Density of states in the quantum-chromodynamic bag model with a nonzero baryon number

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We generalize the analysis of a quark-gluon plasma to the case of a nonzero baryon number. This is done in the context of a bag model upgraded by perturbative QCD corrections. Massive quarks are ignored in the calculation but the full running coupling constant is implemented by an explicit dependence on the temperature and chemical potentials. First, the thermodynamical properties are examined and, in the case of a system of hadrons, the likelihood of a transition from a quark-gluon plasma to hadronic matter is discussed; next, in view of its probable relevance to models of relativistic nucleus-nucleus collisions, the hadronic density of states is derived by a Laplace inversion of the partition function and its form is discussed.

I. INTRODUCTION

In view of the planned nucleus-nucleus collision experiments[1,2] at energies possibly sufficient to trigger a phase transition from hadrons to a quark-gluon plasma, it is of definite interest to know the hadronic mass spectrum of a large bag of quarks and gluons. In addition, recent works[3–5] suggest that in this energy regime, the incoming nuclei will probably stop each other in the center-of-mass frame resulting in the formation of a single excited system possessing a large baryon number.

So far, it has not been possible to calculate all the desired thermodynamical quantities within the single theory of quantum chromodynamics (QCD). Though the lattice approach has been the most fruitful, no clear results have yet been obtained in the presence of valence quarks.[6,7] In this case, one has to rely on, for instance, phenomenological two-phase approaches.

In a previous paper,[8] one of us (R.G.) presented a calculation of the hadronic density of states in the quantum-chromodynamic bag model. Although the effect of a temperature-dependent coupling constant was implemented in the model, baryon-number conservation was not imposed. Under these conditions, the resulting asymptotic density of states assumed the same form as in the statistical bootstrap model.[9]

In this paper, we generalize this calculation to incorporate a nonzero baryon number. In Sec. II, the thermodynamics of the model is presented taking into account a running coupling constant dependent on the temperature and the chemical potential of the system. In Sec. III, the resulting hadronic density of states is obtained. Section IV contains a brief summary.

II. THERMODYNAMICS OF THE MODEL

The basic idea is to regard hadronic matter as a bag of quarks and gluons. What first comes to mind is the free-gas approximation. However, it would be rather naive to argue the pertinence of this model with regard to bound states of quarks. Thus, to be useful at all, this simple picture must be substantially modified and interactions accounted for.

QCD has given us little insight on the properties of these nonperturbative states. Therefore, we must rely on phenomenologically inspired models to describe them. As such, the bag model is particularly suitable. Of course, in a quark-gluon plasma at high energies, some of the interactions can be treated perturbatively. So, we shall assume the following.

First, the class of interactions responsible for confinement can be accounted for by an overall bag pressure B directed towards the inside of the bag. Second, the residual interactions between individual quarks and gluons are assumed to be weak (α_s is small) enough to be described by perturbative QCD.

Then considering, for simplicity, that the quarks are massless and introducing the chemical potential $\boldsymbol{\mu}=(\mu_1,\mu_2,\ldots)$ where μ_i is associated with each quark flavor i through the variables $\gamma_i=\beta\mu_i$ where β is the inverse temperature, the partition function is given by[10,11]

$$\ln Z(\beta,\gamma)=\frac{B}{\beta^3}\{a+c(\gamma)+[b+d(\gamma)]g^2+O(g^4)\}-\beta BV\ . \tag{2.1}$$

Here, V is the volume of the system, B is the bag constant, g^2 is the strong coupling constant, and the coefficients a, b, c, and d are, respectively,

$$a=\frac{\pi^2}{15}(\tfrac{8}{3}+\tfrac{7}{4}n_f),\quad b=-\tfrac{1}{6}(1+\tfrac{5}{12}n_f)\ ,$$
$$c(\gamma)=-2\pi^2 d(\gamma)=\frac{1}{2}\sum_{i=1}^{n_f}\gamma_i^2\left[1+\frac{\gamma_i^2}{2\pi^2}\right]\ , \tag{2.2}$$

with n_f the number of flavors. The perturbative corrections to a free quark model show up through the term of $O(g^2)$ in Eq. (2.1). Following renormalization-group arguments, we replace g^2 by the running coupling constant

$$g^2 \to g^2(M^2) = \frac{48\pi^2}{(33-2n_f)\ln(M^2/\Lambda_{\rm MOM}{}^2)} , \quad (2.3)$$

where M is the momentum scale involved in the quark collisions occurring in the bag and $\Lambda_{\rm MOM}$ is the QCD parameter in the momentum scheme ($\Lambda_{\rm MOM}=100$ MeV).

We digress here momentarily to address the question of the choice of M^2; we will need it in the next section. Strictly speaking, one should compute $g^2(M^2)$ as a weighted average over all momentum transfers in the quark-gluon plasma. There is, however, another way to proceed which carries more physical significance. Indeed, from a statistical point of view, M^2 can be related to the four-momentum transferred to the constituents of the plasma by the bag itself:[12]

$$M^2 = \frac{4}{3} \frac{\sum_i \langle n(\mathbf{k}_i) | \mathbf{k}_i |^2 \rangle}{\sum_i \langle n(\mathbf{k}_i) \rangle} , \quad (2.4)$$

where $\mathbf{k}_i$ is the momentum of each constituent i and the angular brackets mean averaging over $\mathbf{k}_i$. The statistical weights $n(\mathbf{k})$ are the usual Bose or Fermi distributions:

$$n(\mathbf{k}) = (e^{\beta|\mathbf{k}|}-1)^{-1} \text{ for gluons}$$

and (2.5)

$$n(\mathbf{k}) = (e^{\beta|\mathbf{k}|\mp\mu}+1)^{-1} \text{ for quarks} .$$

Here, the minus (plus) sign corresponds to the quark (antiquark) case. M^2 can then be computed exactly to give the analytical expression

$$M^2(\beta,\gamma) = \frac{16}{\beta^2} \frac{N_g \zeta(5) + N_c \sum_i F(\gamma_i, 5)}{N_g \zeta(3) + N_c \sum_i F(\gamma_i, 3)} , \quad (2.6)$$

where ζ is the Riemann ζ function, N_g and N_c are the number of gluons and of colors, respectively, and F is a complicated function whose simplest representation is

$$F(\gamma,m) = 2 \sum_{n=0}^{\infty} \frac{(-)^n}{(1+n)^m} \cosh[(n+1)\gamma] . \quad (2.7)$$

This is unnecessarily complicated for our purpose [we will need the first and second derivatives of $g^2(\beta,\gamma)$ in the next section] and we choose to approximate $M^2(\beta,\gamma)$ by[4]

$$M^2(\beta,\gamma) = \frac{1}{\beta^2}(\tfrac{4}{5}\bar{\gamma}^2 + 15.97) , \quad (2.8)$$

where $\bar{\gamma}^2 = \sum_i \gamma_i{}^5 / \sum_i \gamma_i{}^3$ ($=\gamma^2$ for the case of equal chemical potentials). Indeed, the limits as $\beta \to \infty$ and $\gamma \to 0$ of expression (2.6) leads to

$$M^2 \underset{\beta\to\infty}{\sim} \frac{4}{5\beta^2}\bar{\gamma}^2$$

and (2.9)

$$M^2 \underset{\gamma\to 0}{\to} \frac{15.97}{\beta^2} \quad (n_f=3) .$$

We can now obtain the thermodynamical properties of the model. The pressure of the system is simply given by the equation of state:

$$P = \frac{1}{\beta}\frac{\partial \ln Z}{\partial V} = \frac{1}{\beta^4}\{a + c(\gamma) + [b + d(\gamma)]g^2\} - B . \quad (2.10)$$

According to (2.1), $P<0$ when the bag pressure B exceeds the pressure of the quark-gluon plasma. This can be interpreted as a confinement phase. When $P>0$, the bag must exert a pressure larger than B in order to retain the plasma and deconfinement is achieved. In this model,[8] following Hagedorn and Rafelski,[13,14] a phase transition from hadronic matter to a quark-gluon plasma is said to occur at $P(\beta,\gamma)=0$. This condition determines an n_f-dimensional surface in the β-γ space. The much celebrated critical curve is illustrated in Fig. 1 for a special case. For simplicity, we have assumed that the two flavors u and d have the same chemical potential, μ_q, but that the chemical potential associated with strange quarks is zero. This corresponds to systems having the same net number of protons and of neutrons but no net number of strange or of any other heavier quarks. These are clearly the most useful at the moment. We retain $n_f=3$ in the expression for $g^2(M^2)$ though. $\Lambda_{\rm MOM}$ is taken to be 100 MeV and finally we use $B^{1/4}=190$ MeV as suggested by lattice calculations.[15] Confinement is seen to be impossible for temperatures above the critical temperature T_c,

$$T_c = \left[\frac{B}{a + bg^2(\gamma=0)} \right]^{1/4} , \quad (2.11)$$

and for a chemical potential above the critical value,

$$\mu_{qc} = \left[\frac{4\pi^4 B}{2\pi^2 - g^2(T=0)} \right]^{1/4} , \quad (2.12)$$

where $g^2(\gamma=0)$ and $g^2(T=0)$ correspond to the limits in Eq. (2.9). In general, for unequal chemical potentials, the following relation must be satisfied:

$$\sum_{i=1}^{n_f} \mu_i{}^4 = \left[\frac{8\pi^4}{2\pi^2 - g^2} \right] B . \quad (2.13)$$

In Fig. 1, the baryon density corresponding to the critical value μ_{qc} is 0.70 fm^{-3} (for $n_f=3$). This should be

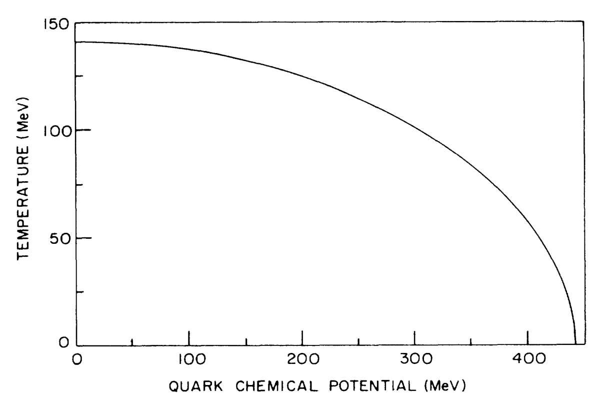

FIG. 1. The critical or phase-transition curve for $n_f=3$. Here, $\Lambda_{\rm MOM}$ is taken to be 100 MeV and $B^{1/4}=190$ MeV.

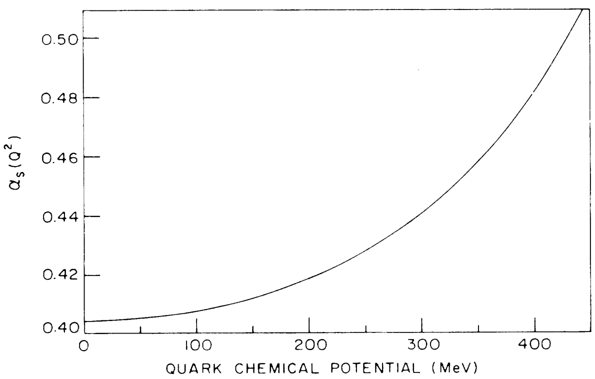

FIG. 2. α_s as a function of μ_q on the critical curve.

compared to a natural baryon density of 0.17 fm^{-3} found in nuclei.

$$E_{\text{av}} = -\frac{\partial \ln Z}{\partial \beta} = \frac{3V}{4B}\{a + c(\gamma) + [b + d(\gamma)]g^2(\beta,\gamma)\} + BV \ . \tag{2.14}$$

Here, we have used the fact that $\partial g^2(\beta,\gamma)/\partial\beta$ is of $O(g^4)$ and can be omitted. Then, on the critical curve (or surface), one has the useful relation[13]

$$E_{\text{av}} = 4BV \ . \tag{2.15}$$

As a consistency check, we plot α_s as a function of μ_q on the critical curve on Fig. 2. Clearly we seem to be in the realm of perturbative QCD with our choice of momentum scale in Eq. (2.4). Although there is a gradual increase of α_s with μ_q, the highest value corresponds to the critical chemical potential above which the bag pressure always dominates that of the plasma. The strong coupling constant is also bounded from below in the limit of a zero chemical potential. In fact, this limit corresponds to the large momentum transfers to the wall of the bag (or high temperature) and ultimately to the deconfinement phase.

The previous description (a quark-gluon plasma maintained inside a bag) leads to a somewhat naive interpretation of the critical temperature. Indeed, the confinement-deconfinement criterion ($P=0$) was identified with the hadron versus quark-gluon plasma phase transition boundary, i.e., for $T > T_c$, confinement cannot be reached and the quark-gluon plasma phase is the only possible one.

However, the description can be improved by using a different model in the low-temperature regime consisting of a gas of extended hadrons.[16] Combining the two approaches, i.e., a low-temperature hadron gas and a high-temperature quark-gluon plasma then results in a shift of the critical temperature to a slightly higher value. The reason is that the pressure of a hadron gas is always positive and will thus equal the quark-gluon plasma pressure at a temperature T_c' greater than T_c. This has the immediate implication that as the temperature decreases, the quarks begin to hadronize before reaching the confinement temperature of the plasma.

Qualitatively, it is easy to imagine how it takes place. Consider a gas of hadrons in a finite volume (this could be nucleons in a large nucleus). Raising the temperature induces the creation of more hadrons whose number is directly related to the hadronic density of states. On the other hand, these hadrons have a size which is believed to increase with excitation energy ($E=4BV$). Then when the density of hadrons gets high, the hadrons will begin to overlap each other up to the point where quarks and gluons no longer pertains to an individual hadron and, a plasma is formed. It is not clear how much this picture can be taken seriously, but it stresses the effect of the density of states on the overlapping of hadrons and on the phase transition.[17]

III. DENSITY OF STATES

We now turn to the evaluation of the density of hadronic states. These can be single hadrons, resonances, highly excited states or fireballs. Moreover, if we ignore the effects of hadronization discussed in Sec. II, i.e., if we assume that the phase transition takes place at $P=0$, it can even be a large nucleus turned into a quark-gluon plasma. Strictly speaking, such a system is unphysical as it would be forbidden to hadronize (by that again we mean, form individual hadrons inside the bag before confinement), but more realistically, we can assume a temperature difference $T_c' - T_c \ll T_c$ and neglect hadronization effects on first approximation.[18]

The partition function can be written

$$Z(\beta,\mathbf{q},\gamma) = \frac{V}{(2\pi)^3}\int dE\, d^3p \left[\prod_{i=1}^{n_f} db_i\right] \exp\left[-\beta E - \mathbf{p}\cdot\mathbf{q} - \sum_j \gamma_j b_j\right] \rho(E,\mathbf{p},\mathbf{b}) \ , \tag{3.1}$$

where ρ is the density of states per unit of energy E at a given total momentum $\mathbf{p}$ with baryonic number $\mathbf{b}$. $\mathbf{q}$ here is proportional to the center-of-mass coordinates of the system which can be placed at the origin without loss of generality when the system is at rest. The density of states is then obtained by Laplace inversion of the partition function:

$$\rho(E,\mathbf{p},\mathbf{b}) = \frac{(2\pi)^3}{V}\frac{1}{(2\pi i)^{4+n_f}}\int_{C-i\infty}^{C+i\infty} d\beta\, d^3q \left[\prod_i d\gamma_i\right] e^{f(\beta,\mathbf{q},\gamma)} \ , \tag{3.2}$$

where f is simply

$$f(\beta,\mathbf{q},\gamma) \equiv \beta E + \mathbf{p}\cdot\mathbf{q} + \sum_{i=1}^{n_f} \gamma_i b_i + \ln Z(\beta,\mathbf{q},\gamma) \ . \tag{3.3}$$

Obtaining an exact analytical form for this inverse Laplace transform with $Z(\beta,\mathbf{q},\gamma_i)$ generalized for $\mathbf{q}\neq 0$ (Refs. 8 and 19): i.e.,

$$\ln Z(\beta,\mathbf{q},\gamma)=\frac{\beta V}{(\beta^2-q^2)^2}\{a+c(\gamma)+[b+d(\gamma)]g^2(\beta,\gamma)\}-\beta BV\ , \tag{3.4}$$

is quite a formidable task. On the other hand, a well-known procedure[20] is particularly suited to approximate Eq. (3.2), the saddle-point method. To first approximation, this leads to

$$\rho(E,\mathbf{p},\gamma)\simeq\frac{(2\pi)^{(n_f+2)/2}}{V}\frac{e^{f(\beta_0,\mathbf{q}_0,\gamma_0)}}{\Delta^{1/2}}\ , \tag{3.5}$$

where β_0, q_0, and γ_{i0} are solutions of

$$\frac{\partial f}{\partial\beta}=\frac{\partial f}{\partial\mathbf{q}}=\frac{\partial f}{\partial\gamma_i}=0,\quad i=\text{flavors} \tag{3.6}$$

and Δ is the determinant of the matrix of second-order derivatives of f evaluated at β_0, $\mathbf{q}_0$, and γ_0:

$$\Delta\equiv\det\left[\left.\frac{\partial^2 f}{\partial\alpha_m\partial\alpha_n}\right|_{\alpha_0}\right],\quad m,n=1,2,\ldots,n_f+4 \tag{3.7}$$

with $\alpha=\beta,\mathbf{q},\gamma$.

Let us now examine the set of conditions in (3.6). First we have

$$\frac{\partial f}{\partial\mathbf{q}}=\mathbf{p}_0+\frac{4V\mathbf{q}_0}{(\beta_0{}^2-\mathbf{q}_0{}^2)^3}\ln Z(\beta_0,\mathbf{q}_0,\gamma_0)=0\ . \tag{3.8}$$

But since we chose to consider the bag at rest, we find that $\mathbf{q}_0$ must be zero; this is consistent with our previous statement. Therefore, all thermodynamical quantities derived in Sec. II remain valid at the saddle point. The second relation gives

$$\frac{\partial f}{\partial\beta}=E-BV-\frac{3V}{\beta_0{}^4}\left[a+c(\gamma_0)+\left[b-\frac{c(\gamma_0)}{2\pi^2}\right]g^2(\beta_0,\gamma_0)\right]+\frac{V}{\beta_0{}^3}\left[b-\frac{c(\gamma_0)}{2\pi^2}\right]\left.\frac{\partial g^2}{\partial\beta}\right|_{\beta_0}=0\ . \tag{3.9}$$

Again, the last term is of $O(g^4)$ and can be neglected. Using similar arguments, we find that the last relation in (3.6) obeys

$$\frac{\partial f}{\partial\gamma_i}=b_i+\frac{V}{\beta_0{}^3}c_i'(\gamma_0)\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right]=0,\quad i=u,d,\ldots \tag{3.10}$$

up to $O(g^2)$. $c_i'(\gamma)$ here is

$$\partial c(\gamma)/\partial\gamma_i=[\gamma_i(1+\gamma_i{}^2/\pi^2)]\ .$$

It is not surprising to see that the roots of Eq. (3.9) obey exactly the equation of state (2.14) involving the average energy of the system. This means that the energy corresponding to the saddle point coincides with the average energy or, to put it crudely, to first approximation, one assigns to each particle an energy $E=E_{\text{av}}$ ($=4BV$ at equilibrium). Similarly, Eq. (3.10) is the same as the relation for the average baryon density, i.e.,

$$\left[\frac{b_i}{V}\right]_{\text{av}}=-\frac{1}{V}\frac{\partial}{\partial\mu_i}\ln Z\ . \tag{3.11}$$

So, it turns out that the first approximation is the simplest one we can imagine and that is to assume that each particle contributes equally to the "macroscopic" properties of the system. Clearly, this picture is improved when the saddle-point approximation is refined to higher orders. But this involves lengthy calculations and anyway, the first order should be accurate to a few percent.[21]

Back to the determination of γ_0 and β_0 in Eqs. (3.9) and (3.10), we must solve n_f+1 equations. At equilibrium, the pressure vanishes and, using (2.15), we obtain

$$\beta_0{}^4B=a+c(\gamma_0)+\left[b-\frac{c(\gamma_0)}{2\pi^2}\right]g^2(\beta_0,\gamma_0) \tag{3.12a}$$

and

$$\beta_0{}^3\left[\frac{b_i}{V}\right]=c_i'(\gamma_0)\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right],\quad i=1,2,\ldots,n_f\ , \tag{3.12b}$$

where the only input parameters are the bag constant, the QCD scale Λ_{MOM}, and the baryon density $b/V\sim 1$ baryon/fm^3. Again, we only consider u, d, and s quarks. Heavier quarks would simply not fit in the masslessness approximation. In fact, they induce a modification in M^2 since the distributions (2.5) involve $E=(|\mathbf{k}|^2+m^2)^{1/2}$ instead of $E=|\mathbf{k}|$. For protonlike matter, the sea-quark distributions are known to be rather small for heavy quarks ($c,b,t,\ldots$) even at relatively high energies. So, there is no need for their introduction at this level anyway. Furthermore, we are interested in nonstrange systems and we shall use $\mu_u=\mu_d$ (or

$b_u/V=b_d/V=2b/3V$) and $\mu_s=0$. Notice that Eq. (3.12a) is the same as (2.10) at $P=0$.

Once β_0 and γ_0 are found (this is much easier for the case of equal chemical potential), we are left with the evaluation of the determinant in (3.7). Using (3.12), we obtain

$$\left.\frac{\partial^2 f}{\partial \beta^2}\right|_0 = \frac{12BV}{\beta_0} ,$$

$$\left.\frac{\partial^2 f}{\partial q_i \partial q_j}\right|_0 = \frac{4BV}{\beta_0}\delta_{ij} ,$$

$$\left.\frac{\partial^2 f}{\partial \gamma_i \partial \gamma_j}\right|_0 = \frac{V}{\beta_0^3}\delta_{ij}\left[1+\frac{3\gamma_{i0}^2}{\pi^2}\right] .$$

For mixed second derivatives, we have

$$\left.\frac{\partial^2 f}{\partial \beta \partial \gamma_i}\right|_0 = -\frac{3V}{\beta_0^4}\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right]\gamma_{i0}\left[1+\frac{\gamma_{i0}^2}{\pi^2}\right]$$

and

$$\left.\frac{\partial^2 f}{\partial \beta \partial q_i}\right|_0 = \left.\frac{\partial^2 f}{\partial \gamma_i \partial q_j}\right|_0 = 0 .$$

The determinant is then

$$\Delta(\beta_0,\gamma_0)\equiv\Delta=192\beta_0^4B^3\left[\frac{V}{\beta_0^3}\right]^{n_f+4}\left[4\beta_0^4B\prod_{i=1}^{n_f}\left[1+\frac{3\gamma_{i0}^2}{\pi^2}\right]-3\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right]\sum_{i=1}^{n_f}\gamma_{i0}\left[1+\frac{\gamma_{i0}^2}{\pi^2}\right]\prod_{j\neq i}\left[1+\frac{3\gamma_{j0}^2}{\pi^2}\right]\right] \tag{3.13a}$$

$$\equiv E^{n_f+4}D(\beta_0,\gamma_0) . \tag{3.13b}$$

Using Eq. (2.15) this expression reduces to

$$\Delta_0=192\beta_0^4B^3\left[\frac{V}{\beta_0^3}\right]^{n_f+4}\left[1+\frac{3\gamma_0^2}{\pi^2}\right]^{n_f-1}\left[4\beta_0^4B\left[1+\frac{3\gamma_0^2}{\pi}\right]-3n_f\gamma_0\left[1+\frac{\gamma_0^2}{\pi^2}\right]\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right]\right] . \tag{3.13c}$$

In the case of identical μ's. Clearly, for nonexotic systems, one can substitute n_f with 2. Obviously, the approximation is valid only when $\Delta\neq 0$. This is certainly the case when the γ_{i0}'s (or μ_{i0}'s) are reasonably small, and then the positive term $4\beta_0^4B$ dominates the term of the right-hand side of (3.12b). Of course, consistency also requires Δ to be positive which can be checked easily.

Combining (3.3) with (3.8) and (3.9), we have, using (2.15),

$$f(\beta_0,\mathbf{q}_0=0,\gamma_0)=\frac{4\beta_0}{3}(E-BV)+\sum_i\gamma_{i0}b_i=\beta_0E+\sum_i\gamma_{i0}b_i . \tag{3.14}$$

Again, this very simple form is quite natural. It is easy to show that (3.14) corresponds to the entropy of the system in the macroscopic limit at vanishing pressure.[17] Comparing (3.14) with (3.3) shows that, in the saddle-point approximation, the partition function does not contribute explicitly to the expression. This is because we are calculating the density of states when the bag is at equilibrium, i.e., when the net pressure is zero. In our model, we can write $\ln Z(\beta,\gamma_i)=VG(\beta\gamma_i)$ and, therefore, using Eq. (2.10), $P=(1/\beta V)\ln Z=0$. It is very interesting to note that this result is independent of the exact form of $G(\beta,\gamma_i)$ and in particular does not depend on the order of perturbation expansion. Obviously, as can be seen from (3.9) and (3.10), the numerical values of β_0 and γ_{i0} depend on the details of the partition function.

Then reassembling the ingredients, we can write the density of states as

$$\rho(E,\mathbf{p}=0,\mathbf{b})\simeq\frac{4B(2\pi)^{n_f/2+1}}{E^{n_f/2+3}D^{1/2}}\exp\left[\beta_0E-\frac{1}{4\beta_0^3B}\sum_i\left[\gamma_{i0}^2+\frac{\gamma_{i0}^4}{\pi^2}\right]\left[1-\frac{g^2(\beta_0,\gamma_0)}{2\pi^2}\right]E\right] , \tag{3.15}$$

where we have used Eq. (3.10) with the volume at equilibrium.

We first notice that ρ is dominated by the flavor i having the largest chemical potential $\mu_{i0}=\gamma_{i0}/\beta_0$ and particles with small μ_{i0} can easily be discarded.

Second, in this approximation, the saddle point falls on the critical curve as obtained in Sec. II. This is due to the fact that the condition of equilibrium of the bag, i.e., the pressure of the constituents equals the "external" pressure, is the same as the condition determining the matching of the two phases in the model $[P(\gamma,\beta)=0]$. Elsewhere, the density is not defined as the bag is not at equilibrium.

Third, we see that the density of states decreases exponentially with the square of the chemical potential (at least for small valence-quark densities). This is because particle production is lower when the valence density is higher. Indeed, annihilation is more favorable with a larger valence density and also, the Pauli principle implies that newly created particles must occupy higher-energy states therefore suppressing their production rate. This reduces the number of degrees of freedom and therefore the density of states.

Fourth, we observe that the power of E drops by a factor of $\frac{1}{2}$ for each conserved flavor. This effect, associated with conserved quantities was previously noted by other authors in a similar context.[9,19]

Finally, as in Ref. 8 and for the same reasons, we neglected finite-bag-size effects and color-singlet requirements. The former have been shown to lead to a modification of the exponent itself.[22] Furthermore, not requiring the plasma states to be color singlets means that additional states were included in the treatment; this should only affect the preexponential factor in (3.15), not the exponent.[23] The hope is that these calculations, which involve different approaches, and our analysis of baryon-number conservation can complement each other.

IV. SUMMARY

In this paper, a quark-gluon plasma described by perturbative QCD at a finite temperature and density supplemented by a bag constant to account for the nonperturbative effects has been studied. A coupling constant depending on the chemical potential and the temperature of the plasma was implemented and, for simplicity, all quarks were assumed to be massless. It was then easy to obtain a critical phase transition curve consistent with previous results.

Next, the asymptotic density of states was derived by Laplace inverting the partition function. Since an exact calculation was not possible, the saddle-point method was used. It was found that the conditions determining the saddle point were exactly the same as those determining the average energy and the average baryonic density of the system. Therefore, the exponent in the density of states coincides perfectly with the critical curve of the phase transition.

Finally, we found that the density of states can be written in a relatively simple form even for the most general case (n_f different chemical potentials) although the numerical values of β_0 and γ_{i0} depend very much on the details of the partition function. A discussion of the results was also presented.

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