(P+q)^2]^{1/2}\geq 2$ GeV (above the first four resonances) and $Q^2\geq 3$ GeV^2, the data are consistent with scaling as defined in Eqs. (9a) and (9b) when $\omega>0.05$.

Now for small Q^2, one can relate the inelastic electroproduction structure functions to the total photon-proton cross section $\sigma_{\gamma p}(\nu)$:

$$\nu W_2(Q^2,\nu) \xrightarrow[Q^2\to 0]{} \frac{Q^2\sigma_{\gamma p}(\nu)}{4\pi^2\alpha}, \tag{10a}$$

$$W_1(Q^2,\nu) \xrightarrow[Q^2\to 0]{} \frac{\nu\sigma_{\gamma p}(\nu)}{4\pi^2\alpha}, \tag{10b}$$

Experimentally,[16]

$$\sigma_{\gamma p}(\nu) \xrightarrow[\nu\to\infty]{} \text{const}. \tag{11}$$

This experimental result is consistent with the usual Regge asymptotic behavior, assuming that the Pomeranchuk trajectory with $\alpha=1$ dominates,[20] so that

$$\lim_{\nu\to\infty;\ Q^2 \text{ fixed}} W_2(Q^2,\nu) = g_2(Q^2)\nu^{\alpha-2}, \tag{12a}$$

$$\lim_{\nu\to\infty;\ Q^2 \text{ fixed}} W_1(Q^2,\nu) = g_1(Q^2)\nu^{\alpha}. \tag{12b}$$

We are going to show the effect of the constraints imposed by Eqs. (9)–(12) on the configuration-space representation of commutators of electromagnetic currents. The space-time representation of commutators of electromagnetic currents is given by[21]

$$\frac{i}{2\pi}\frac{P_0}{M}\langle P|[J_\mu(x), J_\nu(0)]|P\rangle = (g_{\mu\nu}\Box - \partial_\mu\partial_\nu)c_1(x^2, x\cdot P) + \frac{1}{M}[P_\mu P_\nu\Box - (P\cdot\partial)(\partial_\mu P_\nu + \partial_\nu P_\mu) + g_{\mu\nu}(P\cdot\partial)^2]c_2(x^2, x\cdot P), \tag{13}$$

where $c_i(x^2, x\cdot P) = -c_i(x^2, -x\cdot P)(i=1,2)$. Notice that the Fourier transform of $W_{\mu\nu}$ [see Eq. (8)] is a causal commutator, which is zero when $x^2<0$. The connection between Eq. (8) and Eq. (13) is given by

$$W_{\mu\nu} = (-g_{\mu\nu}q^2 + q_\mu q_\nu)C_1(Q^2,\nu) + \frac{1}{M}[-P_\mu P_\nu q^2 + (P\cdot q)(q_\mu P_\nu + q_\nu P_\mu) - g_{\mu\nu}(P\cdot q)^2]C_2(Q^2,\nu), \tag{14}$$

where

$$C_i(Q^2,\nu) = -i\int d^4x e^{iq\cdot x} c_i(x^2, x\cdot P) \tag{15}$$

and

$$C_2(Q^2,\nu) = (Q^2M)^{-1}W_2, \tag{16a}$$

$$C_1(Q^2,\nu) = (-Q^2)^{-1}W_1 + \nu^2(Q^2)^{-2}W_2. \tag{16b}$$

It is interesting to notice that going from Eq. (13) to Eq. (14) or Eq. (8), we have used a four-dimensional integration by parts and we have *neglected* the space-time surface terms. Now Suura and Meyer[21] have shown that $c_i(x^2, x\cdot P)$ are causal using two facts:

(i) $[J_\mu(x), J_\nu(0)] = 0$ when $x^2<0$, i.e., the commutator satisfies microcausality.

(ii) $\lim_{|\vec{x}|\to\infty} c_{1,2}(x^2, x_0\cdot M) = 0$

when x_0 is fixed, i.e., the c_i's have to vanish for infinite spacelike distances [this requirement, by the way, is necessary but not sufficient in order to pass from Eq. (13) to Eq. (14)].

Then, the general structure of the functions $c_{1,2}(x^2, x\cdot P)$ in terms of their light-cone singularities can be taken[9] on the basis of locality and causality to be of the form

$$c_{1,2}(x^2, x\cdot P) = \epsilon(x\cdot P)\Big[\theta(x^2)f_{1,2}(x^2, x\cdot P) + \sum_{n=0}^{N}\delta^{(n)}(x^2)s^{(n)}_{1,2}(x\cdot P)\Big], \tag{17}$$

where N is a finite number,

$$\delta^{(n)}(x^2) = \frac{d^n}{d(x^2)^n}\,\delta(x^2)\ ,$$

and the functions $f_{1,2}$ and $s_{1,2}^{(n)}$ are even in $(x\cdot P)$. Assuming that $f_{1,2}(x^2, x\cdot P)$ and $s_{1,2}^{(n)}(x\cdot P)$ are sufficiently regular and uniform for various manipulations (e.g., interchange of limit and integration), it has been shown[6-9] that nonzero limits of W_1 and νW_2, when $\nu\to\infty$ with ω fixed, exist when the leading light-cone singularity of

$$\begin{Bmatrix} c_1(x^2, x\cdot P) \\ c_2(x^2, x\cdot P)\end{Bmatrix} \text{ is proportional to } \begin{Bmatrix}\epsilon(x\cdot P)\delta(x^2) \\ \epsilon(x\cdot P)\theta(x^2)\end{Bmatrix}.$$

In other words, the presence of singularities such as $\delta^{(n)}(x^2)$ $(n\geq 0)$ in $c_2(x^2, x\cdot P)$, and $\delta^{(m)}(x^2)$ $(m\geq 1)$ in $c_1(x^2, x\cdot P)$, would be incompatible with the scaling of the structure functions $W_1(q^2,\nu)$ and $\nu W_2(q^2,\nu)$, respectively, in the entire physical region: $1\geq\omega\geq 0$. With this result we are able to write

$$\lim_{\nu\to\infty}\nu W_2(Q^2,\nu) = -iQ^2\nu M\int d^4x\, e^{iq\cdot x}\,\epsilon(x\cdot P)\times\theta(x^2)f_2(x^2, x\cdot P) \quad (18)$$

and

$$\lim_{\mathrm{Bj}}\nu W_2(Q^2,\nu) = F_2(\omega) = -4\pi i\omega\int_{-\infty}^{+\infty} d(x\cdot P)e^{i\omega(x\cdot P)}\times f_2(0, x\cdot P)x\cdot P, \quad (19)$$

with $1\geq\omega\geq 0$. A similar expression can be written for $W_1(Q^2,\nu)$.

Let us consider in more detail the space-time structure of $\nu W_2(Q^2,\nu)$ when $\nu\to\infty$. Taking, as in Sec. II, the target proton at rest and choosing $\vec{q}$ along the z axis,[11,18] we can replace in Eqs. (15) and (18) $e^{iq\cdot x}$ by

$$e^{i\nu(x_0-x_3)}e^{-i\frac{1}{2}(Q^2/\nu)x_3}$$

[see Eq. (7)]. Exactly as in Sec. II, the contributing range of x^2 is given by

$$0\leq x^2\leq O\left(\frac{1}{\nu}\frac{4\nu}{Q^2}\right), \quad (20)$$

with $|\vec{x}|\approx|x_0|\lesssim O(1/\omega M)\approx O(2\nu/Q^2)$ and $|x_0-x_3|\lesssim O(1/\nu)$. Hence in the scaling limit, when $\nu\to\infty$, $Q^2\to\infty$, all contributions come from the surface of the light cone, $x^2=0$. That is why, in Eq. (19), $F_2(\omega)$ depends only on the region $x^2=0$. But in the diffractive region (Regge limit) where $\nu\to\infty$ with Q^2 fixed (asymptotic photoabsorption is a special case of this, with $Q^2=0$), the contributing range of x^2 can be not only on the light cone but also inside if $|\vec{x}|\sim O(2\nu/Q^2)$, i.e., if the spatial distance between the two currents which appear in Eq. (8) is infinite, as $\nu\to\infty$. And the higher the value of Q^2,[11] the nearer we come to the light cone. When Q^2 is large enough for the light-cone singularities to dominate – the deep Regge limit[5] – it should be possible to relate scaling and the Regge limit[3-9,11,12]:

$$\lim_{\mathrm{Bj};\,\omega\to 0}\nu W_2(Q^2,\nu) = F_2(0) = g_2(\infty) \quad (21)$$

using Eqs. (9a) and (12a). Now, Eqs. (18) and (19) tell us that (i) if the space-time structure for $\nu W_2(Q^2,\nu)$ really is given by Eq. (18) when $\nu\to\infty$, (ii) if all contributions come from the region $x^2=0$ when Q^2 is finite, and (iii) if Eq. (10a) is taken into account, then

$$\lim_{\nu\to\infty;\,Q^2\,\mathrm{fixed}}\nu W_2(Q^2,\nu)\sim Q^2/\nu$$

and

$$\lim_{\nu\to\infty}\sigma_{\gamma p}(\nu)\sim 1/\nu, \quad (22)$$

in contradiction to the present experimental data [see Eq. (11)] and Regge parametrization [see Eq. (12a)]. For this reason, it has been proposed[7,11,12] that in the Regge limit the leading contributing range of x^2 has to be inside the light cone and therefore $|\vec{x}|\sim|x_0|\sim O(2\nu/Q^2)$ should be the region of interest.

But in Sec. II, we saw that, in the case where the photon is real, it is improbable that we can have such a situation where an infinite spatial distance gives the main contribution, because we must use *localizable* states and not simple plane-wave solutions. We can extend this observation to the case where the photon is virtual in the following way. Strictly speaking – if we want to satisfy the requirements of quantum field theory[19] – the second-rank tensor $W_{\mu\nu}$ in Eq. (8) should have the non-normalizable state $|P\rangle$ replaced by a normalizable and localizable state $|\alpha\rangle$. Furthermore, q_μ, the four-momentum of the virtual photon (the four-momentum transferred from the initial electron to the proton) is not unambiguously defined because the initial and the final electron states are normalizable states defined through wave packets and not plane waves. *Then* $e^{iq\cdot x}$ *should be replaced by a spatially confined wave packet.*

Now we have a reasonable argument for proposing that all asymptotics, especially in inelastic electroproduction and photoabsorption, are *only determined by light-cone contributions*. But we must overcome the difficulty we met in Eq. (22).

To solve this problem, we will take the experimental data and the Regge parametrization seriously, i.e., Eqs. (9)–(12), with the following clarifying point: Scaling behavior is consistent with experimental data for $1>\omega>0.05$. If we assume that scaling behavior is broken for small ω $(\omega\ll 0.05)$, we can replace Eq. (18) by[22]

$$\lim_{\nu\to\infty} \nu W_2(Q^2,\nu) = -iQ^2\nu M\int d^4x\, e^{iq\cdot x}\,\epsilon(x\cdot P)[\theta(x^2)f_2(x^2,x\cdot P)+\delta(x^2)s_2^{(0)}(x\cdot P)]$$

$$= -4\pi i\omega\int_{-\infty}^{+\infty} d(x\cdot P)e^{i\omega(x\cdot P)}f_2(0,x\cdot P)x\cdot P + \pi Q^2\int_{-\infty}^{+\infty} d(x\cdot P)e^{i\omega(x\cdot P)}s_2^{(0)}(x\cdot P), \tag{23}$$

with the condition that

$$\int_{-\infty}^{+\infty} d(x\cdot P)e^{i\omega(x\cdot P)}s_2^{(0)}(x\cdot P)\simeq 0 \tag{24}$$

if $\omega > \omega_0$ ($\omega_0 \ll 0.05$).

With Eqs. (23) and (24), we get perfect agreement with all the constraints represented by Eqs. (9)–(12). Let us notice that, as we have already explained in Sec. II, it is perfectly allowable to use plane waves in Eq. (23), because we are only considering finite spatial distances. An example which satisfies Eq. (24) is

$$s_2^{(0)}(x\cdot P)\sim \frac{\sin\omega_0(x\cdot P)}{x\cdot P} \tag{25}$$

because

$$\int_{-\infty}^{+\infty} d(x\cdot P)e^{i\omega(x\cdot P)}\frac{\sin\omega_0(x\cdot P)}{x\cdot P}\sim\theta(\omega_0{}^2-\omega^2) \tag{26}$$

and ω_0 can be arbitrarily small.

From this analysis, we conclude that the structure functions which scale appear to depend on which kinematical region we are considering:

$$\lim_{\text{Bj}} \nu W_2(Q^2,\nu) = F_2(\omega) \quad \text{if} \quad \omega > \omega_0, \tag{1}$$

$$\lim_{\text{Bj}} M W_2(Q^2,\nu) = G_2(\omega) \quad \text{if} \quad \omega < \omega_0. \tag{2}$$

In the same way, we can get

$$\lim_{\text{Bj}} M W_1(Q^2,\nu) = F_1(\omega) \quad \text{if} \quad \omega > \omega_0, \tag{27}$$

$$\lim_{\text{Bj}} M^2 W_1(Q^2,\nu) = \nu G_1(\omega) \quad \text{if} \quad \omega < \omega_0. \tag{28}$$

To ensure that $\lim_{\text{Bj}}\nu W_2(Q^2,\nu)$ and $\lim_{\text{Bj}} W_1(Q^2,\nu)$ scale everywhere it would have been necessary to choose $\alpha = 0$ in Eqs. (12a) and (12b), as we can see from Eq. (22).

IV. CONCLUSION

The above conclusion could be expressed in the frame of the Deser-Gilbert-Sudarshan (DGS) representation. Following Brandt's work,[7] we should replace his Eq. (2.12) by

$$\int da\,\sigma_2(a,\omega)\begin{cases} =0 & \text{if} \quad \omega>\omega_0\\ \neq 0 & \text{if} \quad \omega<\omega_0\end{cases} \tag{29}$$

to get Eqs. (1), (2), (27), and (28). Furthermore, due to Eq. (29), it is not necessary to have $\sigma_2(a,b) \sim \ln b$, when $b\to 0$ [see Eq. (2.36) of Ref. 7]. An interesting point of the unsubtracted DGS representation is that it requires that $c_i(x^2, x\cdot P)$ cannot have higher light-cone singularities than $\delta(x^2)$ [see Eqs. (2.8) and (3.58) of Ref. 7].[23] It is then straightforward to show that $Q^2C_1(Q^2,\nu) = -W_1 + (\nu^2/Q^2)W_2$ must scale when $\nu\to\infty$, for any value of ω ($1 \geq \omega \geq 0$), i.e.,

$$\lim_{\text{Bj}}[(Q^2)^2\nu^{-1}]C_1 \equiv F_L(\omega).$$

Using (2) and (28), we then have the relation

$$G_1(\omega) = (\nu M/Q^2)G_2(\omega) \quad \text{if} \quad \omega < \omega_0. \tag{30}$$

It is interesting to understand our results in the light of the parton model.[13,14] In the infinite-momentum frame, $P\to\infty$, of the proton, it is possible to treat the interaction between the proton and the virtual photon as the collision between the constituents of the proton, the partons (treated as instantaneously free during the interactions), and the virtual photon. Then ω is the fraction of the longitudinal $\vec{P}$ carried by the parton from which the electron scatters. When ω is very small, we have to deal with very slow partons in the $P\to\infty$ frame, the impulse approximation fails and the basis of the parton concept becomes doubtful.[13] So, it is perfectly possible to propose, in agreement with our results, that the parton model predicts scaling behavior as defined in Eqs. (1) and (27). Then we can interpret ω_0 as the fraction of the longitudinal momentum for which the impulse approximation is not applicable. And when $\omega < \omega_0$, we are in another asymptotic region, where scaling is applied to $W_2(Q^2,\nu)$. It would be important to see if such scaling behavior is consistent with another version of the parton model which could be applied in this kinematical region. Finally, let us notice that it is not necessary to assume that the proton has an infinite number of constituents [if $F_2(\omega)\to$ const as $\omega\to 0$][14] because we do not assume that $\nu W_2(Q^2,\nu)$ scales for very small ω.

Recently, it has been emphasized[15] that the behavior of resonances at high energies is related to that of deep-inelastic electroproduction on photons, i.e., on the average, the resonances are moving along the universal curve $\nu W_2(Q^2,\nu)$ plotted against $1/\omega' = (2M\nu + M^2)/Q^2$. This universal curve is defined in the region beyond the first four resonances and for $Q^2 > 3$ GeV^2. Duality would imply that if the Pomeranchuk exchange were the only contribution related to scaling behavior, these resonances would disappear at high energies, leaving all the room to the background, instead of moving along the universal curve (or "scaling-limit curve"[15]). Therefore, the resonances might even dominate the deep-inelastic region, when $\omega \ll 0.05$. This is another reason to believe that our analysis is consistent.

It was the purpose of this article to show that a diffractive mechanism is *not* related to scaling behavior in deep-inelastic electroproduction. The reason why these two asymptotics are *decoupled* is related to the fact that nucleon and photon states which are involved in deep-inelastic electroproduction have to be spatially confined. Then, when $\nu \to \infty$, all the contributions must come from the light-cone singularities. As we have seen in Sec. II, this result is also valid for hadron-hadron collisions.

Our analysis has been made using general requirements of quantum field theory and experimental facts. We have seen that there exists a value ω_0 above which $\nu W_2(Q^2, \nu)$ and $W_1(Q^2, \nu)$ do scale and below which $W_2(Q^2, \nu)$ and $\nu^{-1}W_1(Q^2, \nu)$ scale. Such a division does not contradict the parton model and is consistent with Regge parametrization and the observed scaling behavior.

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[1]See the invited talks of F. J. Gilman and R. E. Taylor in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970). For the most recent results, see E. D. Bloom *et al*. SLAC Report No. SLAC-PUB-746 (presented to the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970) (unpublished).

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$$f(x) = \int d^4k\, \theta(k)\delta(k^2 - m^2)g(k)e^{ik\cdot x}$$

is proportional to $x_0^{-3/2}$ when x_0 and $|\vec{x}| \to \infty$. Paul Roman, *Introduction to Quantum Field Theory* (Wiley, New York, 1969), pp. 301–331.

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[22]Eqs. (23) and (24) have already been proposed in Ref. 12, footnote 10, without justification. The new ingredient of the present work is the observation that infinite spatial distances can only be considered if *normalizable* – spatially confined – states are used to define a scattering amplitude, and that such states are vanishing for infinite spatial distances.

[23]This is not true for a once-subtracted DGS representation. See Appendix A in Ref. 9.