Dynamical and Kinematical Aspects of Corrections to Scaling Behavior*

Howard J. Schnitzer
Department of Physics, Brandeis University, Waltham, Massachusetts 02154
(Received 2 April 1971)

Nonscaling corrections to the scaling behavior of deep-inelastic scattering are discussed within the context of light-cone analysis. It is shown that there are both kinematical and dynamical corrections to scaling, derived from the leading and nonleading light-cone singularities, respectively. It is suggested that such corrections have already been observed, as evidenced by the improved fit to the SLAC data in the variable $\omega' = -q^2/(2\nu + m^2)$. Theoretical implications of these corrections are discussed.

I. INTRODUCTION

The surprising results of the series of experiments on deep-inelastic electron scattering performed at SLAC[1] have provided a challenge to theorists to interpret these results. Among the several pictures of high-energy processes that have evolved as a result of these experiments is the analysis of deep-inelastic processes in terms of the leading singularities of two current operators near the light cone.[2] Although this analysis merely attempts to accommodate the experimental results in terms of the language of light-cone singularities, this language is particularly interesting to the current-algebraist, since it may provide clues to a generalization of the powerful tools developed from the equal-time commutators of currents. In this paper we extend earlier analyses to provide a framework for the study of the next-to-leading singularities near the light cone in terms of the nonscaling corrections to the scaling behavior observed in the deep-inelastic region. There are at least two theoretical reasons for studying nonleading light-cone singularities:

(1) The leading singularities can be interpreted in terms of operators with canonical dimensions.[2,3] Can the nonleading singularities be so interpreted?

(2) The very speculative arguments of Brandt and Orzalesi[4] relate simple behavior for the nonleading light-cone singularities to a number of selection rules for Regge trajectories. It would be useful to put such ideas to direct experimental test.

In what follows, we discuss a framework for the extension of light-cone analyses to nonleading singularities,[5] with special emphasis on the next-to-leading light-cone singularities, since these corrections to scaling behavior are the most easily accessible to experimental analysis. We also suggest in this paper that corrections to scaling behavior have already been observed, as evidenced by the improved fit to the electron-scattering data of the variable proposed by Bloom *et al.*[6] We then relate experimental corrections to scaling to the nature of nonleading singularities, to the existence of various equal-time commutators of the currents, and to new sum rules. In particular, if the corrections to scaling are of the form $\nu^{-1}G(\omega)$, there exists a class of sum rules of the form $\int d\omega\, \omega^{2n+1}G(\omega)$, analogous to the generalized Callan-Gross integrals.[7] These new sum rules would then imply the existence of an infinite "ladder" of local operators with integral dimensions, associated with the next-to-leading light-cone singularity.

II. CORRECTIONS TO SCALING

A clear presentation of the light-cone analysis of electron scattering experiments can be found in JVW.[2] We find their notation particularly suited for our purposes. For completeness we summarize the principal definitions of that work. The total cross section for forward electron scattering from a proton of momentum p can be described by means of the Fourier transform of the commutator function

$$\begin{aligned} C^{\mu\nu}(x,p) &= i\langle p|[j^\mu(x), j^\nu(0)]|p\rangle \\ &= (g^{\mu\nu}\Box^2 - \partial^\mu\partial^\nu)C_1(x^2, x\cdot p) + [p^\mu p^\nu \Box^2 - p\cdot\partial(\partial^\mu p^\nu + \partial^\nu p^\mu) + g^{\mu\nu}(p\cdot\partial)^2]C_2(x^2, x\cdot p)\,. \end{aligned} \tag{1}$$

In momentum space, one defines

$$C_i(\omega,\nu) = -i\int d^4x\, e^{iq\cdot x}C_i(x^2, x\cdot p), \quad i=1,2 \tag{2}$$

where q is the virtual-photon four-momentum, $\nu = q\cdot p$, and $\omega = -q^2/2\nu$. In terms of other notation to be found in the literature,

$$\begin{aligned}\tilde{F}_2(\omega,\nu)&=2\omega\nu^2C_2(\omega,\nu)=\nu W_2(\omega,\nu),\\ \tilde{F}_L(\omega,\nu)&=\tilde{F}_2(\omega,\nu)-2\omega\tilde{F}_1(\omega,\nu)=4\omega^2\nu C_1(\omega,\nu),\\ \tilde{F}_1(\omega,\nu)&=W_1(\omega,\nu).\end{aligned}\tag{3}$$

The existence of a nontrivial scaling limit implies

$$\lim_{\nu\to\infty;\,\omega\text{ fixed}}\tilde{F}_i(\omega,\nu)=F_i(\omega)\neq\infty,\tag{4}$$

with support properties $F_i(\omega)=0$ for $|\omega|>1$. The results of the light-cone analysis[2] show that the commutator function can be characterized by $F_i(\omega)$ as follows:

$$C_1(x^2,x\cdot p)=\frac{1}{4\pi^2}\epsilon(x\cdot p)\delta(x^2)\int_0^1 d\omega\,\frac{\cos(\omega x\cdot p)}{\omega^2}F_L(\omega)+\epsilon(x\cdot p)\theta(x^2)f_1(x^2,x\cdot p),\tag{5}$$

and

$$C_2(x^2,x\cdot p)=\frac{1}{4\pi^2}\epsilon(x\cdot p)\theta(x^2)\int_0^1 d\omega\,\frac{\sin(\omega x\cdot p)}{\omega x\cdot p}F_2(\omega)+\epsilon(x\cdot p)\theta(x^2)f_2(x^2,x\cdot p),\tag{6}$$

where $f_1(x^2,x\cdot p)=O((x^2)^{-1+\epsilon})$, $\epsilon>0$ near $x^2=0$, and $f_2(0,x\cdot p)=0$. The same techniques which lead to (5) and (6), relate the *corrections* to the scaling limit to the light-cone behavior of the functions $f_i(x^2,x\cdot p)$. For example, if the functions $f_i(x^2,x\cdot p)$ behave as simple powers of x^2 for $x^2\simeq 0$, with

$$\begin{aligned}f_1(x^2,x\cdot p)&=O((x^2)^{-1+a}),\quad a>0\\ f_2(x^2,x\cdot p)&=O((x^2)^{b}),\quad b>0\end{aligned}\tag{7}$$

near $x^2=0$, then the methods of Ref. 2 applied to (5)–(7) imply

$$\tilde{F}_L(\omega,\nu)\xrightarrow[\nu\to\infty;\,\omega\text{ fixed}]{}F_L(\omega)-\frac{1}{2}\frac{m^2\omega^2}{\nu}F_L'(\omega)-\frac{1}{\nu^a}G_L(\omega)+\cdots\tag{8}$$

and

$$\tilde{F}_2(\omega,\nu)\to F_2(\omega)-\frac{m^2\omega}{2\nu}\left[\omega F_2'(\omega)+3F_2(\omega)-2\int_\omega^1\frac{dz}{z}F_2(z)\right]-\frac{1}{\nu^b}G_2(\omega)+\cdots,\tag{9}$$

where

$$F_i'(\omega)=dF_i(\omega)/d\omega.$$

Note the presence of the ν^{-1} terms in (8) and (9) which are *kinematical* corrections to scaling behavior originating in (5) and (6). There are also nonleading terms of *dynamical* origin determined by the behavior characterized by (7), which is what one wishes to extract from the experimental data. To do so the data must be analyzed according to Eqs. (8) and (9), with due regard to the kinematical corrections, in order to decide the nature of the next-to-leading light-cone singularity. [We have neglected possible logarithms in (7) as these are of subsidiary experimental interest.] Also note that Eqs. (7)–(9) are compatible with a single Regge pole dominating the corrections to scaling.

Is it premature to analyze the data this way? Perhaps not, based on the study of Bloom *et al.*[6] They show that the approach to scaling is more rapid when described in terms of the variable $\omega' = -q^2/(2\nu+m^2)$ rather than ω, particularly for low values of ν when ω is near 1. Since $F_2(\omega')$ achieves scaling "sooner" than $F_2(\omega)$, and since

$$F_2(\omega')\xrightarrow[\nu\to\infty;\,\omega\text{ fixed}]{}F_2(\omega)-\frac{m^2\omega}{2\nu}F_2'(\omega)+\cdots,\tag{10}$$

one can interpret the result of Bloom *et al.*[6] as indicating that the deep-inelastic-scattering data can be analyzed for nonscaling corrections even though the effect is small. Naturally, one can not use (10) directly to study the nonleading light-cone singularities since one must separate the kinematical effects from the dynamical corrections as in Eqs. (8) and (9).

III. THEORETICAL IMPLICATIONS

Certain types of behavior of the nonleading singularities characterized by (7) have greater appeal to the theorist than others. Standard Regge behavior suggests that $a=b\simeq\frac{1}{2}$, while the speculations of Brandt and Orzalesi require $a=b=\frac{1}{2}$.[4] Finally, if the operators contributing to the next-to-leading light-cone singularities have canonical dimensions, then $a=b=1$.

A. Light-Cone Singularities

In this section we characterize the new scaling functions $G_i(\omega)$ of (8) and (9) in terms of the nonleading singularities. We assume, consistent with (7), that

$$f_1(x^2, x\cdot p)=(x^2)^{a-1}f_1(x\cdot p)+\tilde{f}_1(x^2, x\cdot p) \tag{11a}$$

and

$$f_2(x^2, x\cdot p)=(x^2)^{b}f_2(x\cdot p)+\tilde{f}_2(x^2, x\cdot p)\,, \tag{11b}$$

with $a, b>0$,[8]

$$[(x^2)^{1-a}\tilde{f}_1(x^2, x\cdot p)]_{x^2=0}=0 \tag{12a}$$

and

$$[(x^2)^{-b}\tilde{f}_2(x^2, x\cdot p)]_{x^2=0}=0\,. \tag{12b}$$

Since the analysis that follows involves only a simple extension of the methods of JVW,[2] we merely state our results without derivation.

The new scaling functions $G_i(\omega)$ are related to the $f_i(x^2, x\cdot p)$ by

$$G_L(\omega)=\frac{8\pi i\omega^2}{(2\omega)^{1-a}}\int_{-\infty}^{\infty}d\alpha\, e^{i\alpha\omega}\alpha(-\alpha^2)^{a-1}f_1(\alpha) \tag{13a}$$

and

$$G_2(\omega)\simeq 4\pi i\omega(2\omega)^b\int_{-\infty}^{\infty}d\alpha\, e^{i\alpha\omega}\alpha(-\alpha^2)^b f_2(\alpha)\,. \tag{13b}$$

Equation (13b) is only an approximation extracted from the asymptotic expansion of $\tilde{F}_2(\omega, \nu)$ by the method of stationary phase. The exact expression that follows from (11b) is considerably more complicated. For the special case $a=1$ and $b=1$, one obtains

$$G_L(\omega)=-16\pi\omega^2\int_0^{\infty}d\alpha\, \alpha f_1(\alpha)\sin\alpha\omega \tag{14a}$$

and

$$G_2(\omega)=4\pi\omega\int_0^{\infty}d\alpha\,\alpha^2 f_2(\alpha)\times[2\cos\alpha\omega+4\alpha\omega\sin\alpha\omega+(\alpha\omega)^2\cos\alpha\omega]\,, \tag{14b}$$

where now (14b) is exact given (8) and (11). The definition of the commutator function implies that the $f_i(x^2, x\cdot p)$ are even under $x^\mu\to -x^\mu$, which implies that

$$G_L(-\omega)=(-1)^a G_L(\omega)\,,\qquad G_2(-\omega)=(-1)^b G_2(\omega)\,. \tag{15}$$

[In deriving (15) from (13), one must recall that $f_i(x^2, x\cdot p)$ has a cut along the real x^2 axis in the timelike region of the x^2 plane.[9]] Equations (13) and (14) may be inverted by the standard methods of Fourier analysis, with the result

$$f_1(x\cdot p)=-\frac{(2)^{1-a}}{8\pi^2}(x\cdot p)^{2(1-a)}\int_0^1 d\omega\,\frac{G_1(\omega)}{\omega^a}\,\frac{\sin(\omega x\cdot p)}{\omega x\cdot p} \tag{16a}$$

and

$$(x\cdot p)^{2b}f_2(x\cdot p)\simeq-\frac{(-2)^{-b}}{4\pi^2}\int_0^1 d\omega\,\frac{G_2(\omega)}{\omega^b}\,\frac{\sin(\omega x\cdot p)}{\omega x\cdot p}\,, \tag{16b}$$

where (16b) is only valid for small $x\cdot p$. Of special interest is the case $a=b=1$, which corresponds to canonical dimension for the next-to-leading singularity. For $a=b=1$, one has

$$f_1(x\cdot p)=-\frac{1}{8\pi^2}\int_0^1 d\omega\,\frac{G_L(\omega)}{\omega}\,\frac{\sin(\omega x\cdot p)}{\omega x\cdot p} \tag{17a}$$

and

$$\begin{aligned}(x\cdot p)^2 f_2(x\cdot p)&=\frac{1}{2\pi^2}\int_0^1 d\omega\,\frac{G_2(\omega)}{(s_1+s_2)\omega}\left[\int_0^1 dz\, z^{s_1}\cos(\omega x\cdot pz)+\int_1^{\infty}dz\, z^{-s_2}\cos(\omega x\cdot pz)\right]\\&\simeq\frac{1}{8\pi^2}\int_0^1 d\omega\frac{G_2(\omega)}{\omega}\,\frac{\sin(\omega x\cdot p)}{\omega x\cdot p}\ \text{for small } x\cdot p\,,\end{aligned} \tag{17b}$$

with

$$s_1=-\tfrac{1}{2}(3+\sqrt{17})\ \text{ and }\ s_2=\tfrac{1}{2}(3+\sqrt{17})\,.$$

At this point we emphasize that in the derivation of Eqs. (13)–(17), we have freely interchanged orders of integration, and have ignored possible $\delta^{(n)}(\omega)$ contributions to the right-hand sides of (16) and (17).[10] These $\delta^{(n)}(\omega)$ contributions occur when one constructs $G_L(\omega)/\omega^{1+a}$ and $G_2(\omega)/\omega^{1+b}$ from (13), as is required in the solution for $f_i(x\cdot p)$. If $G_i(\omega)$ is bounded on $0<|\omega|<1$, then for $0<a\leqslant 1$, and $0<b\leqslant 1$, the integrals in (16) and (17) exist *without* these additional contributions, i.e., without subtractions. On the other hand, for $a>1$, and $b>1$, these complications must certainly be considered. That is, if integrals over ω do not exist in the usual sense, they should be interpreted as generalized functions.[8,10]

B. Equal-Time Commutators

The Fourier transform of the T^* product corresponding to (1) is

$$T^{\mu\nu}(q,p)=i\int d^4x\, e^{iq\cdot x}\langle p|\,T^*\{j^\mu(x),j^\nu(0)\}\,|p\rangle$$

$$=-(q^2g^{\mu\nu}-q^\mu q^\nu)q^{-2}T_L(q^2,\nu)+[\nu^2g^{\mu\nu}-\nu(q^\mu p^\nu+p^\mu q^\nu)+q^2p^\mu p^\nu]\,q^{-2}T_2(q^2,\nu)\,. \tag{18}$$

Generally one assumes that $T_2(q^2,\nu)$ satisfies the fixed-q^2 dispersion relation[8,10]

$$T_2(q^2,\nu)=\frac{2\omega^2}{q^2\pi}\int_0^1 d\omega'\,\frac{\tilde{F}_2(\omega',q^2)}{\omega'^2-\omega^2}\,, \tag{19}$$

while $T_L(q^2,\nu)$ satisfies either

$$T_L(q^2,\nu)=\frac{\omega^2}{2\pi}\int_0^1 d\omega'\,\frac{\tilde{F}_L(\omega',q^2)}{\omega'^2(\omega'^2-\omega^2)} \tag{20a}$$

or

$$T_L(q^2,\nu)=T_L(q^2,0)+\frac{1}{2\pi}\int_0^1 d\omega'\,\frac{\tilde{F}_L(\omega',q^2)}{\omega'^2-\omega^2}\,, \tag{20b}$$

depending on whether T_L requires a subtraction or not. The Bjorken-Johnson-Low (BJL) limit,[11] $q_0\to i\infty$, $\vec{q}$ fixed, defines the equal-time commutators of the currents and their time derivatives. We must discuss the BJL limit of (20a) and (20b) separately.

Case (i): T_L Unsubtracted

From (18)–(20a)

$$T^{ij}(q,p)\underset{\mathrm{BJL}}{\longrightarrow}-\delta^{ij}\frac{1}{2\pi}\int_0^1 d\omega\,\frac{F_L(\omega)}{\omega^2}+\frac{1}{q_0^{\,2}}\left[-q^iq^j\frac{1}{2\pi}\int_0^1 d\omega\,\frac{F_L(\omega)}{\omega^2}-\delta^{ij}\frac{2p_0^{\,2}}{\pi}\int_0^1 d\omega\,F_L(\omega)\right.$$
$$\left.-\frac{m^2\delta^{ij}}{2\pi}\int_0^1 d\omega\,\omega F_L'(\omega)+(p_0^{\,2}\delta^{ij}-p^ip^j)\frac{2}{\pi}\int_0^1 d\omega\,F_2(\omega)\right]$$
$$+\left(\frac{2}{-q_0^{\,2}}\right)^a\delta^{ij}\frac{1}{2\pi}\int_0^1 d\omega\,\frac{G_L(\omega)}{\omega^{2-a}}+O(q_0^{-3})+O((q_0^{\,2})^{-a-1})+O((q_0^{\,2})^{-b-1})\,, \tag{21}$$

with $0<a,b$. If $a<1$, one must sum over all such contributions to $f_i(x^2,x\cdot p)$, in order that (21) be accurate to $O(q_0^{-2})$. The Schwinger term and Callan-Gross integrals have been discussed by many authors,[2,7,8,10] so we need not reproduce those arguments here. On general grounds, one expects the equal-time commutator

$$[j^i(\vec{x},0),j^j(0)]=0\,, \tag{22}$$

if it is finite, which requires either $a>\frac{1}{2}$ or

$$\sum_{0<a\le 1/2}\int_0^1 d\omega\,\frac{G_L(\omega)}{\omega^{2-a}}=0\,, \tag{23}$$

where the sum indicates that *all* contributions to $f_1(x^2,x\cdot p)$ with $0<a\le\frac{1}{2}$ must be included in the sum rule (23). For $0<a<1$, the integral in (23) may not exist, so that one may be required to consider $\delta(\omega)$ contributions to make the sum rule "work." If one requires

$$[\partial_0 j^i(\vec{x},0),j^j(0)] \tag{24}$$

to exist, then either $a\ge 1$ or

$$\sum_{0<a<1}\int_0^1 d\omega\,\frac{G_L(\omega)}{\omega^{2-a}}=0\,. \tag{25}$$

Notice that none of these considerations, involving terms up to $O(q_0^{-2})$, place any restriction on b other than the original requirement of $b>0$.

Case (ii): T_L Subtracted

From (18), (19), and (20b),

$$T^{ij}(q,p)\underset{\mathrm{BJL}}{\longrightarrow}-\delta^{ij}T_L(-\infty,0)+\frac{1}{q_0^2}\left[-q^iq^jT_L(-\infty,0)-\delta^{ij}\frac{2p_0^2}{\pi}\int_0^1 d\omega\,F_L(\omega)+(p_0^2\delta^{ij}-p^ip^j)\frac{2}{\pi}\int_0^1 d\omega\,F_2(\omega)\right]$$
$$+O(q_0^{-3})+O((q_0^2)^{-1-a})+O((q_0^2)^{-1-b})\,. \tag{26}$$

In this case the operator Schwinger term is given by $T_L(-\infty,0)$, so that the Schwinger term is no longer expressible as a sum rule.[8,10] We see that Eqs. (22) and (24) are satisfied without restriction if $a>1$. Thus, if the dispersion relation for T_L is subtracted, one has a *natural* way for $[j^i(\vec{x},0),j^j(0)]=0$ and $[\partial^0 j^i(\vec{x},0),j^j(0)]$ to exist, without imposing strong requirements on the nonleading singularities. Subjectively, this seems to be a reason for favoring a subtracted dispersion relation for $T_L(q^2,\nu)$, although there is as yet no reason to believe that the sum rules (23) or (25) can not be satisfied. If $T_L(q^2,\nu)$ is subtracted, and if (11) is valid, then one can unambiguously state that the equal-time commutator

$$i\langle p|[(\partial^0 j^i(\vec{x},0)-\partial^i j^0(\vec{x},0)),j^j(0)]|p\rangle=(\delta^{ij}\vec{p}^2-p^ip^j)\delta^3(\vec{x})\frac{2}{\pi}\int_0^1 d\omega[F_L(\omega)-F_2(\omega)]$$
$$+p^ip^j\delta^3(\vec{x})\frac{2}{\pi}\int_0^1 d\omega\,F_L(\omega)+\delta^{ij}\delta^3(\vec{x})\frac{2m^2}{\pi}\int_0^1 d\omega[F_L(\omega)-F_2(\omega)]\,. \tag{27}$$

C. Sum Rules

It is known that one can extract a class of sum rules involving[12]

$$\int_0^1 d\omega\,\omega^{2n}F_L(\omega)\text{ and }\int_0^1 d\omega\,\omega^{2n}F_2(\omega)$$

from the equal-time commutator

$$\langle p|[\partial_0^{2n+1}j^i(\vec{x},0),j^j(0)]|p\rangle$$

from the highest-spin components of the commutator function (1), even if, strictly speaking, this commutator does not exist. Similarly, if $a=1$, then one can extract the sum rules $\int_0^1 d\omega\,\omega^{2n-3}G_L(\omega)$ from the same commutators, while if $b=1$, one obtains sum rules involving $\int_0^1 d\omega\,\omega^{2n-3}G_2(\omega)$. As the method is a straightforward extension of (21) or (26), we leave the details to the reader. If, in fact, $a=b=1$, then all corrections to scaling behavior go as ν^{-1}, as is seen from (8) and (9). Further, we have an infinite class of sum rules involving $\int_0^1 d\omega\,\omega^{2n-3}G_i(\omega)$. These new sum rules would imply the existence of an infinite "ladder" of local operators with integral dimensions associated with the next-to-leading light-cone singularity.

It should be apparent to the reader that one can develop this framework further so as to include even less-singular terms in the asymptotic expansions of the electron scattering functions. Here we have concentrated on those aspects most accessible to experimental analysis.

IV. CONCLUSIONS

Let us recapitulate the major points of experimental interest:

(1) If one analyzes deep-inelastic scattering in terms of light-cone singularities,[2] one finds nonscaling corrections to the leading scaling behavior. These corrections are of two types, one of which goes as ν^{-1} and is a kinematical correction derived from the leading light-cone behavior. The other is a dynamical correction, with power behavior of $\nu^{-a}G_L(\omega)$ and $\nu^{-b}G_2(\omega)$, obtained from the next-to-leading light-cone singularity.

(2) The improved fit to deep-inelastic scattering by means of the variable $\omega'=-q^2/(2\nu+m^2)$, proposed by Bloom *et al.*,[6] suggests that the corrections to scaling behavior are observable. Our study shows that a future step in the analysis of these data should be the disentangling of the dynamical and kinematical corrections to the observed scaling behavior.

Notes added in proof. Let us answer some questions raised by colleagues.

(1) It is to be noted that the *kinematical* corrections to (8) and (9) are not identical. The two additional terms in (9) arise from the complete differential operator of Eq. (1) acting on (6), as is required to maintain current conservation. It is *not correct* to consider the kinematical correction to $p^\mu p^\nu\Box^2 C_2(x,x\cdot p)$ alone, if *all* corrections of $O(\nu^{-1})$ are to be computed, contrary to the suggestion of others.

(2) Let us make explicit what is implied by Eq. (10) ff. Since $F_2(\omega)$ is very small near $\omega=1$, the kinematical corrections to (9) account for the *entire* correction observed by Bloom *et al.* This implies that the dynamical correction $G_2(\omega)$ is quite small near $\omega=1$. For $\omega\neq1$ no clear-cut statement can as yet be made. [Compare Eqs. (9)

and (10).]

(3) As a step in obtaining (17b) from (14b) inverse Fourier transforms, one must solve a second-order differential equation. Hence the unexpectedly complicated form of (17b). The approximation to (17b) valid for small $x \cdot p$ takes on a more conventional appearance.

ACKNOWLEDGMENTS

We are grateful to Professor Fred Gilman for his comments on the subject of this paper. We also wish to thank Professor David Boulware for a useful conversation.

---

* Supported in part by the AEC under Contract No. AT (30-1) 3179 M8.

[1]For a summary of the data, see R. E. Taylor, SLAC Report No. SLAC-PUB-677 (unpublished); F. J. Gilman, in an invited talk presented at the Symposium on Hadron Spectroscopy, Balatonfüred, Hungary, 1970 [SLAC Report No. SLAC-PUB-842 (unpublished)].

[2]D. Boulware and L. S. Brown (unpublished); L. S. Brown, in *Lectures in Theoretical Physics*, edited by W. E. Brittin, B. W. Downs, and J. Downs (Interscience, New York, to be published); R. Brandt, Phys. Rev. Letters 23, 1260 (1969); Y. Frishman, Phys. Rev. Letters 25, 966 (1970); B. L. Ioffe, Phys. Letters 30B, 123 (1969); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970); and H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970). See also G. Altarelli, R. Brandt, and G. Preparata, Phys. Rev. Letters 26, 42 (1971). We refer to Jackiw *et al.* in the text as JVW.

[3]K. Wilson, Phys. Rev. 179, 1499 (1969).

[4]R. Brandt and C. Orzalesi, New York University report (unpublished).

[5]See also R. Gatto and P. Menotti, Nuovo Cimento 2A, 881 (1971), which discusses nonleading singularities in the context of scalar currents in ladder models.

[6]E. D. Bloom *et al.*, MIT-SLAC Report No. SLAC-PUB-796, 1970 (unpublished) presented at the Fifteenth International Conference on High-Energy Physics, Kiev, U.S.S.R., 1970. See also E.D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

[7]C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

[8]See R. Jackiw *et al.*, Ref. 2; J. M. Cornwall, J. D. Corrigan, and R. E. Norton, Phys. Rev. Letters 24, 1141 (1970).

[9]A. S. Wightman, Phys. Rev. 101, 860 (1956).

[10]A. Zee, Phys. Rev. D 3, 2432 (1971); D. Corrigan, *ibid.* 4, 1053 (1971).

[11]J. D. Bjorken, Phys. Rev. 148, 1467 (1966); K. Johnson and F. Low, Progr. Theoret. Phys. (Kyoto) Suppl. 37-38, 74 (1966).

[12]D. J. Gross (unpublished); R. Jackiw (unpublished).