Supersymmetry in a two-dimensional interacting fermionic model

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We discuss supersymmetry in a two-dimensional self-interacting fermionic model using the path-integral approach. After analyzing the equivalence between the purely fermionic model and the supersymmetric sine-Gordon model, we show that the supersymmetric transformations, which are nonlinear in fermionic fields, have an associated supercurrent which is conserved only at the quantum level: the noninvariance of the purely fermionic action is compensated by the noninvariance of the fermionic measure. The connection between these results and those arising in free fermionic models are also discussed.

I. INTRODUCTION AND RESULTS

One very interesting aspect of two-dimensional models, relevant in connection with string theories,[1] is the ability of fermions to provide (nonlinear) realizations of the supersymmetry algebra. Indeed, in two dimensions one can hope to represent the full supersymmetry algebra solely in terms of either bosonic or fermionic fields by means of bosonization (fermionization).[2,3]

A first analysis of this property of two-dimensional purely fermionic models was given by Witten[4] in his discussion of a Gross-Neveu model with Majorana fermions and then investigated in some detail by Aratyn and Damgaard.[5]

After the discovery of bosonization rules respecting non-Abelian symmetries,[6] supersymmetry in free fermion systems (equivalent to nonlinear σ models with a Wess-Zumino term) was also investigated, particularly in connection with Kac-Moody algebras and superstrings.[7–12]

In the present work we analyze supersymmetry for two-dimensional self-interacting fermions [we study precisely the O(3) Gross-Neveu[13] model discussed originally by Witten[4,5]] employing the path-integral approach to bosonization.[14]

Using this method we prove in Sec. II the equivalence between the purely fermionic model and the supersymmetric sine-Gordon (SUSY SG) one, showing in a very simple way that the bosonization recipe is also valid in the present case in which a certain mass term in the fermionic version of the theory is in fact x dependent [see Eq. (2.7)].

We then analyze in Sec. III how the supersymmetry invariance of the SUSY SG model translates in the equivalent purely fermionic model. Since bosonization rules do not maintain the same number of bosonic and fermionic degrees of freedom the supersymmetry transformations cannot be linearly realized in both theories; in fact, they look more like an internal fermionic symmetry rather than a proper supersymmetry. Moreover, it is very easy to see that these transformations (which can be guessed by applying the bosonization recipe to the corresponding transformations in the SUSY SG model) do not correspond to a classical symmetry of the fermionic model.

As we carefully show, it is only at the quantum level that the supersymmetric current is conserved: the noninvariance of the fermionic measure cancels out the "classical anomaly." This possibility of having a quantum symmetry which does not appear at the classical level is, in fact, a phenomenon already exploited by Faddeev and Shatashvili[15,16] in their proposal of consistently quantizing anomalous chiral models and the same behavior is encountered in the quantization of strings.[17] (See also Ref. 14.)

It is interesting to note that we give the explicit expression for the SUSY quantum conserved current for arbitrary coupling constant g (it was previously obtained in Ref. 5 for the particular $g=\pi/2$ case). As we discuss, this expression can be related to the one derived by Antoniadis *et al.*[11] for N free Majorana-Weyl fermions in their complete classification of all SUSY theories of free massless two-dimensional fermions.

We briefly describe in the Appendix the evaluation of fermionic Jacobians.

II. THE O(3) FERMIONIC MODEL AND ITS SUSY COUNTERPART

Let us begin by considering the purely fermionic model in two-dimensional (Minkowski) space-time with dynamics defined by the Lagrangian

$$\mathcal{L}=\frac{1}{2}\sum_{j=1}^{3}\bar{\psi}^{j}i\partial\!\!\!/\psi^{j}+g\left[\sum_{j=1}^{3}\bar{\psi}^{j}\psi^{j}\right]^{2}, \tag{2.1}$$

where ψ^j is a three-component Majorana-Fermi field.

Our conventions are

$$(g^{\mu\nu})=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\quad \mu,\nu=0,1\,, \tag{2.2}$$

$$\gamma_0=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix},\quad \gamma_1=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix},$$
$$\gamma_5=\gamma_0\gamma_1=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}. \tag{2.3}$$

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The adjoint $\bar{\psi}$ of a Majorana spinor $\psi=\binom{\psi_1}{\psi_2}$ is

$$\bar{\psi}=\psi^T\gamma^0=(i\psi_2,-i\psi_1)\ . \tag{2.4}$$

This theory, with an obvious O(3) invariance, is in fact the O(3) Gross-Neveu model. Witten[4] was the first to note that this Lagrangian can be shown to be equivalent to the supersymmetric sine-Gordon one. In his proof, two Majorana fermions were combined in a Dirac fermion and the (Abelian) Coleman-Mandelstam[2,3] recipe was then applied. Now the validity of the bosonization recipe can be questioned for the case of Dirac fermions with an x-dependent mass, as turns out to be the case for the Dirac fermion replacing two of the Majorana fields in the Lagrangian (2.1), see below.

Using the path-integral approach[14] we shall prove the equivalence between the model defined by Lagrangian (2.1) and the supersymmetric sine-Gordon model, thus showing the validity of the bosonization recipe also in the present case (where the Dirac fermion mass depends on $\bar{\psi}_3\psi_3$).

Let us start by rewriting (2.1) in terms of a Dirac fermion

$$\psi_D=\frac{1}{\sqrt{2}}(\psi^1+i\psi^2)\ . \tag{2.5}$$

We shall use for that purpose the identities (valid for Majorana fermions)

$$\begin{aligned}&\bar{\chi}\psi=\bar{\psi}\chi\ ,\quad \bar{\chi}\gamma_5\psi=-\bar{\psi}\gamma_5\chi\ ,\\ &\bar{\chi}\gamma_\mu\psi=-\bar{\psi}\gamma_\mu\chi\ .\end{aligned} \tag{2.6}$$

Using (2.5) and (2.6), the Lagrangian (2.1) becomes

$$\begin{aligned}\mathcal{L}=&\bar{\psi}_D i\partial\!\!\!/\psi_D+\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3\\ &-2g(\bar{\psi}_D\gamma^\mu\psi_D)^2+\sigma(x)\bar{\psi}_D\psi_D\ ,\end{aligned} \tag{2.7}$$

where

$$\sigma(x)=4g\bar{\psi}_3\psi_3 \tag{2.8}$$

plays the role of a mass for the Dirac fermion.

In the path-integral approach to bosonization one considers the generating functional

$$Z=\int\mathcal{D}\bar{\psi}_D\mathcal{D}\psi_D\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\exp\left[i\int d^2x\,\mathcal{L}\right] \tag{2.9}$$

and then uses the identity

$$\begin{aligned}&\exp\left[-2ig\int(\bar{\psi}_D\gamma^\mu\psi_D)^2d^2x\right]\\ &\quad=\int\mathcal{D}A_\mu\exp\left[\frac{i}{2}\int(A_\mu A^\mu+4\sqrt{g}\,\bar{\psi}_D\gamma^\mu\psi_D A_\mu)d^2x\right]\end{aligned} \tag{2.10}$$

to rewrite (2.9) in the form

$$Z=\int\mathcal{D}\bar{\psi}_D\mathcal{D}\psi_D\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\mathcal{D}A_\mu e^{iS_F}\ , \tag{2.11}$$

where

$$\begin{aligned}S_F=\int d^2x[&\bar{\psi}_D(i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/)\psi_D+\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3\\ &+\tfrac{1}{2}A_\mu A^\mu+\sigma(x)\bar{\psi}_D\psi_D]\ .\end{aligned} \tag{2.12}$$

The vector field A_μ can be written in terms of two scalar fields ϕ, η in the form

$$A_\mu=-\frac{1}{2\sqrt{g}}(\epsilon_{\mu\nu}\partial^\nu\phi-\partial_\mu\eta) \tag{2.13}$$

and then eliminated from the Dirac operator $D\!\!\!\!/=i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/$ appearing in (2.11) by performing the fermionic transformation

$$\psi_D=e^{i\eta+i\gamma_5\phi}\chi_D\ ,\quad \bar{\psi}_D=\bar{\chi}_D e^{-i\eta+i\gamma_5\phi}\ . \tag{2.14}$$

We shall consider (2.13) and (2.14) as a change of integration variables in (2.11):

$$\mathcal{D}A_\mu=\mathcal{J}_A\mathcal{D}\phi\,\mathcal{D}\eta\ ,\quad \mathcal{D}\bar{\psi}_D\mathcal{D}\psi_D=\mathcal{J}_F\mathcal{D}\bar{\chi}_D\mathcal{D}\chi_D\ . \tag{2.15}$$

Concerning $\mathcal{J}_A$ it gives a trivial contribution

$$\mathcal{J}_A=\frac{1}{4g}\det\nabla^2\ . \tag{2.16}$$

The Jacobian $\mathcal{J}_F$ is evaluated in the Appendix following the techniques reviewed in Ref. 14. The answer is

$$\ln\mathcal{J}_F=-\frac{i}{2\pi}\int d^2x(\partial_\mu\phi\partial^\mu\phi)\ . \tag{2.17}$$

As we discuss in the Appendix, there is an ambiguity in the regularization of $\mathcal{J}_F$ due to the fact that A_μ is not a gauge field and hence there is no gauge principle to invoke when choosing the regularization prescription.

This ambiguity is reflected in a term of the form[14]

$$a\int(A_\mu A^\mu)d^2x$$

with a an arbitrary parameter. However, this arbitrariness can be absorbed in a rescaling of the fields ϕ and η, and hence with no loss of generality we set $a=0$ (Ref. 14).

We then obtain

$$Z=\mathcal{N}\int\mathcal{D}\bar{\chi}_D\mathcal{D}\chi_D\mathcal{D}\phi\,\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\exp\left[i\int\mathcal{L}_{\text{eff}}d^2x\right]\ , \tag{2.18}$$

where

$$\begin{aligned}\mathcal{L}_{\text{eff}}=&\bar{\chi}_D[i\partial\!\!\!/+\sigma(x)e^{2i\gamma_5\phi(x)}]\chi_D\\ &+\left[\frac{1}{8g}-\frac{1}{2\pi}\right]\partial_\mu\phi\partial^\mu\phi+\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3\ .\end{aligned} \tag{2.19}$$

Note that we have absorbed the η-field integration in a normalization constant since it has completely decoupled, playing no role in the resulting effective theory.

In the path-integral approach to bosonization one now considers a perturbative expansion (in the σ term) in the generating functional:[18–20]

$$Z=\int \mathcal{D}\phi\,\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\exp\left\{i\int\left[\left[\frac{1}{8g}-\frac{1}{2\pi}\right]\partial_\mu\phi\partial^\mu\phi+\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3\right]d^2x\right\}Z_F\ , \tag{2.20}$$

$$Z_F=\int \mathcal{D}\bar{\chi}_D\mathcal{D}\chi_D\exp\left[i\int d^2x(\bar{\chi}_D i\partial\!\!\!/\chi_D)\right]\times\prod_{i=1}^{n}\frac{1}{n!}\int dx_i\sigma(x_i)e^{2i\phi(x_i)}\Sigma_+(x_i)\times\prod_{j=1}^{m}\frac{1}{m!}\int dy_j\sigma(y_j)e^{-2i\phi(y_j)}\Sigma_-(y_j)\ , \tag{2.21}$$

where

$$\Sigma_\pm=\bar{\chi}_D\frac{1\pm\gamma_5}{2}\chi_D\ . \tag{2.22}$$

Concerning the fermionic part we have

$$\langle\Sigma_+(x)\Sigma_-(y)\rangle_0\equiv\int\mathcal{D}\bar{\chi}_D\mathcal{D}\chi_D\exp\left[i\int d^2x(\bar{\chi}_D i\partial\!\!\!/\chi_D)\right]\times\Sigma_+(x)\Sigma_-(y)=-\frac{1}{(2\pi)^2}\frac{1}{|x-y|^2}\ , \tag{2.23}$$

$$\langle\Sigma_+(x)\Sigma_+(y)\rangle_0=\langle\Sigma_-(x)\Sigma_-(y)\rangle_0=0\ . \tag{2.24}$$

For the bosonic one we shall use

$$\left\langle\exp\left[i\sum_i\beta_i\phi(x_i)\right]\right\rangle_0=\int\mathcal{D}\phi\exp\left\{i\int\left[\left[\frac{1}{8g}-\frac{1}{2\pi}\right]\partial_\mu\phi\partial^\mu\phi\right]d^2x\right\}\exp\left[i\sum_i\beta_i\phi(x_i)\right]=\exp\left[-\tfrac{1}{2}\sum_{i,j}\beta_i\beta_j[\Delta_F(\mu,x_i-x_j)-\Delta_F(\Lambda,x_i-x_j)]\right]\ , \tag{2.25}$$

where Λ is a large mass introduced to cut off the theory[17] and

$$\Delta_F(\mu,x)=\frac{\lambda^2}{2\pi}K_0(\mu,x) \tag{2.26}$$

with

$$\lambda^2=\frac{4g\pi}{\pi-4g}\ . \tag{2.27}$$

In order to circumvent the trivial infrared problems which arise when one performs a mass perturbation expansion about a massless theory we consider the restricted region[2-17]

$$\mu\,|x_i-x_j|\ll 1\ , \tag{2.28}$$

also

$$\Lambda\,|x_i-x_j|\gg 1\ . \tag{2.29}$$

We then obtain

$$\left\langle\exp\left[i\sum_i\beta_i\phi(x_i)\right]\right\rangle_0=\left[\frac{\mu}{M}\right]^{-(\lambda^2/4\pi)(\sum_i\beta_i)^2}\times\left[\frac{M}{\Lambda}\right]^{-(\lambda^2/4\pi)\sum_i\beta_i^2}\times\prod_{i>j}(c\,|x_i-x_j|)^{-(\lambda^2/2\pi)\beta_i\beta_j}\ , \tag{2.30}$$

where c is the Euler constant, $c=0.577$, and the mass M is included as a normal order in mass.

Note that if $\sum_i\beta_i\neq 0$, (2.30) vanishes in the limit $\mu\to 0$. We thus consider the case $\sum_i\beta_i=0$.

Using (2.21), (2.24), and (2.30) we get, for the generating functional (2.20),

$$Z=\int\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\frac{\int\prod_{i>j}^{k}(M^2c^2\,|x_i-x_j|\,|y_i-y_j|)^{2+2\lambda^2/\pi}\prod_{i,j=1}^{k}\sigma(x_i)\sigma(y_i)d^2x_id^2y_i}{\prod_{i,j=1}^{k}(Mc\,|x_i-y_j|)^{2+2\lambda^2/\pi}}\exp\left[i\int d^2x\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3\right]\ . \tag{2.31}$$

It is easy to see that this generating functional coincides with that arising in the SUSY sine-Gordon model. Indeed, the generating functional for this last model is given by

$$Z_{\rm SG}=\mathcal{N}\int\mathcal{D}\phi\,\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3\exp\left[i\int d^2x\left[\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi+\tfrac{1}{2}\bar{\psi}_3 i\partial\!\!\!/\psi_3+2g\frac{Me^c}{\pi}\bar{\psi}_3\psi_3\cos\beta\phi\right]\right]\ , \tag{2.32}$$

$$\beta=\sqrt{4\pi}\frac{1}{\sqrt{1-4g/\pi}}\ , \tag{2.33}$$

where M is the same regulating mass as in (2.31). Then, performing a perturbative expansion for the cosine term one just gets the same expansion as in (2.31).

It is convenient to rescale variables in the form

$$x \to M'^{-1}x, \quad \psi_3 \to M'^{1/2}\psi_3 , \qquad \phi \to \frac{\phi}{\sqrt{1-4g/\pi}}, \quad M' \to \frac{e^c}{\pi}M . \tag{2.34}$$

In the rescaled variables, the Lagrangian in (2.32) reads

$$\mathcal{L} = \frac{1}{2}\left[1-\frac{4g}{\pi}\right]\partial^\mu\phi\partial_\mu\phi + \tfrac{1}{2}\bar\psi_3 i\partial\!\!\!/\psi_3 + 2g\bar\psi_3\psi_3\cos 2\sqrt{\pi}\phi . \tag{2.35}$$

We then see that the bosonization rules hold as in the normal sine-Gordon-Thirring case (where instead of $\bar\psi_3\psi_3$ playing the role of a mass one has a true mass term).

Note that the Lagrangian (2.35) does not appear as the usual supersymmetric sine-Gordon model which is usually written as

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \tfrac{1}{2}\bar\psi_3 i\partial\!\!\!/\psi_3 + 2A^2\cos^2 B\phi + AB\cos B\phi\bar\psi_3\psi_3 . \tag{2.36}$$

Now, since the $\cos^2$ term can be rewritten as a kinetic term,

$$\cos^2 B\phi = -\frac{B^2}{2\pi^2}\partial_\mu\phi\partial^\mu\phi \tag{2.37}$$

(this relation can be obtained using bosonization), one can see that (2.35) and (2.36) coincide at least for $g=\pi/2$. Now since a renormalization of the kinetic term in (2.35) is always possible (this being equivalent to a rescaling of the field ϕ) one can, in fact, easily add just the Lagrangian (2.35) to the form (2.36) with arbitrary $g^{(4)}$.

So we have proven, using the path-integral bosonization scheme, the equivalence between the O(3) purely fermionic model and the sine-Gordon supersymmetric one.

III. THE SUPERCURRENT IN THE FERMIONIC MODEL

We shall address in this section the question of how the explicit supersymmetry of the model defined by Eq. (2.36) is manifested in the equivalent purely fermionic model with Lagrangian (2.1). Of course the supersymmetry transformation cannot be linearly realized in both theories since the bosonization rules do not maintain the same number of bosonic and fermionic degrees of freedom.

It will become clear from our approach that a certain purely fermionic transformation, which is equivalent to the SUSY transformation in the sine-Gordon model, does not correspond to a symmetry of the classical fermionic action; i.e., it has no associated conserved current. It is at the quantum level (and due to the noninvariance of the fermionic measure under that transformation) that the Noether theorem holds and the symmetry reappears. The same phenomenon already happens in the path-integral quantization of anomalous gauge theories[15,16] and string theory.[17]

Let us begin by recalling the supersymmetry transformations, leaving invariant the SUSY sine-Gordon model with the Lagrangian (2.36):

$$\delta\phi = \bar\epsilon\psi , \qquad \delta\psi = [-i\partial\!\!\!/\phi + \sqrt{\pi}\sin(\sqrt{4\pi}\phi)] , \tag{3.1}$$

where $\bar\epsilon$ is a constant Majorana spinor. (We have specialized to the case $g=\pi/2$, but as we discussed at the end of Sec. II everything holds for arbitrary g.)

We shall now consider the action of the following infinitesimal nonlinear transformation on the purely fermionic model:

$$\psi_D \to \psi'_D = \psi_D + i\sqrt{\pi}\bar\epsilon\psi_3\gamma_5\psi_D , \tag{3.2}$$

$$\psi_3 \to \psi'_3 = \psi_3 + i\sqrt{\pi}\gamma^\mu\bar\psi_D\gamma_\mu\gamma_5\psi_D\epsilon + 2i\sqrt{\pi}\bar\psi_D\gamma_5\psi_D\epsilon \tag{3.3}$$

[these transformations were first considered in Ref. 5 by naive bosonization of (3.1)].

Under transformations (3.2) and (3.3) the fermionic action (2.12) changes as

$$S_F \to S'_F = S_F + \int d^2x\,\{2\pi\bar\epsilon\partial_\nu(\bar\psi_D\gamma^\nu\gamma_5\psi_D)\psi_3 - \sqrt{\pi}\bar\epsilon\partial_\nu[(\gamma^\mu\bar\psi_D\gamma_\mu\gamma_5\psi_D\gamma^\nu + 2\bar\psi_D\gamma_5\psi_D\gamma^\nu)\psi_3]\} . \tag{3.4}$$

Now, setting

$$j_5^\nu = \bar\psi_D\gamma^\nu\gamma_5\psi_D , \tag{3.5}$$

$$\mathcal{J}^\nu = -i\sqrt{\pi}(\gamma^\mu\bar\psi_D\gamma_\mu\gamma_5\psi_D + 2\bar\psi_D\gamma_5\psi_D)\gamma^\nu\psi_3 , \tag{3.6}$$

(3.4) becomes

$$S_F \to S'_F = S_F + \int d^2x\,[2\sqrt{\pi}(\partial_\nu j_5^\nu)\bar\epsilon\psi_3 - i\bar\epsilon\partial_\nu\mathcal{J}^\nu] . \tag{3.7}$$

Obviously (3.2) and (3.3) do not correspond to a symmetry of the fermionic action (2.12) and hence the Noether theorem does not hold at the classical level. In order to study this transformation at the quantum level we consider (3.2) and (3.3) as a change of variables in the generating function (2.11) and then evaluate the associated Jacobian[14]

$$\mathcal{D}\bar{\psi}_D\mathcal{D}\psi_D\mathcal{D}\bar{\psi}_3\mathcal{D}\psi_3=\mathcal{J}_F\mathcal{D}\bar{\psi}'_D\mathcal{D}\psi'_D\mathcal{D}\bar{\psi}'_3\mathcal{D}\psi'_3 \ . \tag{3.8}$$

Since A_μ is coupled solely to ψ_D, only transformation (3.2) (a chiral one) can lead to a nontrivial Jacobian. The answer is [see the Appendix, Eq. (A12)]

$$\mathcal{J}_F=\exp\left[i\sqrt{2}\int d^2x(\bar{\epsilon}\psi_3\epsilon_{\mu\nu}\partial^\mu A^\nu)\right] . \tag{3.9}$$

In terms of the new variables the generating function (2.12) reads

$$Z=\int \mathcal{D}\bar{\psi}'_D\mathcal{D}\psi'_D\mathcal{D}\bar{\psi}'_3\mathcal{D}\psi'_3\mathcal{D}A_\mu\exp\left[iS_F(\bar{\psi}'_D,\psi'_D,\bar{\psi}'_3,\psi'_3,A_\mu)+i\int d^2x(-i\bar{\epsilon}\partial_\nu\mathcal{J}^\nu-2\sqrt{\pi}\bar{\epsilon}j_5^\nu\partial_\nu\psi_3+\sqrt{2}\bar{\epsilon}\psi_3\epsilon_{\mu\nu}\partial^\mu A^\nu)\right] . \tag{3.10}$$

Since Z cannot depend on $\bar{\epsilon}$,

$$\frac{1}{Z}\frac{\partial Z}{\partial\bar{\epsilon}}=0=\langle\partial_\nu\mathcal{J}^\nu\rangle-2i\sqrt{\pi}\langle j_5^\nu\partial_\nu\psi_3\rangle +i\sqrt{2}\langle\epsilon^{\mu\nu}\partial_\mu A_\nu\psi_3\rangle \ . \tag{3.11}$$

Being quadratic, the path integration over A_μ can be exactly performed. In particular one gets, for the third term in (3.11),

$$i\sqrt{2}\langle\epsilon^{\mu\nu}\partial_\mu A_\nu\psi_3\rangle=-2i\sqrt{\pi}\langle\partial_\mu j_5^\mu\psi_3\rangle \ . \tag{3.12}$$

This term cancels out the second one in (3.11) and we thus have

$$\langle\partial_\mu\mathcal{J}^\mu\rangle=0 \ . \tag{3.13}$$

The transformations (3.2) and (3.3) then correspond to a quantum symmetry of the model defined in (2.11) and (2.12); $\mathcal{J}^\mu$ is the associated conserved current.

As we stated above $\mathcal{J}^\nu$ as given by (3.6) corresponds to the particular case $g=\pi/2$. For the general case (obtained by rescaling the ϕ field) the supersymmetric conserved current reads

$$\mathcal{J}^\nu=-i\sqrt{\pi}\gamma^\mu\bar{\psi}_D\gamma_\mu\gamma_5\psi_D\gamma^\nu\psi_3 -4gi\left[\frac{4g-\pi}{8g^2-\pi^2}\right]^{1/2}\bar{\psi}_D\gamma_5\psi_D\gamma^\nu\psi_3 \ . \tag{3.14}$$

An expression of this kind was estimated in Ref. 4 without giving the explicit g dependence, which we have calculated in (3.14).

It is clear from our approach that in the purely fermionic version of the SUSY sine-Gordon model the supersymmetry is realized at the quantum level as a nonlinear transformation that looks more like an internal fermionic symmetry rather than a proper supersymmetry. (It is shown in Ref. 4 that the charges associated with a current of the kind of $\mathcal{J}_\mu$ satisfy the usual supersymmetry algebra.)

It is interesting to note that for $g=0$ (free fermion model) and considering Weyl-Majorana fermions the supersymmetry that we have discussed becomes the one investigated in Refs. 7–12 in the general O(N) case.

Indeed, by considering chiral components for the fermions,

$$\psi^i=\begin{bmatrix}\psi^i_+\\ \psi^i_-\end{bmatrix} , \tag{3.15}$$

it is easy to see that the SUSY transformation (3.2) and (3.3) can be rewritten, for positive-chirality components, in the form

$$\delta\psi^i_+=i\sqrt{\pi}\eta^{ijk}\psi^j_+\psi^k_+\epsilon \ , \tag{3.16}$$

where η^{ijk} is totally antisymmetric in its indices and corresponds to the structure constants of O(3) with a particular normalization. (An analogous expression holds for negative-chirality components.) We then see that the result of Antoniadis *et al.*[11] concerning the fact that the structure constants of semisimple Lie groups exhaust all possible supersymmetries among two-dimensional free fermions also holds for the present interacting model.

In summary we have shown how supersymmetry is manifested in a two-dimensional self-interacting fermion model. The corresponding transformations are nonlinear and have an associated supercurrent which is related to the one arising in the free fermion system; contrary to this last case, it is only conserved at the quantum level; this being due to the noninvariance of the fermionic measure under transformation (3.2). That is, bosonization makes the effective action (2.12) noninvariant under the SUSY transformation (due to anomalies), but the fermionic measure restores this invariance exactly as it happens in chiral gauge models[15,16] and string theories.[17]

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APPENDIX: THE FERMION JACOBIAN

We shall first compute the Jacobian associated with the following infinitesimal chiral transformation:

$$\psi_D(x)=e^{i\delta\phi(x)\gamma_5}\chi_D\ ,$$
$$\bar\psi_D(x)=\bar\chi_D e^{i\delta\phi(x)\gamma_5}\ . \tag{A1}$$

In terms of the new fermions fields χ_D, $\bar\chi_D$ the Lagrangian defined by (2.12),

$$\mathcal{L}(\bar\psi_D,\psi_D,\bar\psi_3,\psi_3,A_\mu)=\bar\psi_D(i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/\,)\psi_D+4g\bar\psi_3\psi_3\bar\psi_D\psi_D \tag{A2}$$

becomes

$$\mathcal{L}'(\bar\chi_D,\chi_D,\bar\psi_3,\psi_3,A_\mu)=\bar\chi_D(i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/+4g\bar\psi_3\psi_3e^{2i\delta\phi\gamma_5})\chi_D\ , \tag{A3}$$

and hence in terms of the transformed variables the generating functional reads

$$Z=\int \mathcal{D}\bar\chi_D\mathcal{D}\chi_D\mathcal{D}\bar\psi_3\mathcal{D}\psi_3\mathcal{D}A_\mu\mathcal{J}_F\exp\left[i\int\mathcal{L}'d^2x\right]\ , \tag{A4}$$

where[21,14]

$$\mathcal{J}_F=\exp\left[i\int d^2x\,\mathrm{tr}(\gamma_5\delta\phi)\right] \tag{A5}$$

and

$$\mathrm{tr}(\gamma_5\delta\phi)=\sum_n\varphi_n^\dagger(x)\gamma_5\varphi_n(x)\ . \tag{A6}$$

The expression (A6) is ill defined and has to be evaluated by using a regularization procedure. Following Fujikawa[21] we consider

$$\sum_n\varphi_n^\dagger(x)\gamma_5\varphi_n(x)\equiv\lim_{M\to\infty}\sum_n\varphi_n^\dagger(x)\gamma_5e^{-D_{\rm reg}^2/M^2}\varphi_n(x)\ , \tag{A7}$$

where

$$D_{\rm reg}=i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/+4g\bar\psi_3\psi_3e^{2i\delta\phi\gamma_5}\ . \tag{A8}$$

Setting

$$\mathcal{J}_F=\exp\left[-\frac{2\sqrt{g}}{\pi}i\int d^2x\,\epsilon_{\mu\nu}\partial^\mu(\delta\phi)A^\nu\right]\ , \tag{A9}$$

where we have used

$$\mathrm{tr}(\gamma_\nu\gamma_\mu\gamma_5)=2\epsilon_{\mu\nu}\ , \tag{A10}$$

the transformation (A1) corresponds to the one defined by Eq. (3.2) if

$$\delta\phi=\sqrt{\pi}\bar\epsilon\psi_3\ . \tag{A11}$$

Hence the Jacobian associated with the infinitesimal transformation (3.2) is

$$\mathcal{J}_F=\exp\left[-\frac{2\sqrt{g}}{\pi}i\sqrt{\pi}\bar\epsilon\int d^2x\,\epsilon_{\mu\nu}(\partial^\mu\psi_3)A^\nu\right]\ . \tag{A12}$$

In order to compute the Jacobian associated with the finite transformation

$$\psi_D=e^{i\eta(x)+i\gamma_5\phi(x)}\chi_D\ ,$$
$$\bar\psi_D=\bar\chi_De^{-i\eta(x)+i\gamma_5\phi(x)}\ , \tag{A13}$$

where ϕ and η are related to A_μ according to (2.13), one builds up the transformation (A13) from infinitesimal ones by introducing a parameter t, $0\le t\le 1$. (See Ref. 14 for details.)

The answer is

$$\mathcal{J}_F=\exp\left[i\int_0^1\mathcal{A}(\phi,\eta;t)dt\right] \tag{A14}$$

with

$$\mathcal{A}=\lim_{M^2\to\infty}\mathrm{tr}\gamma_5\phi e^{-D_{\rm reg}^2/M^2}\ , \tag{A15}$$

where

$$D_{\rm reg}=e^{i(\gamma_5\phi-\eta)t}(i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/+4g\bar\psi_3\psi_3)e^{i(\gamma_5\phi+\eta)t} \tag{A16}$$

or

$$D_{\rm reg}=i\partial\!\!\!/+2\sqrt{g}\,(1-t)A\!\!\!/+4g\bar\psi_3\psi_3e^{2i\gamma_5\phi t}\ . \tag{A17}$$

Expanding the exponential in (A15) one gets

$$\mathcal{A}=\lim_{M^2\to\infty}\frac{1}{2\pi^2}\mathrm{tr}\gamma_5\int\phi\,d^2x\,e^{-k^2/M^2}\times\left[1-\frac{D_{\rm reg}^2}{M^2}+O\left[\frac{1}{M^4}\right]\right]\ . \tag{A18}$$

In two dimensions it is only the $1/M^2$ term contributing in the $M^2\to\infty$. After rescaling variables and integrating over k and t we finally get

$$\mathcal{J}_F=\exp\left[-\frac{i}{2\pi}\int d^2x\,\partial_\mu\phi\partial^\mu\phi\right]\ . \tag{A19}$$

It is important to stress that A_μ is not a gauge field and hence there is no gauge principle restricting the choice of the regularization operator $D_{\rm reg}$. In fact, a more general operator of the form

$$\tilde D_{\rm reg}=e^{i(\gamma_5\phi-\eta)t}\left[i\partial\!\!\!/+2\sqrt{g}\,A\!\!\!/\frac{1-\gamma_5}{2}+2\sqrt{g}\,a\,A\!\!\!/\frac{1+\gamma_5}{2}+4g\bar\psi_3\psi_3\right]e^{i(\gamma_5\phi+\eta)t} \tag{A20}$$

with a an arbitrary parameter which can be employed.

As explained in Ref. 14 this amounts to a change in the Jacobian (A20), which becomes

$$\mathcal{J}_a=\mathcal{J}\exp\left[-\frac{ia}{2\pi}\int A_\mu A^\mu d^2x\right]\ . \tag{A21}$$

For the case of the model we are interested in, this a dependence can be absorbed by a rescaling of the η and ϕ fields.

[1]P. van Nieuwenhuizen, in *Supersymmetry, Supergravity and Superstrings,* edited by B. de Wit *et al.* (World Scientific, Singapore, 1986), and references therein.
[2]S. Coleman, Phys. Rev. D **11**, 2088 (1975).
[3]S. Mandelstam, Phys. Rev. D **11**, 3026 (1975).
[4]E. Witten, Nucl. Phys. **B142**, 285 (1978).
[5]H. Aratyn and P. H. Damgaard, Nucl. Phys. **B241**, 253 (1984).
[6]E. Witten, Commun. Math. Phys. **92**, 455 (1984).
[7]P. di Vecchia, V. G. Knizhnik, J. L. Petersen, and P. Rossi, Nucl. Phys. **B252**, 701 (1985).
[8]T. Curtright and C. Zachos, Phys. Rev. Lett. **53**, 1799 (1984).
[9]R. Rohm, Phys. Rev. D **32**, 2849 (1985).
[10]P. Goddard and D. Olive, Nucl. Phys. **B257** [FS14], 226 (1985).
[11]I. Antoniadis, C. Bachas, C. Kounnas, and P. Windey, Phys. Lett. B **171**, 51 (1986).
[12]P. Windey, Commun. Math. Phys. **105**, 511 (1986).
[13]D. J. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974).
[14]F. A. Schaposnik, in Proceedings of a Lecture Delivered at ELAF, La Plata, 1987 (unpublished), and references therein.
[15]L. Faddeev and S. L. Shatashvili, Phys. Lett. **167B**, 225 (1986).
[16]O. Babelon, F. A. Schaposnik, and C. M. Viallet, Phys. Lett. B **177**, 385 (1986); K. Harada and H. Tsutsui, *ibid.* **183**, 311 (1986).
[17]L. Bonora, M. Martellini, and C. M. Viallet, CERN Report No. TH 4796/87, 1987 (unpublished).
[18]C. M. Naon, Phys. Rev. D **31**, 2035 (1985).
[19]M. I. Eides, Phys. Lett. **153B**, 157 (1985).
[20]H. Dorn, Phys. Lett. **167B**, 86 (1986).
[21]K. Fujikawa, in *Quantum Gravity and Cosmology,* proceedings of the Eighth Kyoto Summer Institute, Kyoto, Japan, 1985, edited by H. Sato and T. Inami (World Scientific, Singapore, 1986), and references therein.