Generalization of Sirlin's asymptotic theorem in a class of gauge theories based on SU(2)×U(1)×G

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We calculate the radiative corrections to semileptonic decays of hadrons, mediated by the W boson, within the context of $SU(2)\times U(1)\times G$ models, proposed by de Groot, Gounaris, and Schildknecht, which allow for an arbitrary number N of neutral weak gauge bosons. In particular we analyze the large $m_W, m_{Z_1}, \ldots, m_{Z_N}$ behavior of the $O(\alpha G_F)$ corrections and we generalize a theorem by Sirlin on these corrections formulated in the standard model.

I. INTRODUCTION

A few years ago de Groot, Gounaris, and Schildknecht[1] proposed a model for the electroweak interaction based on the symmetry group $SU(2)\times U(1)\times G$, where G is an arbitrary Lie group of rank $N-1$. This model allows N neutral weak bosons, and gives, at low momentum transfer, the same predictions as the standard model based on $SU(2)\times U(1)$. However, it is possible that the present low-energy data can give indications of the existence of extra gauge bosons.[2] Besides, some models of grand unification after breaking yield, at low energy, symmetry groups larger than the one of the standard model, and contain at least one extra U(1) gauge factor.

The purpose of this paper is to generalize a result by Sirlin,[3] for the radiative corrections to semileptonic decays of hadrons in the standard model, to the class of models in Ref. 1. Sirlin's result, stated as a theorem, governs the large m_W, m_Z behavior of the $O(\alpha)$ corrections to general semileptonic processes mediated by W. As we will show an analogous theorem, for the group $SU(2)\times U(1)\times G$, governs the large $m_W, m_{Z_1}, \ldots, m_{Z_N}$ behavior of the $O(\alpha)$ corrections in semileptonic decays, and, of course, to purely leptonic decay (e.g., μ decay).

To get our goal we follow the current-algebra formulation of radiative corrections developed extensively by Sirlin in Ref. 4. The paper is constructed as follows. In Sec. II we give the relevant piece of the interaction Lagrangian density and some important relationship between the couplings of the model. In Sec. III we compute the contributions from the neutral-Z_j-boson exchange. The photonic corrections are as given in the standard model. So we take this last result from Ref. 3. Both these corrections are to be combined with the so-called two-current correlation functions coming from γ and W exchange, which result to be a universal multiple of the zeroth-order amplitude, independent of the nature of the initial and final hadrons. When this is compared with the analogous result in μ decay, we find that such contributions cancel and have no physical consequences. This is done in the Appendix. Section IV is devoted to present the final result.

II. MODEL BASED ON SU(2)×U(1)×G

It is assumed[1] that the fermions are singlets under G and transform in exactly the same manner under $SU(2)\times U(1)$ as in the standard model. The relevant part of the interaction Lagrangian density is given by

$$L_{\text{int}} = L_{\text{CC}} + L_{\text{NC}} + L_{\text{VV}} , \tag{1}$$

where

$$L_{\text{CC}} = -\frac{g}{\sqrt{2}}(W_\mu^\dagger J_{(w)}^\mu + \text{H.c.}) \tag{2}$$

is the charged-current Lagrangian density,

$$L_{\text{NC}} = -e A_\mu J_{(\gamma)}^\mu - \sum_{j=1}^{N} f_j Z_j^\mu J_{(Z_j)}\mu \tag{3}$$

is the neutral-current Lagrangian density, and

$$L_{\text{VV}} = -\tfrac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}C_{\mu\nu}C^{\mu\nu} \tag{4}$$

is the vector-boson–vector-boson interaction Lagrangian density.

Introducing the notation $\psi_i = \text{col}(c,u,d,s)$ for quarks [$i=1,2,3$ is the $SU(3)_c$ degree of freedom] and $\chi = \text{col}(\nu_\mu, \nu_e, \mu, e)$ for leptons, we can write, for the charged current,

$$\begin{aligned} J_{(W)}^\mu &\equiv J_{(W;h)}^\mu + J_{(W;l)}^\mu \\ &= \bar{\psi}\gamma^\mu a_- C_- \psi + \bar{\chi}\gamma^\mu a_- c_- \chi , \end{aligned} \tag{5}$$

where[5] $a_- = \frac{1}{2}(1-\gamma_5)$, C_- is a Cabibbo-type matrix, and

$$c_- = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} .$$

For the neutral current we have

$$J^{\mu}_{(\gamma)} \equiv J^{\mu}_{(\gamma;h)} + J^{\mu}_{(\gamma;l)} = \bar{\psi}\gamma^{\mu}Q\psi + \bar{\chi}\gamma^{\mu}Q'\chi \ , \tag{6}$$

$$J^{\mu}_{(Z_j)} \equiv J^{\mu}_{(Z_j;h)} + J^{\mu}_{(Z_j;l)} \ , \tag{7}$$

$$J^{\mu}_{(Z_j;h)} = \tfrac{1}{2}\bar{\psi}\gamma^{\mu}a_{-}C_3\psi - \sin^2\alpha_j J^{\mu}_{(\gamma;h)} \ , \tag{8a}$$

$$J^{\mu}_{(Z_j;l)} = \tfrac{1}{2}\bar{\chi}\gamma^{\mu}a_{-}C_3\chi - \sin^2\alpha_j J^{\mu}_{(\gamma;l)} \ , \tag{8b}$$

where

$$Q = \mathrm{diag}(\tfrac{2}{3}, \tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}) \ ,$$

$$Q' = \mathrm{diag}(0, 0, -1, -1) \ ,$$

$$C_3 = \mathrm{diag}(1, 1, -1, -1) \ ,$$

and

$$\cos\alpha_j = \frac{m_W}{m_{Z_j}} \ ,$$

with m_W and m_{Z_j} the masses of the charged and the neutral weak bosons ($j = 1, 2, \ldots, N$), respectively.

For the vector-boson sector of Eq. (4), we only give the vertex interaction relevant to our problem:

$$\gamma WW \text{ vertex: } \ -ieV_{\mu\alpha\beta} \ , \tag{9a}$$

$$Z_j WW \text{ vertex: } \ if_j \sin^2\alpha_j V_{\mu\alpha\beta} \ , \tag{9b}$$

where $V_{\mu\alpha\beta}$ is the usual three-linear vector-boson tensor of the standard model. The coupling constants of the model satisfy the relations

$$g^2 = \sum_{j=1}^{N} f_j^2 \cos^2\alpha_j \ , \tag{10a}$$

$$e^2 = \sum_{j=1}^{N} f_j^2 \cos^2\alpha_j \sin^2\alpha_j \ , \tag{10b}$$

the connections with the Fermi coupling and the Weinberg angle are

$$g^2/2m_W^2 = 4G_F/\sqrt{2} \quad \text{and} \quad e^2 = g^2\sin^2\theta_W \ .$$

Finally, we give the commutation relation between the hadronic currents:

$$[J^{\mu}_{(W;h)}(x), J^{\nu}_{(\gamma;h)}(x')]_{x^0=x^{0'}} = \delta_{0\rho}S^{\mu\rho\nu\sigma}J_{(W;h)\sigma}(x)\delta(\mathbf{x}-\mathbf{x}') - i\delta_{0\rho}\epsilon^{\mu\rho\nu\sigma}[2\bar{\psi}(x)\gamma_{\sigma}a_{-}QC_{-}\psi(x) + J_{(W;h)\sigma}(x)]\delta(\mathbf{x}-\mathbf{x}') + \mathrm{ST} \ , \tag{11a}$$

$$[J^{\mu}_{(W;h)}(x), J^{\nu}_{(Z_j;h)}(x')]_{x^0=x^{0'}} = \delta_{0\rho}S^{\mu\rho\nu\sigma}\cos^2\alpha_j J_{(W;h)\sigma}(x)\delta(\mathbf{x}-\mathbf{x}') + i\delta_{0\rho}\epsilon^{\mu\rho\nu\sigma}\sin^2\alpha_j[2\bar{\psi}(x)\gamma_{\sigma}a_{-}C_{-}\psi(x) + J_{(W;h)\sigma}(x)]\delta(\mathbf{x}-\mathbf{x}') + \mathrm{ST} \ , \tag{11b}$$

where $S^{\mu\rho\nu\sigma} = g^{\mu\rho}g^{\nu\sigma} - g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho}$, $\epsilon^{\mu\rho\nu\sigma}$ is the totally antisymmetric tensor; ST represents a c-number Schwinger term, which do not contribute to the connected amplitudes.

III. Z_j-BOX CORRECTIONS

These corrections arise from virtual Z_j exchange, as shown in Figs. 1–3. The transition amplitudes from Fig. 1 are

$$M_{(1a)} = \frac{ig^2f_j^2}{8}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^2-m_W^2)(k^2-m_{Z_j}^2)}T^{\lambda\rho}_{(Z_j)}(k)\bar{u}_{\nu}\gamma_{\lambda}\not{k}\gamma_{\rho}a_{-}v_e \ , \tag{12}$$

$$M_{(1b)} = -\frac{ig^2f_j^2}{8}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)(k^2-m_{Z_j}^2)}T^{\lambda\rho}_{(Z_j)}(k)\bar{u}_{\nu}\gamma_{\rho}a_{-}\frac{1}{\not{k}-\not{l}-m}\gamma_{\lambda}(2\sin^2\alpha_j - a_{-})v_e \ , \tag{13}$$

where

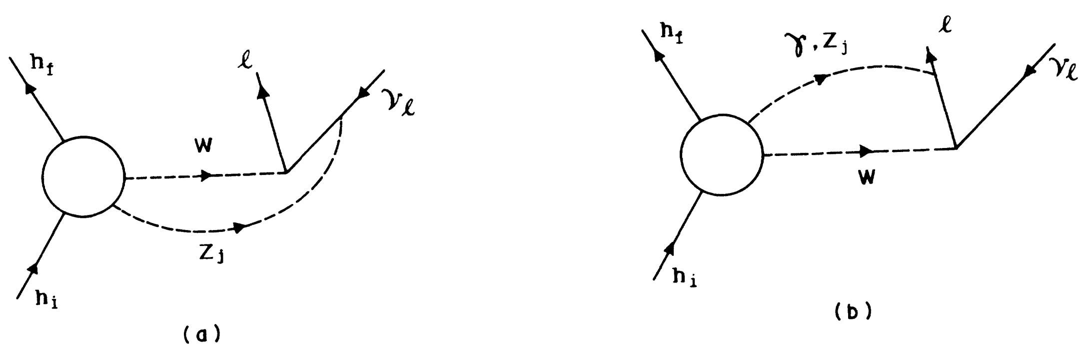

FIG. 1. Feynman diagrams involving the exchange of Z_j or γ between hadrons and leptons. h_i, h_f are the initial and final hadrons of momenta p and p', respectively.

$$T^{\lambda\rho}_{(Z_j)}(k)=\int d^4k\, e^{ik\cdot x}\langle p'\mid T[J^{\lambda}_{(Z_j;h)}(0)J^{\rho}_{(W;h)}(0)]\mid p\rangle\ ,$$

m is the mass of the charged lepton, and $j=1,2,\ldots,N$. In these equations we have set the momentum transfer q equal to zero, because all the contributions proportional to q are finite and $O(G_F^2)$. Adding Eqs. (12) and (13) we get the so-called Z_j-box contribution to the radiative corrections, which after some manipulations is given by

$$M^{\text{box}}_{(Z_j)}=\frac{i}{2}g^2f_j^2L^{\mu}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^2-m_W^2)(k^2-m_{Z_j}^2)}$$
$$\times\{\cos^2\alpha_j[-2i\cos^2\alpha_j\langle p'\mid J_{(W;h)\mu}(0)\mid p\rangle-k_{\mu}T^{\lambda}_{(Zj)\lambda}(k)]-i\sin^2\alpha_j\epsilon_{\lambda\rho\sigma\mu}k^{\sigma}T^{\lambda\rho}_{(Z_j)}(k)\}\ , \quad (14)$$

where $L^{\mu}=\bar{u}_{\nu}\gamma^{\mu}a_{-}v_e$ is the leptonic current. To evaluate this expression we use the Bjorken-Johnson-Low asymptotic limit[6] for $T^{\lambda\rho}_{(Z_j)}$:

$$T^{\lambda\rho}_{(Z_j)}(k)=-\frac{i}{k^2}k_{\tau}\langle p'\mid J_{(W;h)\beta}(0)\mid p\rangle(\cos^2\alpha_jS^{\lambda\tau\rho\beta}-2i\sin^2\alpha_j\bar{Q}\epsilon^{\lambda\rho\tau\beta})\ , \quad (15)$$

where we have used the commutation relation of Eq. (11b), and $\bar{Q}$ is the charge average of u and d quarks. Then, after substitution of (15) in (14) and performing the integration, we obtain

$$M^{\text{box}}_{(Z_j)}=-\frac{i}{32\pi^2}g^2f_j^2L^{\mu}\langle p'\mid J_{(W;h)\mu}(0)\mid p\rangle\left[\frac{1}{m_W^2-m_{Z_j}^2}\ln\frac{m_W^2}{m_{Z_j}^2}\right](\tfrac{5}{4}\cos^4\alpha_j+3\bar{Q}\sin^4\alpha_j)\ . \quad (16)$$

This equation can be rewritten in terms of the zeroth-order amplitude

$$\hat{M}_0=-\frac{i}{2}g^2\frac{L^{\mu}}{m_W^2}\langle p'\mid J_{(W;h)\mu}(0)\mid p\rangle\ , \quad (17)$$

so we have

$$M^{\text{box}}_{(Z_j)}=-\hat{M}_0\frac{1}{16\pi^2}f_j^2\cos^4\alpha_j(\tfrac{5}{2}\cot^2\alpha_j+3\bar{Q}\tan^2\alpha_j)\ln\cos^2\alpha_j\ , \quad (18)$$

where we have used the definition of the angles α_j given in Sec. II.

Contributions from Figs. 2 and 3 are to be combined with the corresponding photonic and W-exchange contributions. The result is a universal multiple of M_0 (see the Appendix).

IV. RESULT

To get the final result we need to add the photonic corrections from Figs. 1(b), 2, and 3. This correction is given in Ref. 3:

$$M_{(\gamma)}=\hat{M}_0\frac{3\alpha}{4\pi}[(1+2\bar{Q})\ln m_W^2+\bar{Q}I_{g_s^2}+U]\ , \quad (19)$$

where U represents universal contributions and $I_{g_s^2}$ represent asymptotic effects induced perturbatively by the strong interactions. Adding Eqs. (17), (18), and (19) we obtain the result we are looking for. It is convenient to compare it with the analogous result for muon decay. In this last case $\bar{Q}=-\frac{1}{2}$ and no $I_{g_s^2}$ contribution appears. Then for muon decay we get, up to $O(\alpha G_F)$,

$$M_{(\mu)}=\hat{M}_{0(\mu)}\left[1-\frac{1}{32\pi^2}\sum_{j=1}^{N}f_j^2\cos^4\alpha_j(5\cot^2\alpha_j-3\tan^2\alpha_j)\ln\cos^2\alpha_j\right]\ , \quad (20)$$

where

$$\hat{M}_{0(\mu)}=-\frac{ig^2}{2m_W^2}(\bar{u}_{\nu_\mu}\gamma^{\alpha}a_{-}u_{\mu})(\bar{u}_e\gamma_{\alpha}a_{-}u_{\nu_e})\ .$$

Defining

$$\frac{1}{\sqrt{2}}G_{(\mu)}=\frac{g^2}{8m_W^2}\left[1-\frac{1}{32\pi^2}\sum_{j=1}^{N}f_j^2\cos^4\alpha_j(5\cot^2\alpha_j-3\tan^2\alpha_j)\ln\cos^2\alpha_j\right]\ , \quad (21)$$

we find for the amplitude for muon decay

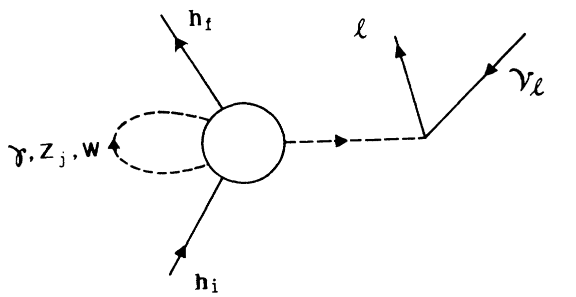

FIG. 2. Diagrams leading to three-current correlations functions.

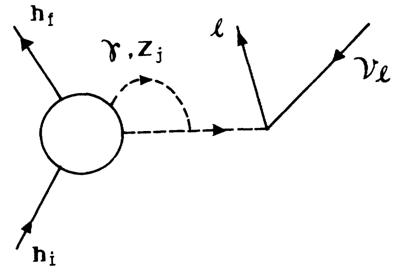

FIG. 3. Diagrams contributing to the Whh'-vertex corrections.

$$M_{(\mu)}=\frac{8m_W^2}{\sqrt{2}g^2}G_{(\mu)} .$$

Thus, calling M the sum of Eqs. (17), (18), and (19), and introducing Eq. (21) we get, for the $O(\alpha)$ radiative corrections to an arbitrary semileptonic process mediated by W,

$$M=M_0\left[1+\frac{3\alpha}{8\pi}(1+2\bar{Q})\left[\ln m_W^2-\sum_{j=1}^{N}\frac{f_j^2}{e^2}\cos^2\alpha_j\sin^2\alpha_j\ln\cos^2\alpha_j\right]+\cdots\right], \tag{22}$$

where

$$M_0=i\sqrt{8}\,G_{(\mu)}\langle p'\,|\,J_{(W;h)\mu}(0)\,|\,p\,\rangle L^{\mu}$$

and the ellipsis refers to the universal renormalization contributions and the induced ones by the strong interaction. The standard-model limit is obtained when $m_{Z_1}=m_{Z_0}$, where Z_0 is the standard neutral weak boson. In this limit $\cos\alpha_1=\cos\theta_W$, $f_1^2=g^2\sec^2\theta_W$, and $f_j=0\forall j\neq 1$. Equation (22) is the generalization for the asymptotic theorem of Ref. 3, and shows that the coefficient of the asymptotic contribution, the one in large parentheses, depends on the average charge $\bar{Q}$ of the u and d quarks but is otherwise independent of the dynamics of the strong interaction. Thus, the result is suitable to any arbitrary semileptonic process, provided the momentum transfer be negligible and the hadronic squared masses are much less than $m_W^2, m_{Z_j}^2$ for all j.

As an application of our result we can generalize also a result by Marciano and Sirlin[7] on the radiative corrections—to differential cross section for the process $\nu_\mu+N\to\mu^-+X$:

$$d\sigma(\nu_\mu+N\to\mu^-+X)=d\sigma^0\left[1+\frac{3\alpha}{4\pi}(1+2\bar{Q})\left[\ln m_W^2-\sum_{j=1}^{N}\frac{f_j^2}{e^2}\cos^2\alpha_j\sin^2\alpha_j\ln\cos^2\alpha_j\right]+\cdots\right], \tag{23}$$

where $d\sigma^0$ is the differential cross section to zeroth order in α but expressed in terms of $G_{(\mu)}^2$ and the ellipsis represents additional nonlogarithmic contributions.

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APPENDIX

Here we show that the combination of contributions from the exchange of γ, W, or Z_j in Figs. 2 and 3 are independent of the nature of the initial and final hadrons in the large m_W, m_{Z_j} limit.

The amplitude for the diagram in Fig. 2 is

$$M_2=\frac{i}{2}g^2L_\mu\langle p'\,|\,J^{\mu}_{(W;h)}(0)\,|\,p\,\rangle'\frac{1}{q^2-m_W^2}, \tag{A1}$$

where $\langle p'\,|\,J^{\mu}_{(W;h)}(0)\,|\,p\,\rangle'$ includes the order-α radiative corrections associated with the emission and absorption of a virtual γ, W, or Z_j from any hadronic line. This correction can be expressed in terms of the Fourier transform of the time-ordered product of three currents,[8] and are handled using Ward identities to reduce them into residual three-current correlation functions and two-current correlation functions. The three-current correlation functions are to be combined with the order-α counterterms, tadpole terms, and tadpole counterterms in the hadronic lines to yield contributions of $O(G_F^2)$, which are neglected, provided that terms of $O(G_F\alpha q)$ are ignored.

The two-current correlation functions, denoted by $V_{(a)}$, are relevant to our discussion. Their contributions to Eq. (A1) are

$$V_{(\gamma)}\equiv V_{(\gamma;1)}+V_{(\gamma;2)} ,$$

where

$$V_{(\gamma;1)}=\frac{i}{2}e^2g^2\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)^2}k^\mu T^\lambda_{(\gamma)\lambda}(k)\,, \tag{A2}$$

$$V_{(\gamma;2)}=-\frac{i}{2}e^2g^2\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{m_W^2}{(k^2-m_W^2)^2}k^\mu T^\lambda_{(\gamma)\lambda}(k)\,, \tag{A3}$$

$$V_{(Z_j)}=\frac{i}{2}g^2f_j^2\cos^2\alpha_j\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_{Z_j}^2)^2}k^\mu T^\lambda_{(Z_j)\lambda}(k)\,, \tag{A4}$$

and

$$V_{(W)}\equiv V_{(W;1)}+V_{(W;2)}\,,$$

where

$$V_{(W;1)}=\frac{i}{2}g^4\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)^2}k^\mu T^\lambda_{(Z_j)\lambda}(k)\,, \tag{A5}$$

$$V_{(W;2)}=\frac{i}{2}g^4\sin^2\alpha_j\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)^2}k^\mu T^\lambda_{(\gamma)\lambda}(k)\,. \tag{A6}$$

Here $T^{\lambda\rho}_{(\gamma)}(k)=\int d^4k\, e^{ik\cdot x}\langle p'\,|\,T[J^\lambda_{(\gamma;h)}(x)J_{(W;h)\lambda}(0)]\,|\,p\,\rangle$. Contribution (A3) gives terms of $O(G_F^2)$ due to the convergence factor $m_W^2/(k^2-m_W^2)$. Contributions (A2), (A4), (A5), and (A6) are to be discussed in conjunction with the two-current correlation functions associated to the vertex $Wh'h$ in Fig. 3. After some algebraic manipulations these last contributions are

$$U_{(Z_j)}=\frac{i}{2}g^2f_j^2\cos^2\alpha_j\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_{Z_j}^2)^2}k^\mu T^\lambda_{(Z_j)\lambda}(k)\,, \tag{A7}$$

$$U_{(\gamma)}=-\frac{i}{2}e^2g^2\frac{L_\mu}{q^2-m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^2-m_W^2)}[2k^\mu T^\lambda_{(\gamma)\lambda}(k)+2i\langle p'\,|\,J^\mu_{(W;h)}(0)\,|\,p\,\rangle]\,. \tag{A8}$$

Combining $V_{(W;1)}$, $V_{(Z_j)}$, and $U_{(Z_j)}$ we get

$$V_{(W;1)}+\sum_{j=1}^{N}(V_{(Z_j)}+U_{(Z_j)})=ig^2\frac{L_\mu}{q^2-m_W^2}\left[\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\sum_{j=1}^{N}f_j^2\cos^2\alpha_j\left[\frac{1}{k^2-m_W^2}-\frac{1}{k^2-m_{Z_j}^2}\right]k^\mu T^\lambda_{(Z_j)\lambda}(k)+\sum_{j=1}^{N}I^\mu_{(j)}\right], \tag{A9}$$

where

$$I^\mu_{(j)}\equiv -if_j^2\cos^4\alpha_j\langle p'\,|\,J^\mu_{(W;h)}(0)\,|\,p\,\rangle\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m_W^2)(k^2-m_{Z_j}^2)}\,,$$

and we have used Eq. (10a).

Similarly

$$V_{(W;2)}+V_{(\gamma;1)}+U(\gamma)=\frac{i}{2}g^2\frac{L_\mu}{q^2-m_W^2}\left[\int\frac{d^4k}{(2\pi)^4}\left[\frac{e^2m_W^2}{k^2(k^2-m_W^2)^2}+\frac{g^2\sin^2\alpha_j}{(k^2-m_W^2)^2}-\frac{e^2}{k^2(k^2-m_W^2)}\right]k^\mu T^\lambda_{(\gamma)\lambda}(k)+I^\mu\right], \tag{A10}$$

where

$$I^\mu=2ie^2\langle p'\,|\,J^\mu_{(W;h)}(0)\,|\,p\,\rangle\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k^2-m_W^2)}\,.$$

The contributions from $I^\mu_{(j)}$ and I^μ are absorbed into a universal renormalization factor of the weak coupling constant g. To evaluate Eqs. (A9) and (A10) we proceed as after Eq. (14). The Bjorken-Johnson-Low asymptotic limit of $T^\lambda_{(\gamma)\lambda}(k)$ is given by

$$T^\lambda_{(\gamma)\lambda}(k)=\frac{2i}{k^2}k_\lambda\langle p'\,|\,J^\lambda_{(W;h)}(0)\,|\,p\,\rangle\,,$$

where we have used Eq. (11a). Thus, we can perform the integrations in (A9) and (A10). The result is

$$(\mathrm{A9})=-\hat{M}_0\sum_{j=1}^{N}\frac{1}{32\pi^2}f_j^2\cos^2\alpha_j\times[2+(1+\cos^2\alpha_j)\csc^2\alpha_j\ln\cos^2\alpha_j]$$

and

$$(\mathrm{A10})=\hat{M}_0\frac{\alpha}{2\pi}\left[1-\frac{1}{2}\left[1-\frac{g^2}{e^2}\sin^2\alpha_J\right]\left[1+\ln\frac{m_W^2}{\Lambda^2}\right]\right].$$

In Eq. (A10) we have introduced an ultraviolet cutoff Λ. These results show that both contributions are multiples of $\hat{M}_0$, independent of the nature of the initial and final hadrons. As is pointed out in Ref. 4 the results for (A9) and (A10) must be multiplied by a factor $1-\bar{g}_s^2(k^2)/4\pi^2$, which for four flavors is approximately -5% relative to the above results.

As in the standard model the corrections to the W propagator and those involving the Higgs scalars are of $O(G_F^2)$. In the last case it is assumed that the couplings of the Higgs scalars to quarks and leptons are proportional to the mass of the elementary fermions.

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