Topology change and monopole creation

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Lorentzian cobordism is considered as a mechanism for topology change in quantum gravity. It is shown that the condition for a topological cobordism to admit an appropriate metric is different in even and odd dimensions. In odd dimensions such a metric exists if and only if the initial and final manifolds have the same Euler characteristic. This means that pair creation of Kaluza-Klein monopoles cannot occur via the mechanism considered. Possible implications of this result are discussed. Lorentzian cobordism in two dimensions is also analyzed briefly.

I. THE IMPORTANCE OF TOPOLOGY CHANGE: INTRODUCTION

Can the topology of spacetime change, and if so how is its change to be described? This question arises in connection with cosmology, with microscopic physics, and, of course, in connection with any attempt to formulate a complete theory of "quantum gravity."

Thus, for example the "big-bang singularity" might be understood as a transition from the "empty topology" to that of an appropriate three-manifold (such as S^3, the three-sphere, or R^3, the Euclidean topology) expressing the large-scale connectivity of three-space at the present time.[1] In the microscopic realm, virtual topological fluctuations (the so-called "foam") could be an essential vacuum process, and if so, would necessarily enter into any investigation of the stability and energy density of the vacuum.[2,3] Indeed, in one specific model (the Abelian Kaluza-Klein theory) the stability of the presumed vacuum metric does appear to depend on whether a certain topological transition is possible or not.[4] Moreover, a closely related topological transition would, as will be discussed below, describe the pair creation and annihilation of monopoles in the same theory. Hence, any attempt to estimate the expected cosmological abundance of such monopoles also leads directly to the question with which this paper began.

In four dimensions as well[5,2]—and even in two[6]—topological degrees of freedom can contribute to the particle spectrum of quantum gravity. Indeed the freedom involved seems broad enough to reproduce both fermion states[7] and internal-symmetry multiplets[8] without leaving the domain of pure gravity. (I do not know whether the multiplets will appear coupled to effective gauge fields or whether the "chiral" requirement of avoiding huge fermion masses could be met. Concerning the latter question, it should at least be noted that existing disproofs do not cover topological excitations, either in four-dimensions or in Kaluza-Klein theories.[9])

There are also indications that a dynamical topology may be needed for the consistency of quantum gravity itself. One such indication relates to the "wave-function collapses" which may be associated with black-hole evaporation and specifically to the possibility of understanding such information loss as due to the bifurcation of space into two or more disconnected three-manifolds.[10] Another indication concerns the properties of the topological particles ("topological geons") alluded to above. Within a canonical framework of *fixed* topology the particle number would be correspondingly fixed and "second quantization" of geons would be blocked, in contradiction with the historical experience that Lorentz-covariant quantum particles can occur consistently only in a second-quantized framework (cf. Ref. 11, see also Ref. 12). Similarly, the fact that with fixed topology there is no necessary connection between geon spin and geon statistics[5,13] also strongly suggests that it is inconsistent to omit topology change in attempting to synthesize quantum theory with general relativity.

To my knowledge the proposed mechanisms by which (spatial) topology might change all entail some sacrifice of features customarily attributed to normal spacetimes. The more radical proposals (see as samples Ref. 14) would sacrifice continuous spacetime itself, of course. For present purposes, let us leave these proposals aside and concentrate on the "path-integral" mechanism, according to which a four-dimensional "path," i.e., a four-dimensional manifold ${}^4M$ provided with a metric $g_{\mu\nu}$, furnishes a contribution to the quantum amplitude for a transition to occur between the three-geometries ${}^3M_0$ and ${}^3M_1$ where $\partial\, {}^4M = {}^3M_0 \cup {}^3M_1$.

Having accepted this mechanism, one is faced with several choices, two of which may be dubbed "Euclidean versus Lorentzian" and "causality versus the equivalence principle." The first choice raises the option of replacing the $-+++$ metric of everyday experience with one of the Riemannian signature $++++$. For present purposes I will ignore that option, except insofar as it may be related to the possibility of allowing some of the signs to degenerate to zeros at isolated spacetime events. From this standpoint Riemannian metrics could be calculationally useful, but not, in any case, physically real.

The second choice is forced on us in the first place by the theorem that any "Lorentzian cobordism" (as an interpolating ${}^4M$ with everywhere Lorentzian metric is known) must contain closed timelike curves.[15] Such a "causality breakdown" can be avoided by allowing the metric to degenerate to zero at isolated points.[16,1] This may be described[17] as a breakdown of the equivalence principle inasmuch as ${}^4M$ would not be locally isomorphic to Minkowski space at such a point.

The remainder of this article will consider primarily the consequences of attempting to *preserve* the equivalence principle by rejecting any degeneration of the metric. We will see first of all that the causality violations *per se* are probably confined to occur harmlessly in microscopic regions. We will then examine the extra conditions imposed on a cobordism ${}^nM$ in order that it admit an everywhere Lorentzian metric $g_{\mu\nu}$ for which ${}^{(n-1)}M_0$ and ${}^{(n-1)}M_1$ are, respectively, an *initial* and a *final* spacelike hypersurface, and we will obtain results which differ in even and odd dimensions. Specifically the even-dimensional condition that the Euler characteristic $\chi(M)$ of the mediating *spacetime* vanish is replaced in odd dimensions by the conditions $\chi(M_0)=\chi(M_1)$ that the topology change conserve the Euler characteristic of the initial *hypersurface.* As applied in the five-dimensional Kaluza-Klein theory, this conservation law will be seen to forbid the creation of monopole-antimonopole pairs, although the necessary *manifold* ${}^5M$ is readily available.

II. TOPOLOGY CHANGE VIA LORENTZIAN COBORDISM

For simplicity let us limit our discussion to closed (=compact without boundary) spacelike hypersurfaces M_0,M_1 and to compact interpolating manifolds M. This might be a genuine limitation in the cosmological context, but as concerns isolated microscopic system it entails no more than assuming that the system in question may be "put in a box." (For example, the five-dimensional Kaluza-Klein vacuum, $R^4\times S^1$. When placed in a "periodic box," becomes $R\times T^3\times S^1$, where $T^3=S^1\times S^1\times S^1$ is the 3-torus.) A *topological cobordism* will then be a smooth compact n-manifold $M={}^nM$ whose boundary ∂M is the disjoint union of two (not necessarily connected *or* nonempty) $(n-1)$-manifolds designated $M_0={}^{(n-1)}M_0$ and $M_1={}^{(n-1)}M_1$. A *Lorentzian cobordism* will be a topological cobordism together with a (nondegenerate) *time-oriented* Lorentzian metric $g_{\mu\nu}$, with respect to which M_0 and M_1 are, respectively, initial and final spacelike boundaries. (The condition that M_0 be initial and M_1 final is not always included, but it seems unnatural to omit it.[1])

Given a pair of $(n-1)$-manifolds M_0 and M_1, one can seek a Lorentz cobordism between them by looking first for a topological cobordism M and then for a suitable metric on M. According to Ref. 18 a topological cobordism will exist if and only if the so-called "Stiefel-Whitney numbers" of M_0 and M_1 are equal. In any dimension $n-1$ there are only a finite number of possibilities for these numbers, whence only a finite number of cobordism classes of closed $(n-1)$-manifolds; for example, there is a single class in dimension three $(=4-1)$ and four classes in dimension four. (This refers to unoriented cobordism. Cobordism can also be defined for oriented manifolds, as would be appropriate if *CP* violation were truly fundamental.) In many situations, moreover, the desired cobordism is physically evident, and this will be the case with the pair creation we will consider below.

Assuming that a Lorentzian cobordism does exist we know that it will necessarily contain closed timelike curves.[15] If confined to microscopic (say Planck sized) regions of spacetime, such causal anomalies would presumably be harmless, or at least not evidently more worrisome than the "noncausality" implicit in the blurring of light cones by quantum fluctuations of the spacetime metric. However, one might well worry that, once admitted into physics, causal anomalies would occur also on detectably large scales. Fortunately, such a possibility is effectively excluded by the following theorem.

Theorem.[19] Any (sufficiently smooth) Lorentzian cobordism which fails to be globally hyperbolic (in particular any one containing a closed, timelike curve) must violate either the weak energy condition or the "Hawking-Penrose generic condition."

(The weak energy condition says $T^{\mu\nu}U_\mu U_\nu\geq 0$ for all timelike U. In fact only its limit for lightlike U, the so-called "null-convergence condition," is actually used in the proof.)

Based on this theorem it is reasonable to believe that a macroscopic causality violation (of the sort required by topology change) could occur only together with negative-energy densities of a macroscopic scale, which of course, are classically excluded. Thus topology change would be classically (though not obviously semiclassically) forbidden, and could occur only as a form of quantum tunneling. (The energy conditions fail in the semiclassical framework, $G_{\mu\nu}=\langle\psi\,|\,T_{\mu\nu}\,|\,\psi\rangle$, because quantum interference can always render the expectation value, $\langle\psi\,|\,T^{00}(x)\,|\,\psi\rangle$, negative. This possibility is "guaranteed" by a theorem in Ref. 20, and simple examples also exist in free field theories.) To the extent that this is so, the closed timelike curves concomitant to Lorentzian topology change would seem to be physically acceptable acceptable.

Let us return, therefore, to the question of whether a given topological cobordism ${}^nM$ admits a metric $g_{\mu\nu}$ which will render it a Lorentzian cobordism. To begin with, we can recall[1] that a metric of the required sort exists if and only if M admits a nowhere vanishing vector field v which is inward directed at M_0 (so that M_0 will be an initial boundary) and outward directed at M_1 (so that M_1 will be a final boundary). (Such a vector field is topologically equivalent to a consistent choice of future light cone throughout M.) Henceforth a vector field obeying the just stated transversality conditions on M_0 and M_1 and with at most a finite number of zeros will be called a "Morse vector (field)."

Now let v be any Morse vector on M. If v had been *outward* directed on M_0 (or if M_0 is empty) we could have related the existence of a nonvanishing v to the vanishing of $\chi(M)$. However, because M_0 is an *initial* (and not a final) boundary with respect to v, we have instead the following.

Lemma. Let v be a Morse vector field on M. Then

$$\mathrm{ind}(v)=\chi(M)-\chi(M_0)\ ,$$

$$\mathrm{ind}(-v)=\chi(M)-\chi(M_1)\ .$$

Here χ is the Euler characteristic and $\mathrm{ind}(v)$ is the sum of the indices of v at its (by definition finite) set of zeros. [The (Euler) index of a vector field at an isolated zero is the degree of the map it defines from a small S^{n-1} surrounding the zero to the S^{n-1} of "unit vectors," see Ref. 21. For example, the index at the origin of an outwardly radial vector field is $+1$.]

Proof. Let $\hat{M}$ be a new manifold obtained by gluing a copy, N, of $M_0\times[0,1]$ onto M_0; in other words $\hat{M}=M\cup N$ where $N=M_0\times[0,1]$ with $M_0\times\{1\}$ identified to $M_0\subseteq\partial M$. Also let $\hat{v}$ be an extension of v to $\hat{M}$ which has isolated zeros and is *outward* directed on $M_0\times\{0\}\subseteq N$. (Such an extension always exists, cf. Chap. XIV, Sec. 4, Theorem II of Ref. 22.) Clearly $\hat{v}$ is outward directed on the entire boundary of $\hat{M}$, and its restriction $\hat{v}_N$ to N is also outward directed on ∂N. For such a vector we have[21]

$$\mathrm{ind}(\hat{v})=\chi(\hat{M}) ,$$

and similarly

$$\mathrm{ind}(\hat{v}_N)=\chi(N) .$$

The desired relation $\mathrm{ind}(v)=\chi(M)-\chi(M_0)$ then follows from the three elementary equalities $\chi(\hat{M})=\chi(M)$, $\mathrm{ind}(\hat{v})=\mathrm{ind}(v)+\mathrm{ind}(\hat{v}_N)$, and $\chi(N)=\chi(M_0)$. The first equality holds because M and $\hat{M}$ are diffeomorphic: $M\simeq\hat{M}$, or even from the fact that they are homotopic as is immediate by shrinking $N=M_0\times[0,1]$ down to M_0; the second equality is true by definition since $\hat{v}$ has no zeros on $M_0=M\cap N$; and the third follows from the homotopy equivalence $N\sim M_0$ just mentioned, or from the general formula $\chi(X\times Y)=\chi(X)\chi(Y)$. Finally $\mathrm{ind}(-v)=\chi(M)-\chi(M_1)$ follows by interchanging the roles of M_0 and M_1.

To derive a criterion for Lorentzian cobordism from the lemma we need only the following fact.

Fact: $\mathrm{ind}(-v)=(-)^n\mathrm{ind}(v)$.

(One way to see this is to observe that inversion of S^{n-1} reverses orientation if and only if n is odd.) We can now prove the following theorem.

Theorem. Let ${}^nM$ be a topological cobordism between M_0 and M_1 and let M be connected. Then a necessary and sufficient condition for the existence of a Lorentzian cobordism based on M is

$$\chi(M)=0 \quad (n \text{ even}) ,$$

$$\chi(M_0)=\chi(M_1) \quad (n \text{ odd}) .$$

Proof. Let v be a Morse vector on M and suppose n is even. In that case $\chi(M_0)=0$ since $\dim(M_0)=n-1$ is odd. [Any closed manifold of odd dimension has $\chi=0$ because, if w is any vector field theorem with isolated zeros then $\chi=\mathrm{ind}(w)$; while also $\chi=\mathrm{ind}(-w)=-\mathrm{ind}(w)$.] The lemma then implies $\mathrm{ind}(v)=\chi(M)$, whence $\chi(M)=0$ if v is nowhere vanishing. Conversely, if $\chi(M)=0$ then $\mathrm{ind}(v)=0$ which means[21] its zeros can be deformed away as long as M is connected.

Now suppose n is odd. In that case the lemma and "Fact" together imply that

$$2\,\mathrm{ind}(v)=\chi(M_1)-\chi(M_0) ,$$

whose vanishing, just as before, is equivalent to the deformability of v into a nowhere vanishing Morse vector field.

III. PAIR CREATION AND OTHER APPLICATIONS

The application of the above theorem to four-dimensional spacetimes leads to the conclusion that *every* topology change is realizable via some Lorentzian cobordism.[15]

In two dimensions, however, the conclusion is almost exactly reversed. The only connected compact manifolds with vanishing Euler characteristic are T^2, $P^2\#P^2$, $D^1\times S^1=D^2\#D^2$, and $P^2\#D^2$. Here $T^2=S^1\times S^1$ is the 2-torus or "sphere with a single orientable handle"; $D^1=[-1,1]$ is a closed interval of R; $P^2=S^2/Z_2$ is the two-sphere modulo antipodal identification, i.e., the projective two-space, also called "cross cap"; D^2 is the unit disk or "two-ball"; and $\#$ is connected sum, constructed by removing disks from each summand and gluing together the S^1 boundaries thereby created.

Of the above four manifolds only the last yields a physically interesting Lorentzian cobordism. The first two, T^2 and $P^2\#P^2$, lack boundaries and can therefore contribute to topology changes only as disconnected "vacuum bubbles." (A better term might be "nothing bubbles" since "vacuum" here refers to the empty *set,* rather than to a spacetime empty of excitations.) The third manifold, $D^1\times S^1$, yields only the identity cobordism taking S^1 to itself, and therefore contributes to the amplitude for evolution *without* topology change.

The final manifold, $P^2\#D^2$ can be pictured, as in Fig. 1, as an annulus with one boundary having been eliminated by antipodal identification of its points. Since the remaining boundary is a single S^1, $P^2\#D^2$ can contribute either to the appearance or disappearance of a single connected component of the Universe. [The figure depicts a possible v (represented as a contravariant vector) corresponding to the case of disappearance.] Since there is no real topological *transition* here, the $P^2\#D^2$ cobordism also seems uninteresting, except possibly for two-

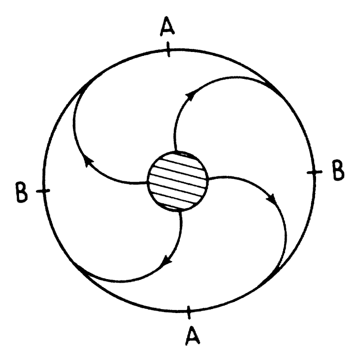

FIG. 1. A Lorentzian cobordism interpolating between the circle and the empty set. The outermost circle of the annulus is one of the closed timelike curves guaranteed by Geroch's theorem. (The letters A and B indicate the antipodal identification which converts the annulus into $P^2\#D^2$.)

dimensional cosmology. Note also that P^2 is nonorientable, which in certain contexts would be a reason for excluding any manifold, such as $P^2\#D^2$, in which it occurred.

In two dimensions then, topology change is essentially impossible via Lorentzian cobordism. The interesting topological cobordisms, such as the "trousers" $(=D^2\#D^2\#D^2)$, which might have mediated a branching off of a satellite universe, all have $\chi<0$, with the consequence that no nonvanishing "Morse vector" can exist on them. For example, the "trousers" itself has an Euler characteristic of minus one.

More interesting than spacetimes of dimension $1+1$ are those of dimension greater than four. In particular the case $n=5$ is both important for its role in the Abelian Kaluza-Klein model, and of interest as an example in which certain topological transitions have an immediate physical interpretation as processes in which particles are either created or destroyed.

Specifically let us consider two possible modifications to the $R^3\times S^1$ (spatial) topology of the Kaluza-Klein vacuum. The first modification will alter the topology to that of a monopole-antimonopole pair, and the second is that responsible for the instability described in Ref. 4. For both these modifications an interpolating ${}^5M$ exists; that for the pair creation is discussed in the Appendix, while that for the vacuum decay is given explicitly in Ref. 4. The only obstacle to the existence of corresponding Lorentzian cobordisms is thus the $\Delta\chi=0$ selection rule which holds in odd dimensions. But in fact $\Delta\chi=+2$ in both cases, as we will now derive. (Incidentally, the fact that creation of a *single* monopole is forbidden by magnetic-charge conservation has an interesting correlate at the level of purely topological cobordism. Namely, $\Delta\chi=+1$ for such a creation, whereas a cobordism can alter χ only by an even integer. Notice, however, that a one-monopole creating cobordism is not really within our present purview since it necessarily alters the the spatial topology at arbitrarily large radii.)

Consider first the vacuum decay, which proceeds via the transition $R^3\times S^1\to R^2\times S^2$. Before the transition

$$\chi=\chi(R^3\times S^1)=\chi(R^3)\chi(S^1)=1\times 0=0\ ,$$

while afterward

$$\chi=\chi(R^2\times S^2)=\chi(R^2)\chi(S^2)=1\times 2=2\ .$$

Hence $\Delta\chi=2$ as claimed.

For the pair creation we can compute χ for the final state by dividing the four-manifold ${}^4M_1$ into three pieces, a submanifold A containing the monopole, a similar submanifold B containing the antimonopole, and a third submanifold C having the structure of a U(1) bundle over $R^3\#D^3\#D^3$ (which is just R^3 with two balls removed). Both A and B are diffeomorphic to a region cut out of the one-monopole manifold, which itself is just R^4; so $A\simeq B\simeq D^4$ and $\chi(A)=\chi(B)=1$. Also $\chi(C)=0$ because C is locally a product $S^1\times R^3$ and $\chi(S^1)=0$. Finally $\chi(A\cap C)=\chi(B\cap C)=0$ because $A\cap C\simeq B\cap C\simeq\partial D^4=S^3$, which is a closed manifold of odd dimension. Using then the general relation $\chi(X)+\chi(Y)=\chi(X\cup Y)+\chi(X\cap Y)$, and noting that $A\cap B=\varnothing$, we easily obtain

$$\begin{aligned}\chi(M_1)&=\chi(A\cup B\cup C)\\&=\chi(A)+\chi(B)+\chi(C)-\chi(A\cap C)-\chi(B\cap C)\\&=1+1+0-0-0=2\ ,\end{aligned}$$

as claimed.

It follows that neither vacuum decay *nor* monopole pair creation is possible via Lorentzian cobordism. Taken alone this restoration of vacuum stability could seem a blessing, but the concomitant blocking of the physically crucial processes of particle-antiparticle creation and annihilation can only be viewed as extremely unnatural. It would appear therefore that either the Abelian Kaluza-Klein theory (as well probably as two-dimensional gravity) is unphysical, or Lorentzian cobordism is not an adequate mechanism for change of topology.

Is it possible to widen the notion of Lorentzian cobordism in such a way that it can encompass processes like the pair creation just discussed? One thought might be to drop the requirement that $g_{\mu\nu}$ be time oriented with M_0 an initial and M_1 a final manifold. This could make a difference for some cobordisms, but not for those such that $\pi_1(M)=0$, and therefore not for the pair-creation cobordism described in the Appendix.

A different way around the $\Delta\chi=0$ selection rule is suggested by adopting the attitude that not the metric, but rather the associated causal structure is what is physically fundamental. From this standpoint [whose motto might be "the causal structure is $\frac{9}{10}$ of the metric (in four dimensions)"] time nonorientability, and even closed timelike curves would be rejected, but singularities in $g_{\mu\nu}$ might be acceptable as long as a well-defined causal structure were preserved.[1,23,16] It turns out, in fact, that *every* topological cobordism admits a metric which renders M_0 initial and M_1 final, which endows M with a well-defined causal structure, and which has Lorentzian signature except at a finite number of points. At such points the metric remains a smooth tensor field, but its inverse $g^{\mu\nu}$ fails to exist.

It may be, of course, that still more general possibilities should be considered, but they would be required only to describe processes, such as changes in the *dimensionality* of spacetime, to which not even a topological cobordism corresponds.

Note added in proof. I have recently learned that the theorem proved in Sec. II was given previously in B. L. Reinhart, Topology 2, 173 (1963).

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APPENDIX: THE TOPOLOGY OF PAIR CREATION

The spacetime for a single Kaluza-Klein monopole has been described in Ref. 24. Globally, it is just R^5, but the

Kaluza-Klein fibering presents it as a circle bundle over R^4, except along the one-dimensional submanifold $x=y=z=0$ (the "world line of the monopole"), where the circular fibers shrink down to single points.

The antimonopole spacetime is obtainable from that of the monopole by reversing the spatial orientation (e.g., by using new coordinates $x'=-x, y'=-y, z'=-z$) or equivalently by reversing the orientation of the U(1) fibers. (The latter reversal represents charge conjugation and the former either parity or *CP* depending on whether the monopole is identified as magnetic or electric.)

Now return to the monopole spacetime and consider the effect of "bending the monopole's world line into the shape of a U." This produces a manifold which at "early times" is vacuum (the monopole having been "moved away") and at "late times" contains both a monopole and (because the bending reverses the spatial orientation of the world line's "lower half") an antimonopole, corresponding to the two branches of the "*U*."

To express this analytically consider the one-monopole spacetime, R^5, as given in Ref. 24, and let t,x,y,z be four (not five) "coordinate" functions on it related in the usual way to the coordinates t,r,θ,ϕ of that reference. Then introduce, say, the function $t':=t^2/2-z$ and take ${}^5M$ to be the region $-1\leq t'\leq 1$, with initial and final boundaries ${}^4M_0$ and ${}^4M_1$ corresponding to $t'=-1$ and $t'=+1$, respectively. The manifold with boundary ${}^5M$ provides a topological cobordism (in the sense of this paper) between the vacuum topology $R^3\times S^1$ and the monopole-antimonopole topology discussed in Sec. III.

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