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Muon-Pair Production in High-Energy Proton-Proton Collisions*

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Using a current-algebra sum rule derived from an equal-time commutator between charge densities, we have set a lower bound on the muon-pair production cross sections. In spite of the fact that our estimate is conservative, the lower bound turns out to be very large at energies larger than ~100 GeV ($\gtrsim 0.50\times 10^{-30}$ cm^2 at 500 GeV, the maximum available energy at the National Accelerator Laboratory). Comparing it with recent experimental data at lower energies, we predict that the muon-pair production cross section will keep increasing until it turns over owing to higher-order electromagnetic interactions through unitarity.

I. INTRODUCTION

The Fubini sum rule with a two-particle scattering state as a target is applied to muon-pair production in high-energy proton-proton collisions.[1,2] At presently available energies ($\lesssim$30 GeV in lab energy), our sum rule needs knowledge of unphysical regions. But if the energy becomes as high as 100–500 GeV at the National Accelerator Laboratory (NAL), or even higher, up to ~60 GeV in the center-of-mass system (~1900 GeV in the lab frame) at the Intersecting Storage Ring machine,[3] the inequality resulting from our sum rule will be tested with physical amplitudes only. The inequality has two main features: (1) The sum of the muon-pair-production cross sections with the proton target and with the neutron target will increase linearly in the lab energy, until higher-order electromagnetic interactions make them damp through unitarity at extremely high energies. (2) The lower bound of cross sections is very large numerically. Comparing the results from our inequality with a recent experiment,[4] we predict that the muon-pair production cross section must increase *very rapidly* as the energy increases from 30 GeV to energies available at NAL. We have drawn curves of our lower bound for a few typical values of the incident energy as a function of the invariant muon-pair mass.

II. BOUND ON CROSS SECTIONS

As in the case of the original Fubini sum rule or the Cabibbo-Radicati sum rule derived from it, we start from the sum rule for charged vector currents. We start by introducing

$$W^{\pm}_{\mu\nu}=\tfrac{1}{2}\int d^4x\, e^{-iqx}\langle \vec{p}_1\vec{p}_2^{\,\rm in}|J^{\pm}_{\mu}(x)J^{\mp}_{\nu}(0)|\vec{p}_1\vec{p}_2^{\,\rm in}\rangle \tag{2.1}$$

$$=\left(\frac{m_N^{\,2}}{p_{10}p_{20}}\right)\Big[(-\delta_{\mu\nu}q^2+q_\mu q_\nu)W_1^{\pm}+X_\mu X_\nu W_2^{\pm} + Y_\mu Y_\nu W_3^{\pm}+\tfrac{1}{2}(X_\mu Y_\nu+X_\nu Y_\mu)W_4^{\pm}, + \tfrac{1}{2}(X_\mu Y_\nu - X_\nu Y_\mu)W_5^{\pm}\Big], \tag{2.2}$$

where the state vectors are states of two nucleons with momenta as designated. Other notations are as follows:

$-q_\mu$ = "photon" momentum,

$P_\mu=(p_1+p_2)_\mu$, $\Delta_\mu=(p_1-p_2)_\mu$,

$X_\mu=P_\mu-q_\mu(P\cdot q)/q^2$, and $Y_\mu=\Delta_\mu-q_\mu(\Delta\cdot q)/q^2$.

W_i are functions of

$\nu_1=-(P\cdot q)$, $\nu_2=-(\Delta\cdot q)$,

$s=-P^2$, and $-q^2$.

$J^{\pm}_{\mu}(x)$ are the charged components of isospin vector currents so normalized as to obey an equal-time commutation relation

$$[J_0^+(x),J_0^-(0)]\delta(x_0)=J_0^3(0)\delta^4(x), \tag{2.3}$$

where $J^3_\mu(x)$ is the third component of the isospin currents.

By repeating the arguments developed in Ref. 2, we can derive a Fubini-like sum rule which leads us to an inequality

$$\frac{1}{\pi}\left(\int_{-\nu_0}^{-\nu_0+m^*\sqrt{s}}d\nu_1 W_2^+ + \int_{-\nu_0}^{-\nu_0+m^*\sqrt{s}}d\nu_1 W_2^-\right)\geq F(s), \tag{2.4}$$

where

$$F(s)=\sigma_T(\infty)/4m_N^{\,2}, \tag{2.5}$$

$\sigma_T(\infty)$ being the high-energy limit of the nucleon-

nucleon total cross section, $\nu_0 = \frac{1}{2}(s - q^2 - 4m_N^2)$, and m^* is the cutoff parameter discussed previously in detail ($\simeq$ a few GeV).[2] Note that the negative sign at ν_1 indicates emission of the "photons" instead of absorption.

Because of the charge independence, we have a relation in differential cross sections in q_μ:

$$2d\sigma(pn\gamma_V) + d\sigma(pp\gamma_V) \geqslant d\sigma(pp\gamma^+) + d\sigma(pp\gamma^-)\,, \tag{2.6}$$

where γ_V stands for the isovector part of the photon, and p and n stand for the proton and the neutron, respectively. The muon-pair-production cross sections are written linear in W_i's. Keeping in mind the inequality (2.6) and using a relation between the cross section and the W_i's,[5] we find from (2.4)

$$\frac{2d\sigma_{pn}}{d|q^2|} + \frac{d\sigma_{pp}}{d|q^2|} \gtrsim \frac{\alpha^2}{12\pi|q^2|^2}\sigma_T(s)I(s)s\,, \tag{2.7}$$

with

$$I(s) = \{[1 - 2m^*/\sqrt{s} + (|q|^2 - 4m_N^2)/s]^2 - 4|q^2|/s\}^{3/2}\,, \tag{2.8}$$

where $d\sigma_{pn}$ and $d\sigma_{pp}$ are the muon-pair-production cross sections through the isovector photon in the proton-neutron and proton-proton collisions, and $\alpha = e^2/4\pi$ $(= \frac{1}{137})$. Strictly speaking, the cross sections in (2.7) are those for muon pairs whose momenta in the center-of-mass frame of the N-N system are larger than

$$\left[\left(\frac{s + |q^2| - 4m_N^2}{2\sqrt{s}} - m^*\right)^2 - |q^2|\right]^{1/2}.$$

Just as in the case of the W-meson production, we have introduced one physical assumption on the angular distribution of the virtual photons; namely, it has been assumed that virtual timelike photons are emitted more into either a forward or background cone than perpendicularly to the collision axis in the center-of-mass frame.[6]

A few comments are in order. Although the cross sections in (2.7) are those through the isovector photon, we expect that inclusion of the isoscalar photon would not change the values of the cross sections drastically; the isoscalar photon coupling with the eighth component of the unitary spin current is weaker by $1/\sqrt{3}$ than the isovector photon coupling, and the isoscalar photon does not excite the nucleon to $I = \frac{3}{2}$ states. Further, in order to get an order-of-magnitude estimate, we shall put

$$d\sigma_{pn} \approx d\sigma_{pp}$$

in the relevant energy region. Under these additional physical assumptions, we obtain finally

$$\frac{d\sigma_{pp}}{d|q^2|} \gtrsim \frac{\alpha^2}{36\pi|q^2|^2}\sigma_T(s)I(s)s\,. \tag{2.9}$$

This lower bound will be evaluated and discussed in the following.

III. NUMERICAL RESULTS

First we note that our inequality is insensitive to the cutoff parameter m^* when the collision energy is sufficiently high and the invariant mass of a muon pair is small. The value m^*, although it cannot be fixed precisely from presently available experimental knowledge, is expected to be a few GeV.[2] For energies for which

$$[1 - 2m^*/\sqrt{s} + (|q^2| - 4m_N^2)/s]^2 - 4|q^2|/s$$

is negative, we need knowledge of the unphysical region, and (2.7) does not hold valid.

A measurement has been done for $1.1\text{ GeV} < \sqrt{|q^2|} < 6.7\text{ GeV}$ between 22 GeV and 29.5 GeV in the lab energy.[4] By comparing it with our formula we see that the incident energy is still not high enough for our formula to be applicable. We have drawn the lower bounds on the muon-pair-production cross section predicted by (2.9). The cutoff m^* has been chosen 3 GeV and, for the sake of comparison, 5 GeV. It seems to be unlikely that m^* is larger than 5 GeV, so that the bound obtained with $m^* = 5$ GeV will be even more a conservative one. The high-energy total cross section of the p-p collision has been put equal to $\simeq$30 mb. The target has been treated as a free proton.

Our lower bound (2.9) is fairly a conservative estimate in two ways: (1) The cross sections are, in fact, those for muon pairs whose momenta are larger than a certain limit,[7] and (2) in actual processes virtual photons are likely to be emitted more into a forward or backward cone. This is the case in the experiment in the range of 22–29.5 GeV. In spite of these facts, we find in Figs. 1 and 2 that the lower bounds of the production cross section are very large.

The Columbia group has fitted the production cross section at 29.5 GeV with[4]

$$\frac{d\sigma}{dm_{\mu\mu}} \approx \frac{10^{-32}}{m_{\mu\mu}^{5}}\ \text{cm}^2\,\text{GeV}^{-1}\,, \tag{3.1}$$

with $m_{\mu\mu}(=\sqrt{|q^2|})$ being the invariant muon-pair mass in GeV. If one takes the smallest value of $m_{\mu\mu}$ they measured,

$$\left.\frac{d\sigma}{dm_{\mu\mu}}\right|_{m_{\mu\mu}=1.1\text{ GeV}} \simeq 1.61\times10^{-32}\ \text{cm}^2\,, \tag{3.2}$$

with 24% of random errors and 65% of systematic errors. We can calculate from (2.9) our lower bounds on the same cross section at higher energies as

$$\left.\frac{d\sigma}{dm_{\mu\mu}}\right|_{m_{\mu\mu}=1.1\ \mathrm{GeV}} \begin{cases} =5.7\times10^{-31}\ \mathrm{cm}^2\,\mathrm{GeV}^{-1} \text{ at } \omega_{\mathrm{1ab}}=100\ \mathrm{GeV},\\ =2.3\times10^{-30}\ \mathrm{cm}^2\,\mathrm{GeV}^{-1} \text{ at } \omega_{\mathrm{1ab}}=200\ \mathrm{GeV},\\ =1.0\times10^{-29}\ \mathrm{cm}^2\,\mathrm{GeV}^{-1} \text{ at } \omega_{\mathrm{1ab}}=500\ \mathrm{GeV},\\ =5.5\times10^{-28}\ \mathrm{cm}^2\,\mathrm{GeV}^{-1} \text{ at } \omega_{\mathrm{1ab}}=1900\ \mathrm{GeV}. \end{cases} \quad (\sqrt{s}=60\ \mathrm{GeV}) \tag{3.3}$$

These are much larger than the measured value (3.2) at $\omega_{\mathrm{1ab}}=29.5$ GeV. As was emphasized above, our lower bounds are fairly modest. It would not be very surprising that experimental results will come out larger by a factor of several than our lower bounds.

It is impossible to extrapolate the data at 29.5 GeV to higher energies, but it has been observed in the same experiment[4] that the total production cross section increases quite rapidly as the incident proton energy increases from 22 to 29.5 GeV. The increase of the cross section there is claimed to be *a factor of 5*. This indicates that the production cross section will increase enormously as one goes to higher energies. The rapid increase observed in the experiment can not be fitted even with the square of ω_{1ab} (the incident proton energy) in the region between 22 and 29.5 GeV. This is evidence that the production processes do not yet reach smooth monotonic regions of sufficiently high energies. If our estimate is correct, the rapid increase (faster than ω_{1ab}^2) in the cross section will eventually turn over somewhere to obey a linear increase, and the linear increase will be observed until higher-order electromagnetic interactions start competing. We predict for instance that the muon-pair production cross section will continue to grow and exceed $\sim0.5\times10^{-30}$ cm^2 ($m_{\mu\mu}\geqslant 1$ GeV) at 500 GeV, the maximum energy available at NAL.

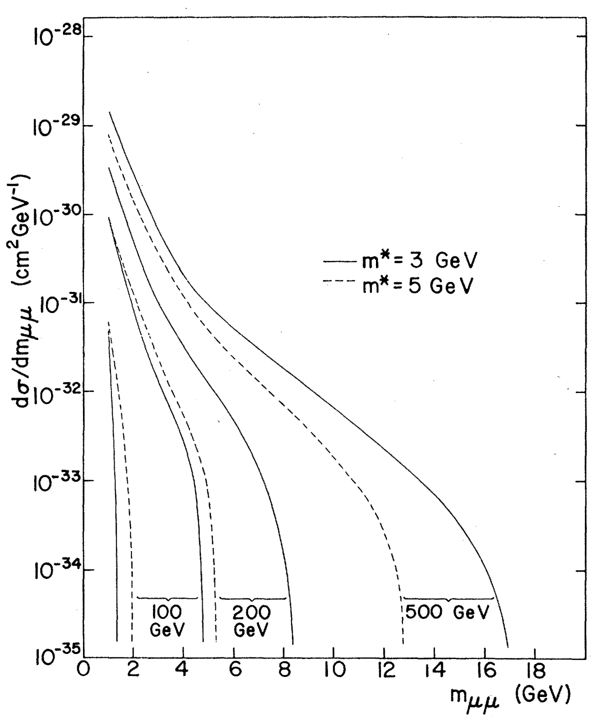

FIG. 1. The lower bounds on the muon-pair production cross section derived from (2.9). The two values are chosen for the cutoff parameter m^*. The invariant mass of the muon pair $\sqrt{|q^2|}$ is denoted by $m_{\mu\mu}$.

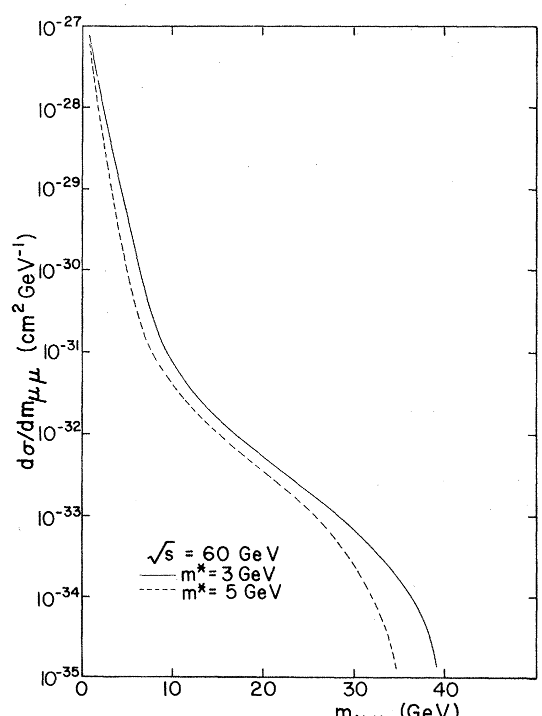

FIG. 2. The lower bounds at $\sqrt{s}=60$ GeV.

IV. POSSIBLE SCALING RULES

Different scaling rules have been proposed from a parton-type model[8] and a heuristic analysis of singularities near the light-cone.[9] The former predicts in the limit of $|q^2|$ and s being infinite, with $s/|q^2|$ fixed,

$$\frac{d\sigma}{dq^2} \sim \left(\frac{1}{q^2}\right)^2 f\left(\frac{s}{q^2}\right), \tag{4.1}$$

while the latter indicates

$$\frac{d\sigma}{dq^2} \sim g\left(\frac{s}{q^2}\right) + \frac{1}{q^2} h\left(\frac{s}{q^2}\right), \tag{4.2}$$

where f, g, and h are arbitrary functions, yet undetermined from theory. Our lower bound is valid when one lets both s and $|q^2|$ to infinity with $s/|q^2|$ kept fixed (not too small). In this asymptotic area, (2.9) leads us to

$$\frac{d\sigma}{dq^2} \gtrsim \frac{1}{q^2} \tilde{f}\left(\frac{s}{q^2}\right), \tag{4.3}$$

where $\tilde{f}$ is another undetermined function of s/g^2. This is contradictory with the prediction of a parton model, but not inconsistent with the analysis of light-cone singularities. However, our formula will be valid only well above 100 GeV (in lab energy), and in case that the smooth scaling region is realized at fairly lower energies, our analysis would not have a predictive power there.

V. COMMENTS

Our current-algebra sum rule has led to very large lower bounds on the muon-pair production cross section. How should we interpret this result if our lower bounds turn out to be false? The first possibility is to say that the equal-time commutator between the charge densities is wrong. However, since this is one of the basic hypotheses in the current algebra which leads to other sum rules tested by experiment, we are reluctant to abandon it.[10] Then we come to consider possible modifications due to anomalous thresholds that we have not considered in deriving our sum rule with a two-particle state as a target. Finally we should ask whether the cutoff parameter m^* might be larger than 5 GeV. The "proton-proton-γ" scattering amplitudes are expanded in fractional powers in the Regge asymptotic region. From our experience in two-body scattering, it is not likely that such a high-energy tail makes an appreciable contribution to isospin-antisymmetric sum rules. However, the analogy of two-body scattering may not apply to three-body amplitudes if emission of timelike photons is enormously suppressed for large transverse momenta. It is still true that the ratio of the isospin-antisymmetric amplitudes to the isospin-symmetric amplitudes is largest near the branch point at $\nu_1 = \nu_0$, but we are not sure that the magnitude of the antisymmetric amplitudes is largest there. The current-algebra sum rule was written originally for scattering amplitudes in which bra and ket vectors are under different boundary conditions.[2] The forward limits of these amplitudes may be quite different from the W amplitudes to which all intermediate states contribute in constructive interference. The assumption concerning the upper limit of the integral over ν_1 in (2.4) thus rests on a behavior of amplitudes which are hard to access physically. Observation of the angular distribution of the timelike photons would not be able to test validity of the assumption, although it may give some insight into how plausible the assumption is.[11,12]

Note added in proof. If the transverse momenta of virtual photons are effectively cut off at a finite value, we should use a modified sum rule which holds for emission in the direction near the collision axis (see Ref. 12). This sum rule leads us to a muon-pair production cross section approaching a constant at high energies and to the same scaling rule as that of the parton model [see (4.1)].

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[1]For the Fubini sum rule, see S. L. Adler, Phys. Rev. 143, 1144 (1966); S. Fubini, Nuovo Cimento 43A, 475 (1966); R. F. Dashen and M. Gell-Mann, Phys. Rev. Letters 17, 340 (1966).

[2]Its extension to a "target" of a two-particle state is made by A. I. Sanda and M. Suzuki, Phys. Rev. D 3, 2019 (1971).

[3]An ISR experiment to search for muon pairs is scheduled to run in mid 1971. L. M. Lederman (private communication).

[4]J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, B. G. Pope, and E. Zavattini, Phys. Rev. Letters 25, 1523 (1970).

[5]The kinematics is exactly the same as in the case of the W production in Ref. 2 so that we have avoided repetition.

[6]To the extent that the photon behaves like a hadron or a superposition of hadrons, that is justified from observed angular distributions of secondary hadrons at superhigh energies.

[7]See a remark below (2.8).

[8]S. D. Drell and T. M. Yan, Phys. Rev. Letters 25,

316 (1970).

[9]G. Altarelli, R. A. Brandt, and G. Preparata, Phys. Rev. Letters 26, 42 (1971).

[10]Some modifications have been proposed for the Fubini sum rule. See G. W. Barry, G. L. Gounaris, and J. J. Sakurai, Phys. Rev. Letters 21, 941 (1968); H. Harari, *ibid.* 22, 1078 (1969).

[11]According to unpublished data of the Columbia-Brookhaven group, the transverse momenta of the timelike photons do not fall off very rapidly. This may indicate that our assumption is not bad, or else that the falling off looks slower because of the large masses of the photons. We thank Professor L. M. Lederman for making available to us some of their data prior to publication.

[12]We can generalize the current-algebra sum rule to apply to forward emission of the timelike photons with a few plausible assumptions. Our assumption concerning the upper limit of the integral over ν_1 will be much better for the forward emission in the case in which the transverse momenta damp rapidly. We shall discuss this generalization in a separate paper.