

Polarization of Cosmic-Ray Muons

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We have measured the polarization of cosmic-ray muons arriving at sea level with different energies and from different zenith angles. The polarization of positive muons from the zenith with energies near 1.0 GeV was found to be -0.33 ± 0.04 , where the polarization in the direction of flight was measured and the negative sign indicates that the spin was antiparallel to the direction of flight. Muons from the zenith with energies near 3.5 GeV at sea level had polarizations of -0.38 ± 0.04 , and the polarization of muons with energies near 4.5 GeV incident at an angle of 45° with the zenith was found to be -0.34 ± 0.04 . These polarizations are consistent with the polarizations calculated on the basis that the muons are derived almost wholly from the decay of pions produced by the interaction of primary and secondary cosmic-ray hadrons with the nuclei of the atmosphere.

INTRODUCTION

The first firm evidence for the existence of "cosmic radiation," obtained by Hess in his balloon flights, was derived from the measurements of ionization produced by particles which we now know were primarily muons. Modern analyses of this muon flux proceed basically from the assumption that it results from the decay of mesons produced in the interaction of the primary cosmic-ray protons and nuclei with atmospheric nuclei, and in the interactions of secondary hadrons from these collisions with other atmospheric nuclei. To the extent that this view is correct and that we understand the fundamental processes which contribute to the chain of production and decay which results in the muon flux, the character of the flux is well defined and can be understood in terms of well-known characteristics of the primary nucleon flux and the interactions of elementary particles. Conversely, a deviation from the predictable character of the muon flux is evidence for the existence of processes which we do not now understand. Recent measurements¹ of the variation of the flux of very high-energy muons with zenith angle suggest strongly that there are properties of the primary cosmic-ray flux and its interaction with the atmosphere which are not within the compass of our present understanding. We were interested in measuring the polarization of cosmic-ray muons in recognition of the possibility that deviations of the polarization from the predictions of well-defined calculations might occur, and such deviations would again suggest the existence of mechanisms other than those now known. Previous measure-

ments²⁻⁷ of the polarization had not completely resolved this question.

In the conventional view of muon production, the muons are derived primarily from the two-body decay of pions to muons and neutrinos and, to a lesser extent, from the two-body decays of charged *K* mesons to muons and neutrinos. One expects rather small contributions from the three-body decays of charged and neutral mesons and very small contributions from the electromagnetic production of muon pairs. In the case of the dominant two-body decays of the mesons, the muon is completely polarized in the direction of motion of the muon in the rest frame of the meson; the polarization is positive, or parallel to the direction of motion, for negative muons, and is negative for positively charged muons. The degree of polarization of the muon in the laboratory system depends, however, on the velocity of the parent meson and the direction of emission of the muon with respect to the direction of motion of the meson.

A beam of muons, derived from the decays of a beam of mesons, contains muons emitted from the mesons in all directions in the mesons' rest system. Since the total spin in any direction of all muons decaying from an assembly of mesons at rest must be zero, in the laboratory system the total polarization in the beam direction of all muons from the decay of a pion beam must also be zero. Of course, all muons are not detected; typically, only muons in a certain energy range are detected. Positive muons which are emitted forward by the decaying meson in the rest system of the meson will have negative polarization in the beam direction and will have nearly the same energy as the

original meson. On the other hand, positive muons emitted backwards by the decaying meson, in the meson rest system, will have positive polarization in the laboratory system. These positively polarized muons will result from the decays of mesons with much larger energy in the laboratory system. Then, if the energy spectrum of the original pion beam is such that the intensity falls off with energy, muons of a definite energy E will have a net negative helicity. Those muons with negative helicity will result from the decays of mesons with a laboratory energy only a little greater than E on the average, while those muons with positive helicity come from the backward center-of-mass decays of mesons with a laboratory energy much greater than E . Since there is a smaller flux of high-energy mesons than lower-energy mesons, there will be more muons emitted forward from a substantial population of mesons with energy near E , than emitted backwards in the meson center-of-mass system from the smaller population of mesons with energies very much larger than E . There will then be a net negative polarization of the positive muon flux.

Hayakawa⁸ has considered quantitatively the polarization of muons from the decay of such a meson beam and concluded that for a meson beam spectrum which can be described as

$$N(E)dE = N_0 E^{-\alpha} dE,$$

the average polarization of muons depends only upon the factor α and on β , the velocity of the decay muons in the rest system of the meson. For relatively large values of β ($\beta \approx 1$), the polarization in the laboratory system will be the same as the polarization in the center-of-mass system (as is the case for neutrinos). Since the value of β for K -meson decays is much larger than for pion decays, the laboratory polarization of muons from K decays will be much larger than for muons from pion decays. Indeed, the different velocities of muon emission lead to predicted polarizations of muons from K decays which are nearly three times that predicted from pion decays (though the inclusion of the minor decay modes, some of which lead to muons with opposite polarizations, reduces this factor somewhat).

Since the factor α is reasonably well known (largely from knowledge of the muon energy spectrum), if there is no anomalous source of muons, the measured muon polarizations can be used as indications of the proportion of the muons derived from the decay of pions and the portion produced through decays of K mesons. In turn, since this K/π ratio is determined by the character of meson production in the interaction of cosmic-ray hadrons,

the polarization measurements provide information concerning the ratio of K mesons to pions produced in these interactions.

The polarization of the muons is determined using a delayed coincidence technique such that muons in the cosmic-ray flux are stopped in an absorber, as determined by a set of coincidence counters, and after an appropriate delay the direction of the decay positron is recorded as well as the time of the decay. The polarization of the muon beam is then determined from the asymmetry of the decay positrons. Since it is possible to observe relatively large numbers of muon decays at rest only for muons with energies at sea level less than 10 GeV, most of our knowledge of the polarization of cosmic-ray muons is limited to muons of this energy or less.

The present experiment incorporates several features which make it different from past experiments in this area. In order to determine the absolute magnitude of the cosmic-ray muon polarization, the analyzing power of the apparatus must be known, preferably from a calibration experiment using a beam of muons of known polarization. For the apparatus used in this experiment, we made such a calibration at the Brookhaven AGS.⁹

Systematic errors introduced by instrumental asymmetries were further reduced by measuring the decay directions of the positrons as a function of time. The muons that stopped in the detector precessed in a small magnetic field produced by coils around the detector. Of course, the instrumental asymmetries are time independent while the asymmetry resulting from the muon polarization oscillated at the Larmor frequency.

In addition to these features, the apparatus had a sufficiently large aperture and energy acceptance to make it feasible to observe relatively large numbers of muon decays, typically on the order of 100 000 decays at each of three energies below 5 GeV, and thus to reduce the statistical error in our knowledge of the polarization.

APPARATUS AND PROCEDURE

The configuration of the apparatus is indicated schematically in Fig. 1. The energy of the muons was selected and degraded by using variable thicknesses of iron degrader above the detector. After slowing down in the degrader, the muons decayed at rest in the detector and the direction of the decay positron and the time of the decay were recorded. The detector itself consisted of 12 rectangular slabs of plastic scintillator, 24 in. $\times$ 36 in. $\times$ $\frac{1}{4}$ in. alternating with 12 slabs of aluminum, 24 in. $\times$ 36 in. $\times$ 4 in. Each scintillator was viewed by two RCA

tion. To find the total upward-escape probability, n_u , this spectrum must be weighted by the escape probability $W_u(X, \cos\theta, Z)$ for an electron of energy X originating at a depth Z with angle θ to the slab normal and then integrated over these variables. The result is

$$n_u = \int dZ d\cos\theta dX W_u(X, \cos\theta, Z) \rho(X, \cos\theta). \quad (2)$$

This integral can be performed analytically in a "straight-line" approximation, where it is assumed that the electrons travel a distance proportional to their energy in a straight path. Then the factor $W_u(X, \cos\theta, Z)$ of Eq. (2) is unity whenever $r(X) \geq Z/\cos\theta$ and is zero otherwise. However, for thick slabs, multiple scattering of the decay electron can significantly smear its angular distribution so that the resulting asymmetry is less than that predicted by the straight-line approximation. Accordingly, a Monte Carlo calculation was made which incorporated the effects of multiple scattering and bremsstrahlung on the electron trajectory.

To obtain an absolute measurement of the polarization, it is necessary to know the up-down asymmetry for a given polarization in the geometry actually used. This information can be obtained either from the evaluation of the integral in Eq. (2) or from our calibration experiment with muons of known polarization. Both methods were used, but the calibration experiment is the more reliable means of determining the absolute polarization. Additional details of the calibration are discussed in the following section.

RESULTS AND ANALYSIS

In calculating the polarization from the experimental data, the forms

$$U(t) = \left[A_U + BP_0 \frac{\sin ft}{ft} \cos \omega_L(t + t_0) \right] \times e^{-t/\tau_+} + C_U + Fe^{-t/\tau_-}, \quad (3)$$

$$D(t) = \left[A_D - BP_0 \frac{\sin ft}{ft} \cos \omega_L(t + t_0) \right] \times e^{-t/\tau_+} + C_D + Fe^{-t/\tau_-}, \quad (4)$$

were fitted to the up and down time distributions of the decay electrons. These forms allow for several backgrounds and corrections. We assume that the oscillatory amplitudes B are the same for up and down distributions, but allow A_U and A_D to differ. The reason is that muons stopping in an aluminum slab in the detector cannot be distinguished from those stopping in the scintillator immediately above the slab. When these latter muons decay, they simulate up decays from the aluminum. Assuming 100% detection efficiency for these elec-

trons, the expected ratio A_U/A_D is 1.23; the measured ratio is 1.40 ± 0.03 . To circumvent this problem, BP_0/A_D is regarded as the best estimate of the true asymmetry. This does not mean a loss in statistics by using only down decays to determine the polarization. On the contrary, up and down decays were simultaneously fitted to the functions $U(t)$ and $D(t)$ by the criteria of least squares, and the fractional error in B is much larger than that of A_D .

The factor $(\sin ft)/ft$ allows for depolarization of the muons. A small depolarization is expected as a result of macroscopic magnetic field nonuniformities. The reason for expressing the depolarization in this form is that the polarization must be averaged over a frequency spectrum of width $\Delta\omega$, corresponding to the variation of the magnetic field. At a given frequency ω the polarization is $P(t) = P_0 \cos \omega t$. Averaging over a uniform distribution of frequencies in the interval $\Delta\omega$ gives the factor $(\sin ft)/ft$, where $f = \frac{1}{2}\Delta\omega$. The best fit to the data was obtained with $\Delta\omega/\omega = (24 \pm 8)\%$. This is to be compared with the calculated variation in the magnetic field of about 15%. It should also be noted that positive muons are not depolarized by the microscopic magnetic field in aluminum.¹¹

The terms C_U and C_D allow for time-independent backgrounds such as photomultiplier noise and room background. The ratios C_U/A_U and C_D/A_D were typically 0.006 and 0.012. Finally, a negative-muon background term of the form Fe^{-t/τ_-} is included. The negative-muon lifetime in aluminum is about 0.865 μsec , so that this background is distinguishable from the positive muon signal. The negative muons enter atomic orbits and are rapidly depolarized after stopping; in addition, about 60% of the negative muons are captured by the nucleus before decaying, so we assume no asymmetry in this term. Negative muons stopping in the scintillator have a lifetime close to τ_+ and should be adequately represented by the term containing A_U .

A time delay t_0 is included to account for the delay in the electronics, and a value of 100 nsec was found by measurement; this is consistent with the delay found in the fit to the data.

As an additional check on the consistency of the analysis the positive muon lifetime was allowed to vary. The analysis program determined a best value of $2.28 \pm 0.16 \mu\text{sec}$, which is consistent with the accepted value¹² of $2.1983 \pm 0.0008 \mu\text{sec}$.

The character of the data is suggested by the graphs of Fig. 2, which show the values of the ratio $R = (U - D)/(U + D)$ as a function of time for the data taken near 3.5 GeV. The data is separated into bins 0.3 μsec wide and the errors represent statistical standard deviations. For comparison, an undamped sine wave of the same period, 2.57

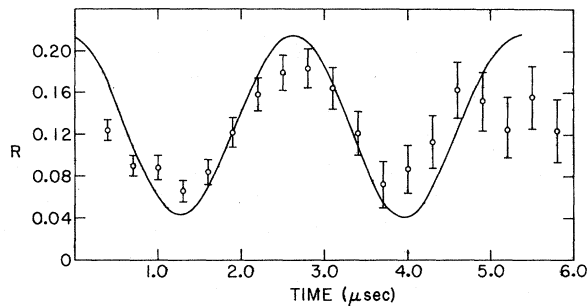


FIG. 2. The measured asymmetry in the distribution of the decay positrons, $R(t) = [U(t) - D(t)] / [U(t) + D(t)]$, where $U(t)$ and $D(t)$ are the number of positrons observed in the up and down directions relative to the direction of the incident muons, is shown as a function of time for the data near 3.5 GeV. For comparison, an undamped cosine curve of the same frequency is plotted. The amplitude of data oscillation is less than that of the curve since the polarization is damped due to nonuniformities in the precession field.

μsec , and amplitude is indicated. The observed damping of the sine wave is attributed to the variation in the magnetic field over the detector and the offset of the ratio from the line $R = 0$ is consistent with the effect to be expected from muons which stop in the scintillator and are then registered as decaying in a backwards direction according to our method of analysis.

In order to determine the absolute magnitude of the polarization from the measured asymmetry, the ratio B/A_D must be determined for a known polarization. This ratio was determined in the calibration experiment at the Brookhaven AGS where the up-down asymmetry was measured for a beam of muons of known polarization. The calibration experiment gave $B/A_D = 0.228 \pm 0.023$. In both the present experiment and in the calibration experiment, comparable thicknesses of steel were used to degrade the energy of the muon before it entered the detector. Therefore, the depolarization due to multiple Coulomb scattering in the degrader and in the aluminum absorber should be the same in both experiments. Moreover, such effects as energy loss due to bremsstrahlung, pair production, and the multiple scattering of the positrons, as well as certain asymmetries in the electronics, are corrected for by the calibration. All of these effects tend to reduce the asymmetry.

The polarization of positive muons was measured at three energies, corresponding to three thicknesses of steel degrader, 440, 1839, and 2575 g/cm². The corresponding numbers of stopped muons observed at each energy are 119 040, 124 149, and 101 376. Measurements at the two lowest energies were taken at zenith angles of 0° and the highest-energy measurement was at 45°.

The final polarizations with statistical errors are indicated in Table I, where each measurement is represented as an average over an interval of about 0.5 GeV, the energy acceptance of the detector. Also shown are the ratios of the measured χ^2 to the expected χ^2 , the number of experimental time bins minus the number of parameters in the fitting function, for the three fits to the experimental data.

The two assumptions, that the muons come entirely from pions or entirely from K mesons, give two quite different limits on the muon polarization. These limits may then be used with the measured polarization to discuss the K/π ratio of the parent mesons. The polarization expected on the assumption that the muons are produced entirely by pions has been calculated by Hayakawa⁸ to be about 0.27. In a more recent article by Osborne,¹³ the polarization of muons, where only pion decays are involved, has been calculated to be 0.33, with a small error resulting from the limitations on our knowledge of the energy dependence of the muon spectrum while the polarization of the muons would be 0.54 if only K -meson decays contributed to the muon flux. The value of the polarization, if the origin were solely K mesons, is derived assuming that the neutral K flux is equal to the charged flux and all decay modes which eventually produce a muon are considered. The results of the present experiment are shown as a function of sea-level energy in Fig. 3, where some of the measurements summarized by Osborne are indicated along with the limits imposed by kaon or pion production of the muons. Our polarization measurements, averaged over energy, give a K/π ratio of 0.095 ± 0.11 for the parent mesons if the polarization varies linearly with the K/π ratio. While this will be the case only if the ratio of K^+ to K^- mesons is the same as the ratio of π^+ to π^- mesons, the number so derived does suggest the magnitude of K -meson contribution though this restriction be invalid.

The results of other measurements are shown in Fig. 3. The underground polarization measurements of Bradt and Clark² cover the widest energy range and have relatively small errors. Their measurements indicate no variation in the muon polarization with muon energy in the region 1–10 GeV, but the absolute values of polarization which

TABLE I. The final polarization with statistical errors.

Energy (GeV)	Polarization	$\chi^2/\chi^2_{\text{ex}}$
0.75–1.17	-0.33 ± 0.04	1.00
3.13–3.61	-0.38 ± 0.04	0.92
4.41–4.95	-0.34 ± 0.04	1.00

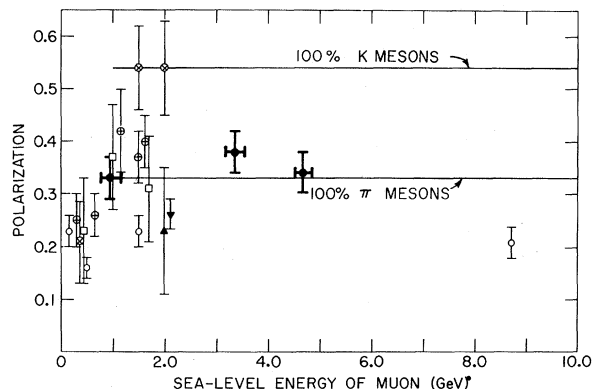


FIG. 3. The polarizations measured in the present experiment are shown as a function of energy. The results of other experiments are shown along with the limits imposed by K - and π -meson production of the muons: Ref. 2, $\circ$; Ref. 3, $\square$; Ref. 4, $\blacktriangle$; Ref. 5, $\blacktriangledown$; Ref. 6, $\oplus$; Ref. 7, $\otimes$; present experiment, $\bullet$.

they report may not be as reliable as their relative values since they did not calibrate their apparatus with a beam of muons of known polarization.

CONCLUSIONS

The results of the measurements suggest that the polarization of cosmic-ray muons is approximately constant over the range of energies, measured at sea level, of from 1–5 GeV. Moreover, it would seem that there is no large variation with zenith angle. Furthermore, the values of polarization which are measured are consistent with the hypothesis that the muons are derived almost wholly from the decay of pions produced by the primary and secondary interactions of primary cosmic-ray nucleons with nuclei in the upper atmosphere. Certainly there is no evidence of any anomalous effects such as were suggested by some of the previous measurements.

It is interesting to examine this information in terms of the character of the production of mesons in high-energy nucleon-nucleon interactions. Since the muons lose energy in passing through the atmosphere, the energy of the muons at production will be somewhat higher than the energy at sea level. We can express the initial muon energy E_i adequately using a relation

$$E_i = E_s + (2.3 \text{ GeV}) \sec \theta,$$

where E_s is the muon energy at sea level, θ is the angle of incidence measured from the zenith, and the energy loss through 1 atmosphere is about 2.3 GeV. Using this relation, we conclude that we have examined the polarization of muons with initial energies of up to 8 GeV derived from the decay of pions of mean energy at production of about

10 GeV.

While we have an adequate knowledge of the energy spectrum of the primary nucleons, we cannot determine the effective nucleon energy for the interactions which produce 10-GeV pions without a better knowledge of meson production processes than we now possess. Nevertheless, it is useful to make an estimate of the effective nucleon energy. We take the multiplicity of mesons produced by nucleon-nucleon interactions as¹⁴

$$N = 1.5 \ln(s/M_p^2) = 1.5 \ln(2E_p/M_p),$$

where s is the square of the center-of-mass energy and M_p is the proton mass. This relation appears to fit the knowledge we have concerning multiplicities at high energy. The nucleon inelasticity is known to be about 0.40 so we can assume that the mesons take up about 40% of the energy, E_p , of the incident proton and we find that the average energy of a meson produced in a high-energy nucleon-nucleon collision is

$$E_{av} = 0.4 E_p / [1.5 \ln(2E_p/M_p)].$$

Taking E_{av} as 10 GeV, we find that the muons which we measure are derived from pions produced in the interactions of 250-GeV nucleons with the nucleons in the atmospheric nuclei. Of course, the spectrum of contributions to the production which we observe is very broad, extending to very high energies, and the estimate made here is presented only to suggest the range of incident nucleon energies involved.

Our results concerning the K/π ratio of the parents of the muons which we observe cannot be equated with the K/π ratio at production. It is probably best to discuss the relation between production ratios and the ratio relevant to the parentage of the muons which are observed, by considering a simple example rather than an analysis of a more complex general situation. Let us assume that the production mechanism was such that the pions and K mesons were produced with equal intensities and equal energy distributions in the laboratory system, that is, the K/π ratio at production, in every range of laboratory energies, was equal to unity. Since the muons take off a larger fraction of the pion energy than of the K -meson energy upon decay of the meson, the K/π ratio of the parents of the muons lying in some energy interval will not necessarily be the same as the production ratio. Indeed, if we use the observed muon spectrum to fix the form of the meson production spectrum in the laboratory system as

$$\frac{dN}{dE} = A E^{-a},$$

where E is the laboratory energy of the mesons and A and a are constants which have the same values

for pion and K -meson production, we can estimate the K/π ratio of the parents of the muons observed in any definite energy range as

$$K/\pi = (R'/R'')^{a-1},$$

where R' and R'' are the ratios of the mean muon energy to meson energy for the K meson and pion, respectively. Since the ratio R'/R'' is about equal to $\frac{1}{2}$, and since $a-1$ is about equal to 1.67, we find that a K/π ratio of unity, upon production, would lead to a ratio of about 0.31 for the parents of the muons which are detected in a specific energy range. We have implicitly assumed here that the ratio we considered was the ratio of all K mesons to charged pions. From charge independence, we know that about one-half as many neutral pions are produced as charged pions and the K/π ratio for all K mesons and all pions would be 0.46. Using this result, our measured ratio of 0.095 ± 0.12 for the parents of the muons which we observe, suggests a K/π ratio at production of about 0.20 ± 0.25 . We recognize, however, the large uncertainties which follow from the physical assumptions it was necessary to make.

However, it should be noted that the K/π ratio which we deduce is consistent with that observed in accelerator experiments. In an experiment by Baker *et al.*¹⁵ at the Brookhaven AGS, 10–30-GeV protons were incident on Al and Be targets and the K^+/π^+ , K^-/π^- ratios were observed at various pro-

duction angles. Their results, averaged over all production angles, give a K^+/π^+ ratio of $\sim 15\%$. However, for cosmic rays the energy of the primary proton is not well defined as in accelerator experiments, and the K/π ratio which we measure is in a sense an average over a wide spectrum of primary energies. Given the energies of the parent pions and kaons, the calculation of the primary energies is model-dependent. For example, if we consider the "pionization-isobar" model of Pal and Peters,¹⁶ there may be two K/π ratios, one associated with the pionization component and one associated with the decay of the isobar component; these two components vary in different ways with the primary energy. Koshiba¹⁷ claims to have observed two such K/π ratios in a study of primary interactions in nuclear emulsion. However, our experiment determines only an average property of cosmic rays.

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