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Two-Body Weak Reactions of Hadrons at Very High Energies*

Mark Dubovoy, Paul Langacker,† and Mahiko Suzuki
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California 94720
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Two-body hadronic scattering through weak interactions is investigated theoretically in the energy region accessible at the National Accelerator Laboratory. In the ordinary current ×current form of weak interactions, the following quantities are calculated for charge-exchange weak hadronic reactions: (i) the differential cross section, the polarization of a final baryon, and the asymmetry in the angular distribution of scattering from a polarized target for strangeness-changing processes, and (ii) the interference of weak-interaction amplitudes with strong-interaction amplitudes for processes to which both weak and strong interactions contribute. The interference with strong amplitudes is discussed by means of a straight extrapolation of existing Regge analysis at lower energies (less than 30 GeV). Differential cross sections near the forward direction are typically of the order of 10^{-38} cm^2 in $d\sigma/d\cos\theta$ in the center-of-mass frame. The interference between weak and strong amplitudes is unfortunately only one part in 10^4–10^5 for the πN charge-exchange scattering in spite of the different high-energy behavior. All calculations are supposed to be valid at high energies up to absorption corrections due to a Regge cut generated by the Pomeranchukon and a fixed pole at $J=1$ from the weak amplitude. Inelastic scattering is also briefly mentioned. Charge-nonexchange processes are discussed in connection with a test for the existence of neutral weak currents.

I. INTRODUCTION

The structure of weak interactions is fairly well known in the low-momentum-transfer region through semileptonic and purely leptonic decay processes.[1] In contrast, purely hadronic weak interactions are less understood, partly because of complications caused by strong interactions. For instance, neutral weak currents have frequently been discussed in connection with the $\Delta I=\frac{1}{2}$ rule, CP violation, and so forth; while their absence is now established to a very good accuracy in semileptonic and leptonic interactions, little can be said about purely hadronic processes.

A cutoff momentum characteristic of weak interactions has been extensively discussed from various viewpoints, recently by means of current algebras.[2-4] Some current-algebra calculations indicate a surprisingly low value of the cutoff momentum as compared with a unitarity cutoff (~300 GeV), although they are partly based on a somewhat controversial technique. It is very interesting to see how our weak-interaction theory, which has successfully described low-energy decay phenomena aside from CP-violating phenomena, must be modified at a short distance. Reactions induced by neutrinos have been thoroughly discussed,[5] since one can develop fairly straightforward arguments similar to those used for electroproduction as measured at SLAC. However, purely hadronic interactions have seldom been considered seriously. The purpose of this paper is to report results of our calculation of various quantities which may possibly be measured in future experiments on purely hadronic weak interactions at high energies.

We shall first consider two-body reactions. The current×current theory of weak interactions gives rise to a fixed pole at $J=1$ in the t-channel complex angular momentum plane.[6] The fixed pole appears in the t channels for reactions with $I=1$, $S=0$, and $|Q|=1$ and with $I=\frac{1}{2}$, $|S|=1$, and $|Q|=1$. We shall not take into account higher orders of weak interactions so that the fixed pole is not Reggeized. At high energies the largest correction to this fixed-pole exchange is a contribution of a Regge cut induced by two-Reggeon exchange of the Pomeranchukon and the fixed pole. Although there is no reliable way to estimate the magnitude of a discontinuity across a cut, it is usually believed that the discontinuity is small enough (of the order of $\frac{1}{10}$ as compared with a pole) to be dominated by the pole term near the forward direction.[7] We may, therefore, safely neglect the cut as well as lower J-plane singularities, since we are interested in incident energies of greater than 100 GeV. Based on numerical results obtained for two-body reactions, we shall develop qualitative arguments on the effects of parity violation in deep-inelastic particle production.

In Sec. II, we shall summarize the conventional current×current picture of weak interactions on which our calculation is based. In Sec. III, we calculate for strangeness-changing processes the differential cross section near the forward direction, the polarization of a final baryon in scattering from an unpolarized target, and the asymmetry in the angular distribution of scattering from a polarized target. We consider baryon-baryon scattering as well as meson-baryon scattering. In Sec. IV, we consider a class of two-body processes in which both weak- and strong-interaction amplitudes exist and interfere. By extrapolating an existing Regge analysis of high-energy scattering, we estimate interference effects between them. Then in Sec. V we discuss the possibility of detecting weak interactions in inelastic hadron production processes. A brief argument is made in Sec. VI on charge-nonexchange weak-interaction processes. The possibility of a test of neutral weak currents in hadronic processes is considered. In Sec. VII the results of our calculation are summarized and the feasibility of experiments is considered.

II. THEORY OF WEAK INTERACTIONS AND REGGE PICTURE OF HIGH-ENERGY REACTIONS

A. Current × Current Theory

Let us summarize the current×current theory of weak interactions.[1] A weak-interaction Hamiltonian is given as

$$H_W = g(J_\mu^\dagger + j_\mu^\dagger)W_\mu + \text{H.c.} \tag{2.1}$$

in a weak-boson version, and

$$H_W = (G/\sqrt{2})(J_\mu^\dagger + j_\mu^\dagger)(J_\mu + j_\mu) + \text{H.c.} \tag{2.2}$$

in a contact-interaction version. The difference between these interactions consists only of an additional weak-boson propagator. The present experimental lower bound on the mass of the weak boson, if any, is 2 GeV,[8-10] so in the momentum-transfer region of interest ($\lesssim 1$ GeV), they lead us to practically identical results. We shall, therefore, not distinguish between them in the present paper. One may say, in other words, that there is little chance in *purely hadronic two-body reactions* to determine the mass of the weak boson or even to explore its existence. The two currents J_μ and j_μ stand for hadronic and leptonic currents, respectively, and the hadronic current J_μ is written in terms of vector and axial-vector unitary-spin currents $\mathfrak{F}_{i\mu}$ and $\mathfrak{F}_{j\mu}^5$ $(i,j = 1, 2, \ldots, 8)$ as

$$J_\mu = (\mathfrak{F}_{1\mu} + i\mathfrak{F}_{2\mu})\cos\theta_V + (\mathfrak{F}_{1\mu}^5 + i\mathfrak{F}_{2\mu}^5)\cos\theta_A + (\mathfrak{F}_{4\mu} + i\mathfrak{F}_{5\mu})\sin\theta_V + (\mathfrak{F}_{4\mu}^5 + i\mathfrak{F}_{5\mu}^5)\sin\theta_A, \tag{2.3}$$

where θ_V and θ_A are vector and axial-vector Cabibbo angles, respectively. Although the assumption $\theta_V = \theta_A$ is very attractive, an analysis of semileptonic weak decays indicates[11] that

$$\sin^2\theta_V = 0.0442 \pm 0.0015, \tag{2.4}$$

$$\sin^2\theta_A = 0.0700 \pm 0.0007. \tag{2.5}$$

We do not distinguish between meson and baryon Cabibbo angles. The weak-coupling constant G is known to be

$$Gm_p^2 = 1.023 \times 10^{-5}, \tag{2.6}$$

m_p being the proton mass.

The matrix elements of the SU(3)×SU(3) currents taken between elements of the baryon octet are

$$\langle j | \mathfrak{F}_{i\mu}(0) | k \rangle = \left(\frac{m_j m_k}{E_j E_k}\right)^{1/2} \bar{u}^j(p') \left\{ i f_{ijk} i\gamma_\mu F_1(t) + [\alpha_M d_{ijk} + (1-\alpha_M) i f_{ijk}] \frac{\mu_n - \mu_p}{2M_N} i\sigma_{\mu\nu} q_\nu F_2(t) \right\} u^k(p), \tag{2.7}$$

$$\langle j | \mathfrak{F}_{i\mu}^5(0) | k \rangle = \left(\frac{m_j m_k}{E_j E_k}\right)^{1/2} \bar{u}^j(p') g_A [\alpha d_{ijk} + (1-\alpha) i f_{ijk}][i\gamma_\mu\gamma_5 F_A(t) + \gamma_5 q_\mu F_P(t)] u^k(p), \tag{2.8}$$

where

$$
\begin{aligned}
&q_\mu = p'_\mu - p_\mu, \quad t = -q^2,\\
&\alpha_M = \tfrac{3}{2}\mu_n(\mu_n - \mu_p)^{-1}, \quad \alpha = 0.665 \pm 0.018,\\
&\mu_p = 1.793, \quad \mu_n = -1.913,\\
&g_A = 1.18 \pm 0.025,
\end{aligned} \tag{2.9}
$$

and the form factors are normalized as

$$F_1(0) = F_2(0) = F_A(0) = 1. \tag{2.10}$$

$F_p(0)$ is determined through consideration of the hypothesis of partially conserved axial-vector current.

A few additional assumptions are implicitly made above: (i) SU(3)-breaking terms are all neglected. This will be a reasonable approximation when we are not interested in obtaining very accurate numbers. (ii) The γ_μ part of the vector current may contain a term proportional to d_{ijk} which vanishes at $q^2=0$. Any such term has been neglected. It will be absent if one-vector-meson dominance holds. This approximation will not lead us to a large error near $t(=-q^2)=0$ which is the region in which we are interested.

The matrix elements between the octet ($\frac{1}{2}^+$) and decuplet ($\frac{3}{2}^+$) baryons are written as[12]

$$
\begin{aligned}
\langle l|\mathfrak{F}_{i\mu}(0)|j\rangle = C(10,l|8,j;8,i)\left(\frac{m_l m_j}{E_l E_j}\right)^{1/2} G_V(t)&\\
\times\left\{H(t)\bar{u}^l_\mu(p')\gamma_5 u^j(p) + \left(\frac{p_\nu}{m_j}\right)\left[\left(\frac{p'_\mu}{m_l}\right)\bar{u}^l_\nu(p')\gamma_5 u^j(p) - i\bar{u}^l_\nu(p')\gamma_\mu\gamma_5 u^j(p)\right]\right\}&,
\end{aligned} \tag{2.11}
$$

$$\langle l|\mathfrak{F}^5_{i\mu}(0)|j\rangle = C(10,l|8,j;8,i)\left(\frac{m_l m_j}{E_l E_j}\right)^{1/2} G_A(t)\left[H(t)\bar{u}^l_\mu(p')u^j(p) + \frac{p_\nu p'_\mu}{m_j m_l}\bar{u}^l_\nu(p')u^j(p)\right], \tag{2.12}$$

where $C(10,l|8,j;8,i)$ is an SU(3) Clebsch-Gordan coefficient[13] of the 8-8-10 coupling, so normalized that $\sqrt{2}C(10,l|8,j;8,i)$ is unity for the $\Delta^{++}\to p\pi^+$ coupling; $u_\mu(p)$ is the Rarita-Schwinger spinor of a spin-$\frac{3}{2}$ particle; and

$$H(t) = \frac{(m_l+m_j)^2 - t}{2m_l m_j}. \tag{2.13}$$

The normalizations of $G_V(t)$ and $G_A(t)$ are fixed in the following way. The decay width of Δ^+(1236 MeV) into $p\gamma$ is known to be

$$\Gamma(\Delta^+\to p\gamma) = 0.65 \pm 0.02 \text{ MeV}. \tag{2.14}$$

Calculating the decay width by use of (2.11), one obtains

$$\Gamma(\Delta^+\to p\gamma) = \frac{(m_\Delta+m_N)^2 p^3}{9\pi m_\Delta^2 m_N^2} G_V{}^2(0), \tag{2.15}$$

from which $G_V(0)$ is determined[14] to be

$$G_V(0) = 3.53. \tag{2.16}$$

To fix the scale of $G_A(q^2)$, we resort to the partially conserved axial-vector relation

$$\phi_i(x) = (m_\pi^2 C_\pi)^{-1}\partial_\mu \mathfrak{F}^5_{i\mu}(x).$$

Through this relation, the coupling constant g for the decay of the decuplet into the octet baryon and the pion, defined through

$$C(10,l|8,j;8,i)g\psi^l_\mu\psi^j\partial_\mu\phi^i, \tag{2.17}$$

is related to $G_A(0)$ by

$$g = \frac{m_\Delta + m_N}{m_N C_\pi} G_A(0). \tag{2.18}$$

Using the value of g determined from the $\Delta^{++}\to p\pi$ decay width, one obtains[15]

$$G_A(0) = 1.13. \tag{2.19}$$

Although g is a coupling constant extrapolated off the pion mass shell to $t=0$, $G_A(0)$ would hardly be affected by this extrapolation.

We also need the matrix elements between octet pseudoscalar mesons. They are simply given as

$$
\begin{aligned}
&\langle j|\mathfrak{F}_{i\mu}(0)|k\rangle = (4\omega_j\omega_k)^{-1/2}\\
&\qquad\times[if_{ijk}(p'_\mu+p_\mu)f_+(t) + (p'_\mu - p_\mu)f_-(t)],\\
&\langle j|\mathfrak{F}^5_{i\mu}(0)|k\rangle = 0,
\end{aligned} \tag{2.20}
$$

where $f_+(0)=1$. The SU(3)-symmetry-breaking term $f_-(t)$ has been controversial in the K_{l3} decays, but fortunately we find by explicit calculation that $f_-(t)$ does not contribute to leading terms at high energies. Therefore, even if a future experiment concludes that the ξ parameter in K_{l3} is large, it does not affect our results.

Finally, we must give the t dependence of the form factors, $F_1(t)$, $F_2(t)$, $F_A(t)$, $G_V(t)$, $G_A(t)$, and $f_+(t)$ [$F_p(t)$ does not enter any leading terms]. The vector form factors are determined from the two electromagnetic form factors of the nucleon, $G_E(t)$ and $G_M(t)$, directly measured in experiment. We choose a fit

$$G_E(t)/G_E(0)=G_M(t)/G_M(0)$$
$$=[1-t/(0.70\ \mathrm{GeV}^2)]^{-2}, \qquad (2.21)$$

where $G_E(0)=1$, $G_M(0)=1+\mu_p$. F_1 and F_2 are related to G_E and G_M by

$$G_E(t)=F_1(t)+\frac{t\mu_p}{4m_N^2}F_2(t),$$
$$G_M(t)=F_1(t)+\mu_p F_2(t). \qquad (2.22)$$

For $F_A(t)$ we shall use a result from the theoretical analysis of electroproduction of pions,[16]

$$F_A(t)=[1-t/(1.34\ \mathrm{GeV})^2]^{-2}. \qquad (2.23)$$

There is no accurate measurement of $G_V(t)$ or $G_A(t)$ [and practically none for $G_A(t)$]. We simply choose

$$G_V(t)/G_V(0)=G_M(t)/G_M(0), \qquad (2.24)$$

$$G_A(t)/G_A(0)=F_A(t)/F_A(0). \qquad (2.25)$$

We may in principle determine $f_+(t)$ from the electromagnetic form factor of the pion and from the K_{l3} decay data, but these measurements are somewhat obscure. As a compromise, we choose

$$f_+(t)=F_1(t). \qquad (2.26)$$

B. Model of High-Energy Scattering

Since the weak currents are vector and axial-vector currents, the lowest-order weak interactions give rise to a fixed pole at $J=1$ in the complex angular momentum plane in the t channel. This is easy to understand in the W-meson picture since the W meson behaves like an elementary particle with spin one (not Reggeized) in the lowest order, but it is true also in the four-fermion interactions.

The weak currents contain charged currents only, so that this fixed pole probably dominates in the charge-exchange processes such as $\pi^-p\to\pi^0\Lambda$. There is a class of diagrams to which the fixed pole does not contribute at all; one example is a diagram in which purely strong interactions take place between two hadrons after or before one of the hadrons makes a strangeness-changing transition (drawn in Fig. 1). The weak processes in those diagrams are known from low-energy weak decays. For strangeness-changing processes, the leading trajectory for those diagrams is the ρ trajectory. The ρ Regge parameter $\alpha(t)$ is $\simeq 0.5$ at $t=0$, and its slope is $\simeq 1\ \mathrm{GeV}^{-2}$. The ratio of the energy dependence in the amplitude of these diagrams to that of fixed-pole diagrams is therefore $s^{\alpha(t)-1}$. A precise numerical factor in front of the s dependence is not easy to calculate unless one adopts some model. However, we can make an order-of-magnitude estimate of this class of diagrams from the specific diagrams drawn in Fig. 1. Parity-conserving pole transitions are calculated through soft-pion techniques. Their contributions to strangeness-changing amplitudes are typically

$$\sim 1\times 10^{-7} M_{\text{c.e. strong}}. \qquad (2.27)$$

This is a small correction ($\approx\frac{1}{10}$ above $\omega_{\text{lab}}\approx 100\ \mathrm{GeV}$) to the fixed-pole amplitude (see Sec. III).

We therefore take the model of fixed-pole dominance in this paper. The model is accurate enough at high energies, and the main corrections will come from absorption effects due to iteration of J-plane poles.

III. STRANGENESS-CHANGING TWO-BODY REACTIONS

A. Differential Cross Sections

We consider three classes of two-body reactions: $PB\to PB$, $BB\to BB$, and $BB\to BD$, where P is any member of the octet of the pseudoscalar mesons, B is a member of the baryon ($\frac{1}{2}^+$) octet, and D is a member of the baryon ($\frac{3}{2}^+$) decuplet.

First, let us define new form factors $\hat{F}_1(t)$, $\hat{F}_2(t)$, $\hat{F}_A(t)$, etc., which are equal to the corresponding form factors times the following: (i) all SU(3) Clebsch-Gordan coefficients, (ii) the appropriate Cabibbo angle, (iii) g_A for $F_A(t)$ and $F_P(t)$, and (iv) $(\mu_n-\mu_p)/2m_N$ for $F_2(t)$.

PB→PB Reaction

The scattering amplitude for $PB\to PB$ reactions is given in the fixed-pole-dominance approximation by

$$\langle p'q'|H_w|pq\rangle=(G/\sqrt{2})\langle p'|J_\mu^\dagger|p\rangle\langle q'|J_\mu|q\rangle$$

or $\qquad (3.1)$

$$(G/\sqrt{2})\langle p'|J_\mu|p\rangle\langle q'|J_\mu^\dagger|q\rangle,$$

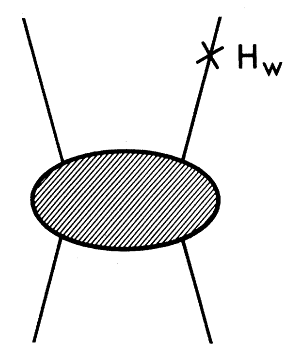

FIG. 1. A diagram in which the weak transition takes place on one of the external lines.

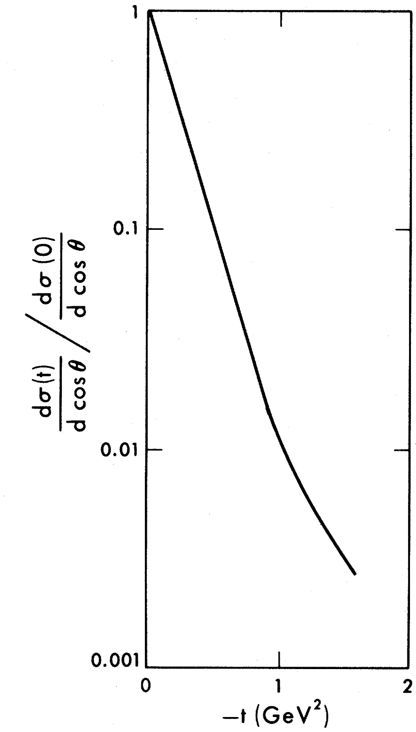

FIG. 2. Momentum-transfer dependence of $d\sigma(t)/d\cos\theta$ in the center-of-mass frame. The dependence is almost identical for all processes (up to 20% for large $-t$) within the range of momentum transfer plotted here.

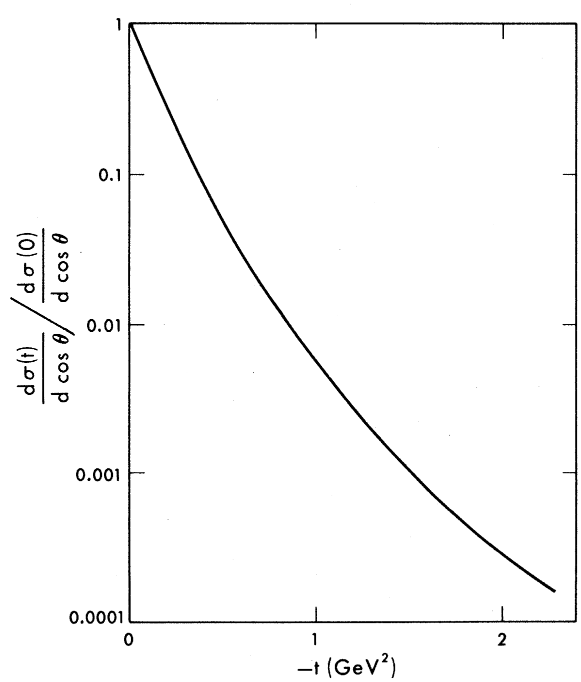

FIG. 3. Momentum-transfer dependence of $d\sigma(t)/d\cos\theta$ for $pn \to \Lambda p$. A curve for $pn \to \Sigma^0 p$ is almost identical.

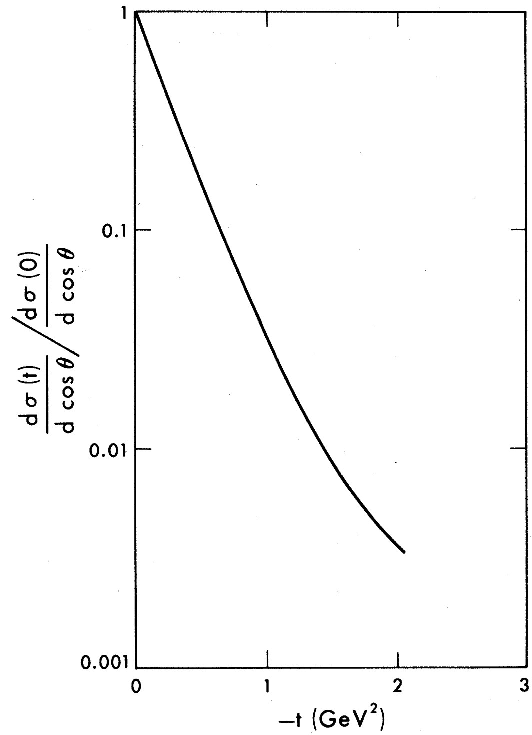

FIG. 4. Momentum-transfer dependence of $d\sigma(t)/d\cos\theta$ for $pp \to \Delta^{++}\Lambda$. A curve for $pp \to \Delta^{++}\Sigma^0$ is almost identical.

where p and p' are the baryon momenta and q and q' are the meson momenta in the center-of-mass system.

The absolute squares of these quantities may be evaluated using (2.3), (2.7), (2.8), and (2.20). In the limit $s \gg -t, m_N^2, \mu^2$ (s and t are the usual Mandelstam variables, m_N is the baryon mass, μ is the meson mass[17]), the differential cross section is[18]

$$\frac{d\sigma}{d\cos\theta} = \frac{s}{16\pi} G^2 \hat{f}_+(t)^2 \{[\hat{F}_1(t)]^2 - t[\hat{F}_2(t)]^2 + [\hat{F}_A(t)]^2\}, \tag{3.2}$$

where θ is the center-of-mass scattering angle. $\hat{f}_-(t)$ and $\hat{F}_P(t)$ do not appear.

We will now consider the reactions (a) $\pi^- p \to \pi^0 \Lambda$, (b) $\pi^- p \to \pi^0 \Sigma^0$, and (c) $\pi^- p \to K^0 n$. Cross sections for other reactions are related by

$$\begin{aligned}
&\sigma(\pi^- n \to \pi^0 \Sigma^-) = 2\sigma(\text{b}),\\
&\sigma(\pi^+ n \to \bar{K}^0 p) = \sigma(\text{c}),\\
&\sigma(K^- p \to \bar{K}^0 \Lambda) = \tfrac{1}{2}\sigma(\text{a}),\\
&\sigma(K^- p \to \bar{K}^0 \Sigma^0) = \tfrac{1}{2}\sigma(\text{b}),\\
&\sigma(K^- p \to \pi^0 n) = \tfrac{1}{2}\sigma(\text{c}),\\
&\sigma(K^+ n \to \pi^0 p) = \tfrac{1}{2}\sigma(\text{c}).
\end{aligned} \tag{3.3}$$

The center-of-mass differential cross sections for (a), (b), and (c) at $t=0$, $s=100$ GeV^2 are: (a) 2.2×10^{-38} cm^2, (b) 0.53×10^{-38} cm^2, and (c) 1.1 $\times10^{-38}$ cm^2. For other energies, multiply these numbers by $s=2m_N\omega_{\rm lab}$, expressed in units of 100 GeV^2, where $\omega_{\rm lab}$ is the meson lab energy.

The t dependence of the cross sections is shown in Fig. 2. The very rapid falloff is due to the damping of the form factors.

$BB\to BB$ Reactions

For reactions of the type $BB\to BB$, let us label the initial baryons as a and b, and the final baryons as c and d. Then, for example,

$$\langle cd|H_w|ab\rangle=(G/\sqrt{2})\langle d|J_\mu^\dagger|b\rangle\langle c|J_\mu|a\rangle. \quad (3.4)$$

If a and b or c and d are identical, we should, in principle, add interfering amplitudes such as

$$(G/\sqrt{2})\langle c|J_\mu^\dagger|b\rangle\langle d|J_\mu|a\rangle. \quad (3.5)$$

However, these extra terms, which become large near backward scattering, are negligible because of the damping of the form factors. When we write the form factors of $\langle c|J_\mu|a\rangle$ as $\hat F_1(t)$, $\hat F_2(t)$, $\hat F_A(t)$, and $\hat F_P(t)$, and those of $\langle d|J_\mu^\dagger|b\rangle$ as $\hat g_1(t)$, $\hat g_2(t)$, $\hat g_A(t)$, and $\hat g_P(t)$, the differential cross section is given in the high-energy limit as

$$\frac{d\sigma}{d\cos\theta}=\frac{s}{16\pi}G^2\{[\hat g_1(t)]^2-t[\hat g_2(t)]^2+[\hat g_A(t)]^2\}$$
$$\times\{[\hat F_1(t)]^2-t[\hat F_2(t)]^2+[\hat F_A(t)]^2\}. \quad (3.6)$$

Again, neither $\hat F_P$ nor $\hat g_P$ enters.

The only reactions which might be measured experimentally are (a) $pn\to\Lambda p$ and (b) $pn\to\Sigma^0 p$. At $s=100$ GeV^2 the forward cross sections are (a) 2.6 $\times10^{-38}$ cm^2 and (b) 0.63×10^{-38} cm^2. The t dependence is shown in Fig. 3. These reactions have the obvious advantage over the meson-baryon reactions of utilizing the primary proton beam. Also, the cross section does not decrease as fast at large $-t$. This is because $[F_A(t)]^2$, which appears quadratically in this cross section, falls relatively slowly compared to $[F_1(t)]^2\approx[f_+(t)]^2$ and $-t[F(t)]^2$. Of course, it is a major disadvantage to require a neutron for the target particle, because a neutron in any nucleus, even the deuteron, has a large momentum distribution which complicates the identification of an event.

$BB\to BD$ Reactions

Finally, for $BB\to BD$ reactions, we have

$$\frac{d\sigma}{d\cos\theta}=\frac{s}{96\pi}G^2\frac{(m_N+m_D)^2-t}{(m_Nm_D)^2}\{[\hat F_1(t)]^2-t[\hat F_2(t)]^2+[\hat F_A(t)]^2\}\{[\hat G_A(t)]^2[(m_N+m_D)^2-t]-t[\hat G_V(t)]^2\}, \quad (3.7)$$

where m_D is the mass of the decuplet baryon. The $\hat F$ form factors refer to the $\langle B|J|B\rangle$ current and the $\hat G$'s to $\langle D|J|B\rangle$. For $pp\to\Lambda\Delta^{++}$ at $s=100$ GeV^2, $t=0$, $d\sigma/d\cos\theta=1.8\times10^{-38}$ cm^2 while for $pp\to\Sigma^0\Delta^{++}$, $d\sigma/d\cos\theta=0.43\times10^{-38}$ cm^2. The t dependence is shown in Fig. 4.

B. Polarized-Target Experiments

In reactions of the type $PB\to PB$, if the target baryon is a proton polarized at a lab angle $\pi-\chi$ to the beam direction [see Fig. 5(a)], then the center-of-mass cross section is

$$\frac{d\sigma}{d\cos\theta}=(s/16\pi)G^2[\hat f_+(t)]^2\{[\hat F_1(t)]^2-t[\hat F_2(t)]^2+[\hat F_A(t)]^2\}$$
$$-P(s/8\pi)G^2[\hat f_+(t)]^2\hat F_1(t)\hat F_A(t)\cos\chi-P(s/8\pi)\sqrt{-t}\,\cos\varphi\, G^2[\hat f_+(t)]^2\hat F_2(t)\hat F_A(t)\sin\chi. \quad (3.8)$$

Here, P is the degree of target polarization and φ is the azimuthal angle of the final meson relative to the x-z plane. For $\sin\chi=1$ we have an up-down asymmetry when $\hat s$ is in the scattering plane [Fig. 5(b)]:

$$\frac{1}{P}\left(\frac{d\sigma}{d\cos\theta}(\cos\varphi=1)-\frac{d\sigma}{d\cos\theta}(\cos\varphi=-1)\right)\Big/\left(\frac{d\sigma}{d\cos\theta}(\cos\varphi=1)+\frac{d\sigma}{d\cos\theta}(\cos\varphi=-1)\right)$$
$$\simeq\frac{1}{P}\left(\frac{d\sigma}{d\cos\theta}(\cos\varphi=1)-\frac{d\sigma}{d\cos\theta}(\cos\varphi=-1)\right)\Big/2\left(\frac{d\sigma}{d\cos\theta}\right)_{\rm unpol}$$
$$=\frac{-2\sqrt{-t}\,\hat F_2(t)\hat F_A(t)}{[\hat F_1(t)]^2-t[\hat F_2(t)]^2+[\hat F_A(t)]^2}, \quad (3.9)$$

where $(d\sigma/d\cos\theta)_{\rm unpol}$ is the unpolarized cross section given in (3.2). This unfamiliar asymmetry is allowed because parity is not conserved.

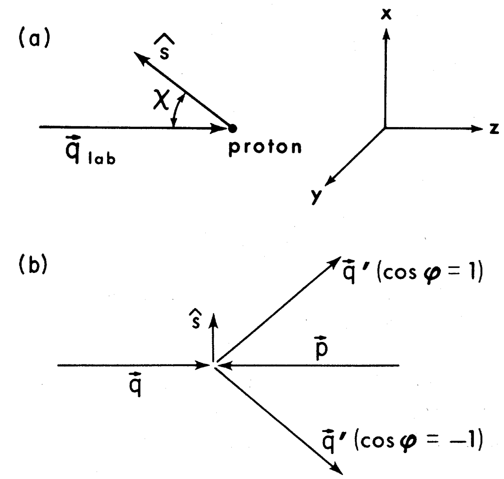

FIG. 5. (a) Definition of χ. $\hat{s}$ is a unit vector along the direction of polarization, and $\vec{q}_{\rm lab}$ is the laboratory momentum of the incident meson. The polarization is in the x-z plane. (b) Up-down asymmetry of the final meson. $\hat{s}$ is a unit vector perpendicular to the beam in both the lab and the c.m. frames.

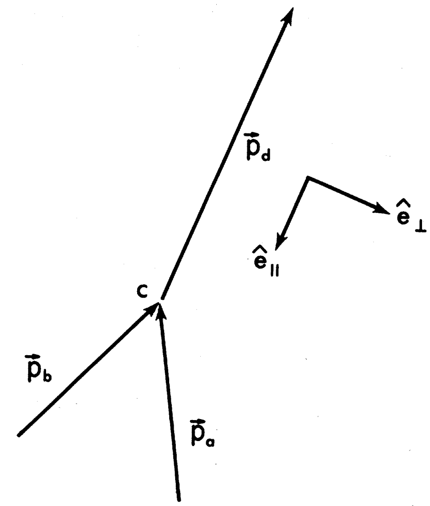

FIG. 6. Definition of polarization directions for a final baryon C in its own rest frame. $\hat{e}_\perp$ is not changed when transforming back to the c.m. frame; there, $\hat{e}_\perp = (\vec{p}_a \times \vec{p}_c) \times \vec{p}_c / |(\vec{p}_a \times \vec{p}_c) \times \vec{p}_c|$.

The second term in (3.8) leads to a change in the cross section when the polarization is changed from antiparallel to the beam to parallel:

$$\frac{1}{P}\left(\frac{d\sigma}{d\cos\theta}(\cos\chi = 1) - \frac{d\sigma}{d\cos\theta}(\cos\chi = -1)\right) \Big/ 2\left(\frac{d\sigma}{d\cos\theta}\right)_{\rm unpol} = \frac{-2\hat{F}_1(t)\hat{F}_A(t)}{[\hat{F}_1(t)]^2 - t[\hat{F}_2(t)]^2 + [\hat{F}_A(t)]^2}\,. \tag{3.10}$$

This asymmetry (which, incidentally, does not vanish when $t=0$) is another manifestation of the parity violation of weak interactions.

The left-right asymmetry for $\hat{s}$ perpendicular to the scattering plane familiar in strong processes (the only asymmetry allowed when parity is conserved) is absent here because of time-reversal invariance and Hermiticity (i.e., the transition operator is Hermitian to lowest order).

The same considerations apply to $BB \to BB$ reactions. If particle a is a polarized target, the differential cross section becomes

$$\frac{d\sigma}{d\cos\theta} = \left(\frac{d\sigma}{d\cos\theta}\right)_{\rm unpol} - P\left(\frac{s}{8\pi}\right) G^2 \{[\hat{g}_1(t)]^2 - t[\hat{g}_2(t)]^2 + [\hat{g}_A(t)]^2\} \hat{F}_A(t)[\hat{F}_1(t)\cos\chi + \sqrt{-t}\cos\varphi\, \hat{F}_2(t)\sin\chi]\,. \tag{3.11}$$

The expressions for the asymmetries and their interpretations are identical to those for $PB \to PB$ scattering.

Finally, consider $BB \to BD$. If the incident baryon is connected by one current to the final decuplet particle, and the target baryon is coupled to the final octet baryon, the asymmetries are again given by (3.9) and (3.10), where the form factors are those of the baryon-baryon current. The modification of the cross section (3.7) should now be obvious. The cross section has not been calculated for the case of a polarized target proton coupled to the final decuplet. The magnitudes of (3.9) and (3.10) will be discussed in the next section.

Polarization

We have also calculated the polarization of a final baryon when the target is unpolarized. It is well known that when parity is conserved and time-reversal invariance holds, a final baryon can only be polarized perpendicular to the scattering plane when the initial baryon is unpolarized; furthermore, the degree of polarization is equal to the left-right asymmetry that would be produced in the same reaction if the initial baryon were 100% polarized perpendicular to the scattering plane.

No corresponding theorem holds in weak scattering processes. However, we have found by explicit cal-

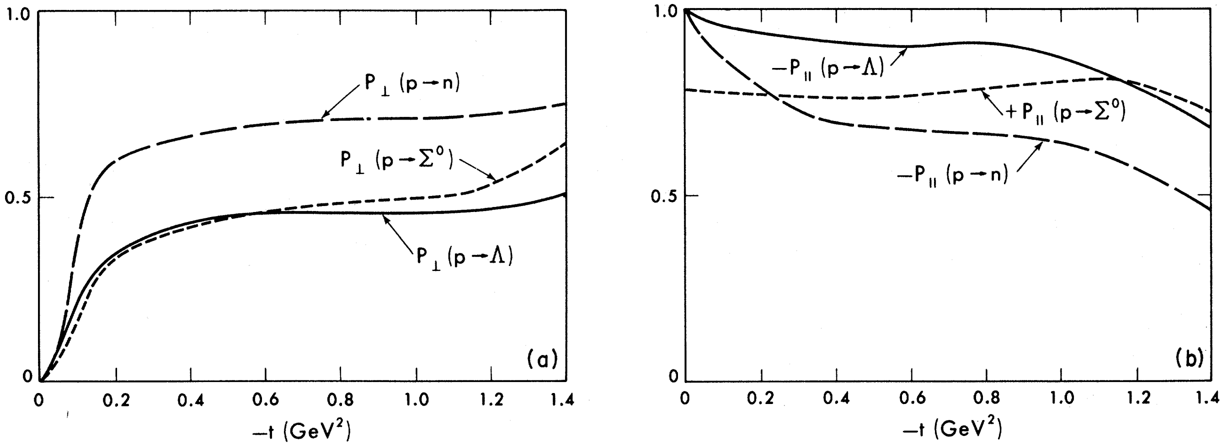

FIG. 7. (a) $P_\perp \equiv \vec{\mathrm{P}}_C \cdot \hat{e}_\perp$ [see (3.9) and (3.12)]. (b) $P_\parallel \equiv \pm\vec{\mathrm{P}}_C \cdot \hat{e}_\parallel$ [see (3.10) and (3.12)].

culation that to highest order in s, the final baryon has a polarization in its own rest frame, lying in the scattering plane. It is given by

$$\vec{\mathrm{P}}_C = \frac{-2\hat{F}_A(t)}{[\hat{F}_1(t)]^2 - t[\hat{F}_2(t)]^2 + [\hat{F}_A(t)]^2}[\hat{F}_1(t)\hat{e}_\parallel + \sqrt{-t}\,\hat{F}_2(t)\hat{e}_\perp], \tag{3.12}$$

where $\hat{e}_\parallel$ and $\hat{e}_\perp$ are defined in Fig. 6. This is true for all three types of reactions, and can be applied to either of the final baryons in $BB \to BB$ reactions.

We notice that $\vec{\mathrm{P}}_C \cdot \hat{e}_\parallel$ does not vanish for forward scattering ($t=0$). This is perfectly all right since parity is not conserved. Also, we observe that $\vec{\mathrm{P}}_C \cdot \hat{e}_\parallel$ is equal to the asymmetry (3.10) for a 100% polarized target. Similarly, $\vec{\mathrm{P}}_C \cdot \hat{e}_\perp$ is identical to the asymmetry (3.9). These equalities are only true to highest order in s. However, they would be exactly true for $PB \to PB$ and $BB \to BB$ scattering in the limit of perfect SU(3) symmetry [i.e., $f_-(t)=0$ and all mass splittings zero] and no absorption corrections. This is best described by an example. For $\pi^-p \to \pi^0\Lambda$, the asymmetries due to a polarized proton are obviously equal to the corresponding components of polarization of the proton in $\Lambda\pi^0 \to \pi^-p$, because of Hermiticity. But all quantities for $\Lambda\pi^0 \to \pi^-p$ and $\pi^-p \to \Lambda\pi^0$ reactions are the same if perfect SU(3) symmetry is assumed. It also turns out that there is an exact equality of the same kind in $BB \to BD$ scattering if all masses are set equal.

The magnitudes of the right-hand sides of (3.9) and (3.10) are shown in Fig. 7 for the currents (a) $\langle\Lambda|J_\mu|p\rangle$, (b) $\langle\Sigma^0|J_\mu|p\rangle$, and (c) $\langle n|J_\mu|p\rangle$ (they are independent of the other current). The values for $\langle\Sigma^-|J_\mu|n\rangle$ are equal to those for (b) and those for $\langle p|J_\mu|n\rangle$ equal those for (c).

For $\langle\Lambda|J_\mu|p\rangle$ and $\langle n|J_\mu|p\rangle$ the magnitude of $\vec{\mathrm{P}}$ is nearly unity, at least up to $t=-1$ GeV^2; for $\langle\Sigma^0|J_\mu|p\rangle$ it is around 0.9. The direction of $\vec{\mathrm{P}}$ is $\hat{e}_\parallel$ for $t=0$ and rotates in the scattering plane as $-t$ increases.

IV. STRANGENESS-CONSERVING TWO-BODY REACTIONS

We have calculated the cross section for strangeness-conserving $PB \to PB$ reactions, in particular, $\pi^-p \to \pi^0 n$, assuming a scattering amplitude $M = M_{\text{strong}} + M_{\text{weak}}$. M_{weak} is given by H_w in (2.2), while we take[19]

$$M_{\text{strong}} = \left(\frac{m_f m_i}{E_f E_i}\right)^{1/2} \frac{1}{(4\omega_f\omega_i)^{1/2}} \bar{u}^f(p')[-A(s,t) + i\gamma\cdot QB(s,t)]u^i(p), \tag{4.1}$$

where $Q = \frac{1}{2}(q+q')$, q and q' being the four-momenta of the initial and final mesons.

Even at very high energies, extrapolations of Regge-pole fits to M_{strong} give $|M_{\text{strong}}/M_{\text{weak}}| \approx 10^5$, so we ignore the contribution of M_{weak} to the unpolarized cross section. In the limit $s \gg -t, m_N^2$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{\text{unpol}} = \frac{1}{64\pi s}[2s^2|B|^2 + 8m_N s\,\mathrm{Re}B^*A + 4|A|^2(2m_N^2 - \tfrac{1}{2}t)]. \tag{4.2}$$

If the target proton is polarized along a direction at an angle $\pi - \chi$ to the beam direction, then

$$\frac{d\sigma}{d\cos\theta}=\left(\frac{d\sigma}{d\cos\theta}\right)_{\rm unpol}+\frac{P}{8\pi}\sqrt{-t}\cos\varphi\frac{G}{\sqrt{2}}\hat{F}_A(t)\hat{f}_+(t)({\rm Re}A)\sin\chi-\frac{P}{8\pi}\frac{G}{\sqrt{2}}\hat{F}_A(t)\hat{f}_+(t)(s{\rm Re}B+2m_N{\rm Re}A)\cos\chi$$
$$-\frac{P}{8\pi}\sqrt{-t}\,\sin\varphi\,{\rm Im}\left[\left(\tfrac{1}{2}B^*+\frac{G}{\sqrt{2}}\hat{f}_+(t)[\hat{F}_1(t)+2m_N\hat{F}_2(t)]\right)\left(-A+2\frac{G}{\sqrt{2}}f_+(t)\hat{F}_2(t)s\right)\right]\sin\chi\,. \qquad (4.3)$$

In (4.3) we have assumed that $|A|\gg m_N|B|$ while $m_N|A|$ and $s|B|$ are comparable.. This is reasonable at very high energies because according to Regge models, A contains an extra factor of s relative to B. An extrapolation of $(d\sigma/d\cos\theta)_{\rm unpol}$ to $\omega_{\rm lab}=200$ GeV is given by Rarita *et al.*[20] for $\pi^- p$ charge exchange. Using their fits to A and B, one finds for the parity-violating asymmetries [found from the second and third terms in (4.3) with $P=1$]

$$\approx\begin{Bmatrix}0\\2\end{Bmatrix}\times 10^{-5} \text{ at } t=0,$$

increasing to

$$\approx\begin{Bmatrix}3\\1\end{Bmatrix}\times 10^{-4} \text{ at } t=-1\text{ GeV}^2,$$

for $\omega_{\rm lab}=200$ GeV. If the ρ Regge pole continues to dominate over a possible Regge cut, the asymmetries will increase like $s^{1-\alpha(t)}$ [$\alpha(t)$ is the ρ trajectory] as the energy increases. At the maximum energy expected for the NAL machine (~500 GeV in $\omega_{\rm lab}$), they will be

$$\approx\begin{Bmatrix}0.8\\0.2\end{Bmatrix}\times 10^{-3} \text{ at } t=-1\text{ GeV}^2.$$

Since they are still smaller than 0.1%, it may be awfully difficult to measure them in experiment, even if a target of high polarization becomes available. It is difficult to obtain precise numerical values of the asymmetries because they depend rather sensitively on the details of the Regge extrapolation.

The last term in (4.3) is the ordinary left-right asymmetry term for polarization perpendicular to the scattering plane. The purely strong contribution ImB^*A would vanish unless more than one Regge singularity is taken into account, but the weak correction terms are probably overwhelmed by other strong effects, such as a ρ-Pomeranchukon cut.

In the same approximations used in deriving (4.3) [or exactly if we ignore $f_-(t)$ and mass splittings], the three components of the polarization vector of the final baryon are again equal to the asymmetries derived from (4.3) for $P=1$. $\vec{\rm P}\cdot\hat{e}_\perp$ corresponds to the second term in (4.3), $\vec{\rm P}\cdot\hat{e}_\parallel$ to the third, and the component normal to the scattering plane to the last.

V. INELASTIC PRODUCTION PROCESS

Most of the events in extremely high-energy strong-interaction processes will be highly inelastic. It is practically impossible to detect the strangeness of all secondary particles, so one cannot see any evidence for weak interactions by measuring the conservation of strangeness. A less formidable measurement will be to detect the polarization of a secondary particle in an inclusive experiment in which the rest of the particles are not measured. In πN charge-exchange scattering, the polarization effects of weak interactions are, unfortunately, too small to detect (see Sec. IV). However, if one carefully chooses a specific configuration in which strong interactions rarely take place, for example, emission of a baryon in the 90° direction in the center-of-mass frame, there might be some chance that the weak amplitude is enhanced enough relative to the strong amplitude to make weak interactions detectable. Whether this is possible or not depends on the angular distribution of secondaries in weak processes, of which we know nothing at this moment.

VI. CHARGE-NONEXCHANGE SCATTERING

Since the weak currents are ordinarily assumed to be charged, charge-nonexchange weak scattering occurs through an interaction of the fixed pole at $J=1$ and the ρ trajectory. Although there is no reliable way to calculate the Regge cut produced by these poles, one may estimate it roughly as follows: The magnitude of the Regge cut produced by a Regge pole and the Pomeranchukon pole is nearly one order of magnitude smaller than that of the pole. This seems to be true in πN charge-exchange scattering. We thus have little reason to expect that the discontinuity of a cut is enhanced. We may estimate the cut amplitude, apart from a possible factor of ln$s$, as[21]

$$M_{\text{cut}} \approx (s/s_0)^{\alpha(0)-1} M_{\text{fixed pole}}, \tag{6.1}$$

where $\alpha(t)$ is the ρ-trajectory parameter and $s_0 \simeq 1\ \text{GeV}^2$. The s dependence in (6.1) makes the cut amplitude smaller than the fixed-pole amplitude. We can estimate the class of diagrams in which no fixed singularity of the weak interactions is relevant (see Sec. II B). A process of practical interest is

$$p+p \to p+\Sigma^+. \tag{6.2}$$

If we calculate this process through pole transitions between p and Σ^+, we obtain

$$M = \frac{2\Delta m}{m_\Sigma - m_N}[M_{\text{strong}}(p\Sigma) - M_{\text{strong}}(pp)], \tag{6.3}$$

where Δm is the weak pole transition between p and Σ^+ ($\propto G$), and $M_{\text{strong}}(p\Sigma)$ and $M_{\text{strong}}(pp)$ are the strong elastic $p\Sigma$ and pp amplitudes. The soft-pion calculation gives the parity-conserving transition strength

$$\Delta m = 1.2 \times 10^{-4}\ \text{MeV}. \tag{6.4}$$

If we put[22]

$$\frac{M_{\text{strong}}(p\Sigma) - M_{\text{strong}}(pp)}{m_\Sigma - m_N} \approx \frac{M_{\text{strong}}(pp)}{m_N}, \tag{6.5}$$

then

$$M \approx 1 \times 10^{-7} M_{\text{strong}}(pp). \tag{6.6}$$

This indicates that the cross section integrated over the scattering angle for $p+p \to p+\Sigma^+$ is roughly equal to

$$10^{-14}\sigma_{\text{el}}(pp), \tag{6.7}$$

where $\sigma_{\text{el}}(pp)$ is the elastic pp cross section. The angular distribution is the same as the two-body strong interaction. The iteration of the fixed weak pole and the ρ trajectory will be a correction to (6.7).

Because of the smallness of (6.7), it looks possible, in principle, to test the existence of neutral weak currents if their magnitude is as large as that of the charged currents. If one introduces neutral currents so as to incorporate the $\Delta I = \frac{1}{2}$ rule for nonleptonic processes, one obtains

$$\sigma(p+p \to p+\Sigma^+) = \sigma(p+n \to p+\Sigma^0), \tag{6.8}$$

where the Σ^+ is emitted near the forward direction on the left-hand side. $(d\sigma/d\cos\theta)_{\text{c.m.}}$ calculated from (6.8) is of the order of $10^{-38}\ \text{cm}^2$ near the forward direction.

However, we would like to make the following comment: We have considered the fixed poles at $J=1$ in the channels where the W mesons, if any, are supposed to appear. If the weak currents [chiral SU(3)×SU(3) currents] consist of fermion fields (for instance, SU(3)-triplet quark fields) and if the current×current interaction is local ($m_W = \infty$), the weak-interaction Hamiltonian is transformed under the Fierz transformation into a sum of products of neutral currents to give rise to fixed poles at $J=1$ in the neutral channels. If the weak interaction is local with high accuracy ($m_W \gg s$), we would have effective neutral current×current interactions. If the charged currents are strictly of $V-A$ type, the strangeness-changing part of the effective neutral interactions is given as

$$(\tfrac{1}{2})^{1/2} G \cos\theta \sin\theta (J_{6\mu} + iJ_{7\mu})[J_{3\mu} + (\tfrac{1}{3})^{1/2} J_{8\mu} + (\tfrac{2}{3})^{1/2} J_{0\mu}], \tag{6.9}$$

where $J_{i\mu}$ ($i = 0, 1, \ldots, 8$) are $V-A$ currents ($\mathfrak{F}_i + \mathfrak{F}_i^5$). The unitary singlet current introduces new parameters, but they are estimated on reasonable assumptions. The differential cross section at $t=0$ for $p+p \to p+\Sigma^+$ turns out to be

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{\text{c.m.}}(p+p \to p+\Sigma^+) \approx 8\left(\frac{d\sigma}{d\cos\theta}\right)_{\text{c.m.}}(p+n \to \Sigma^0+p) \approx 5.0 \times 10^{-38}\ \text{cm}^2. \tag{6.10}$$

It is not easy to see quantitatively how the right-hand side of (6.10) is affected if $m_W \ll s$. One plausibility argument leads to a factor of $\approx (m_W^2/s)^2$. Because of this reason, even if one detects in future experiments the $p+p \to p+\Sigma^+$ of the order of $10^{-38}\ \text{cm}^2$, one can not conclude uniquely the existence of neutral weak currents. It may be a consequence of highly local four-fermion interactions. The process $p+p \to p+\Sigma^+$ is thus very interesting to explore.

TABLE I. Cross sections in the center-of-mass frame at $s = 100\ \text{GeV}^2$, $t=0$ [see also (3.3)]. The cross sections are linear in s ($\simeq 2m_N\omega_{\text{lab}}$) at high energies.

Reaction	$d\sigma/d\cos\theta$ (in $10^{-38}\ \text{cm}^2$)
$\pi^- p \to \pi^0 \Lambda$	2.2
$\pi^- p \to \pi^0 \Sigma^0$	0.53
$\pi^- p \to K^0 n$	1.1
$pn \to \Sigma^0 p$	0.63
$pn \to \Lambda p$	2.6
$pp \to \Delta^{++}\Lambda$	1.8
$pp \to \Delta^{++}\Sigma^0$	0.43

VII. SUMMARY AND FEASIBILITY OF EXPERIMENTS

All of the purely weak cross sections are proportional to s and fall off very rapidly with momentum transfer. At $s=100$ GeV^2 and $t=0$ they are typically 10^{-38} cm^2 (see Table I), ten orders of magnitude smaller than strong cross sections. At $t=-1$ GeV^2, they have fallen to 10^{-40} cm^2.

The largest cross sections occur, then, for large s and small $-t$. However, at high energies $\cos\theta = 1+2t/s$, so the smallest feasible value of $-t$ increases with s. For example, for $s=2m_N\omega_{\text{lab}} = 1000$ GeV^2 and $t=-1$ GeV^2, $\theta \approx 3.6°$. The laboratory scattering angle, given by

$$\cos\theta_{\text{lab}} = 1 + 2m_N{}^2 t/s^2 ,$$

is about 0.12°. Hence, not only are the cross sections very low, but also the scattering angles are very small.

To detect an unpolarized reaction, it would almost certainly be necessary to detect both final particles. It would be quite disadvantageous to not know the initial momenta accurately, so reactions involving a target neutron (in deuterium) are not promising. The most likely $PB \to PB$ reactions are $\pi^- p \to \pi^0 \Lambda$ (which has the largest cross section), $K^- p \to \pi^0 n$, and $\pi^- p \to K^0 n$. The reaction $pn \to \Lambda p$ suffers from requiring a neutron target, but could utilize the much higher flux, purity, and (if desired) energy of the primary beam.

It should here be remarked that the lab energy of the final baryon (the baryon connected by a current to the target) can be quite small for small $-t$. It is

$$E_f = \frac{m_f{}^2 + m_i{}^2 - t}{2m_i} . \tag{7.1}$$

For $m_f = m_\Lambda$, $m_i = m_N$, $t=-1$ GeV^2, this is only 1.7 GeV. Also, the laboratory angle (relative to the beam) of this particle can be quite large.

The reaction $pp \to \Delta^{++}\Lambda$ is perhaps the best possibility. In this case one might be able to get away with detecting only the low energy Λ if the beam energy were known and regulated well enough. The mass of the undetected Δ^{++} would be considerably below that of $p+K^+$, the lightest strangeness-conserving final state (background from $p+\pi^+$, $p+2\pi$, etc., would still be due to purely weak processes).

Final baryons are polarized nearly 100% in the scattering plane (even at $t=0$). A measurement of this polarization for the low-energy final baryon, through a double-scattering experiment (for p or n) or the angular distribution of the decay products (for hyperons) would be evidence for weak scattering. This could be done very near $t=0$ if the other particle were not detected. Of course, one would then run into the difficulty of having a large background of similar particles produced in strong reactions.

Interesting asymmetries could result from the use of a polarized target. However, the asymmetries are reduced, relative to final polarizations, by the target polarization P. 80% polarization of the hydrogen protons may be possible, but only about 25% of the protons in a polarized target are in hydrogen. Hence $P \approx 0.2$ is more likely, and the asymmetries become a small correction to a rare process. If one looked for small asymmetries while only detecting one particle, he would again face a large background from strong reactions.

Parity-violating asymmetries and polarizations in strangeness-conserving reactions are extremely small (10^{-4}–10^{-5}) and are probably unmeasurable in the near future.

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[21]If $\alpha(t) \equiv 1$ for at least one of the Regge poles, the cut produced by it has a branch point independent of t. Recall that $\alpha_{\rm cut}(t) = \mathrm{Max}[\alpha_1(t_1) + \alpha_2(t_2) - 1]$, where $2tt_1 + 2tt_2 + 2t_1t_2 - t^2 - t_1^2 - t_2^2 \geq 0$. We have taken $\alpha_P(t) \equiv 1$.

[22]If SU(3) symmetry were exact, $M_{\rm strong}(p\Sigma) = M_{\rm strong}(pp)$ in the high-energy limit. However, $m_\Sigma = m_N$ in this limit. In the charge-exchange process there is no obvious cancellation among different diagrams.

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Complex Regge Trajectories*

Reinhard Oehme
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
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Conditions are explored which require the presence of leading positive- or negative-signature trajectories of the form $\alpha(t) = 1 + \mathrm{const}\sqrt{t} + O(t)$. General forms of continued partial-wave amplitudes with such trajectories are given, and their implications for high-energy limits are evaluated. The restrictions on the character of the complex singular surfaces are discussed. Explicit examples are given for amplitudes with complex Regge trajectories.

I. INTRODUCTION

From a semiclassical point of view, diffraction scattering looks like one of the more straightforward and simple phenomena. Nevertheless, since the notions of dispersion theory and complex angular momenta appear to form a reasonable framework for the descriptions of fundamental particles, it is relevant to look for a description of diffraction scattering in terms of crossed-channel properties. The features that are relevant to the asymptotic expansion of an amplitude in the s channel are the singularities in the complex angular momentum plane for the t channel.

The Regge-pole trajectories associated with physical particles and resonances are usually assumed to be approximately linear functions in the neighborhood of $t=0$, and so are the branch-point trajectories associated with these Regge poles. However, the singular surfaces directly related to diffraction scattering may well be of a quite different type. Independent of their character – i.e., whether they are poles, square-root branch points, etc. – we may ask whether there are reasons to think that these trajectories, as analytic functions of t, have a singular point at $t=0$. Many years ago, we introduced two-valued singular surfaces of the form

$$\alpha(t) = \alpha(0) \pm \mathrm{const}\sqrt{t} + O(t), \tag{1.1}$$

where, of course, the continued partial-wave amplitude $F(t,\lambda)$ must not inherit the branch point at $t=0$.[1] Trajectories of this type appear in Schrödinger theory with repulsive r^{-2} potentials, and in several field-theoretical calculations which are in principle related to the Schrödinger case.[2,3]

Since high-energy limits of amplitudes are often considered to be calculable by iteration schemes involving s-channel unitarity, we also have considered trajectories of the form

$$\alpha(t) = 1 \pm \mathrm{const}\sqrt{t} \tag{1.2}$$

in the neighborhood of $t=0$, because they are a solution of the bootstrap-type condition[4,5]