Spectrum of Hawking radiation and the Huygens principle

Hirosi Ooguri*
Department of Physics, Faculty of Science, Kyoto University, Sakyo-ku, Kyoto 606, Japan
(Received 7 October 1985)

It has been established that the Minkowski vacuum observed by a uniformly accelerating detector can be equivalently described by a canonical ensemble of states. However, recent calculations have revealed that in a spacetime of odd dimensions free massless scalar particles are detected with the Fermi-Dirac distribution instead of the Planck one. The purpose of this paper is to solve this apparent puzzle. It is concluded that the cause of this phenomenon is the lack of the Huygens principle in odd dimensions: the expectation value of the commutator of massless fields does not vanish in the timelike region. The case of a spinor field is treated equally. Other spacetimes, de Sitter space and Schwarzschild space, are also discussed.

I. INTRODUCTION

It was discovered by Hawking[1] that particles obeying thermal distributions are created near a black hole under the influence of its strong gravitational field. Subsequently, quantum field theories in a variety of background geometries have been studied.[2] Among others the usual Minkowski vacuum observed by a uniformly accelerating detector[3] is recognized as a convenient laboratory which contains the essential feature of this phenomenon.

For a long time most of these discussions had been restricted to the case of free field theories in two or four dimensions. It was proven by Sewell[4] on a rigorous axiomatic basis that, in arbitrary dimensions independently of whether or not particles are interacting, a uniformly accelerating detector observes the Minkowski vacuum as if it were in the Kubo-Martin-Schwinger (KMS) state.[5] This means that the state observed can be equivalently described by a canonical ensemble of states.

Since the detector sees the "thermal" state in the sense described above, one might naively expect that the observed spectrum is characterized by the standard Planck distribution or the Fermi-Dirac distribution depending on whether the particle is bosonic or fermionic.

Recently Takagi[6] has shown by explicit calculations that, contrary to this naive expectation, free massless scalar particles in *odd* dimensions are found to be in the Fermi-Dirac distribution instead of the Planck one. The purpose of this paper is to clarify the physical meaning of his result by reproducing it on more general grounds including the case of a spinor field and to answer the question of what happens in odd dimensions.

In Sec. II the quantum field theory at finite temperature is briefly reviewed focusing on the analytical properties of two-point functions. The intuitive explanation of Sewell's theorem is given in Sec. III. In Sec. IV it is shown that, in the case of the accelerating observer, the Huygens principle should be satisfied for the detector response function to give the standard thermal spectrum in the short-wavelength limit. The massless field theories in odd dimensions do not satisfy the Huygens principle. The final section is devoted to understanding the physical origin of this phenomenon.

When revising this manuscript, the author was informed by S. A. Fulling that, independently of Takagi, Stephens[7] has also noticed the interchanging of the Planck and the Fermi-Dirac spectra in odd dimensions.

II. PROPERTIES OF TWO-POINT FUNCTIONS AT FINITE TEMPERATURE

To begin with, let us recapitulate the properties of two-point functions at zero temperature in a curved space. In the following we will consider cases where there exist a timelike Killing vector field and spacelike sections orthogonal to it. In this case the notions of temperature, energy, and the vacuum state are well defined. At this stage it does not matter whether fields are interacting or not.

Given a complex quantum field $\phi(x)$, we can define the following two types of functions:

$$G_+(x,y)\equiv\langle 0\,|\,\phi(x)\phi^\dagger(y)\,|\,0\rangle\ , \tag{2.1a}$$

$$G_-(x,y)\equiv\langle 0\,|\,\phi^\dagger(y)\phi(x)\,|\,0\rangle\ . \tag{2.1b}$$

Here $|0\rangle$ denotes the vacuum state of the theory. When $\phi(x)$ is a spinor field, $G_+(x,y)$ and $G_-(x,y)$ in the above carry spinorial indices with respect to the local Lorentz frames at x and y. Since the spacetime has a timelike Killing vector, these two-point functions depend on the corresponding Killing coordinates x_0 and y_0 only through their difference $t=x_0-y_0$:

$$\begin{aligned} G_+&=G_+(t;\mathbf{x},\mathbf{y})\ ,\\ G_-&=G_-(t;\mathbf{x},\mathbf{y})\ . \end{aligned} \tag{2.2}$$

Here $\mathbf{x}$ and $\mathbf{y}$ are spacelike coordinates.

The vacuum expectation value of the (anti)commutator between ϕ and $\phi^\dagger$, $[\phi,\phi^\dagger]_-$ $([\phi,\phi^\dagger]_+)$, is given by

$$\left.\begin{aligned} G_C(t;\mathbf{x},\mathbf{y})\\ G_A(t;\mathbf{x},\mathbf{y})\end{aligned}\right\}\equiv\langle 0\,|\,[\phi(x),\phi^\dagger(y)]_\mp\,|\,0\rangle$$
$$=G_+(t;\mathbf{x},\mathbf{y})\mp G_-(t;\mathbf{x},\mathbf{y})\ . \tag{2.3}$$

When x is in the spacelike direction from y, $G_C(x,y)$ or $G_A(x,y)$ vanishes depending on whether $\phi(x)$ is bosonic or fermionic and

$$G_+(t;\mathbf{x},\mathbf{y})=\begin{cases}+G_-(t;\mathbf{x},\mathbf{y}) & (\phi\text{: boson})\,,\\ -G_-(t;\mathbf{x},\mathbf{y}) & (\phi\text{: fermion})\end{cases} \tag{2.4}$$

$(x,y\text{: spacelike})$.

The dependence of the field $\phi(t,\mathbf{x})$ on t can be factorized as

$$\phi(t,\mathbf{x})=e^{iHt}\phi(t=0,\mathbf{x})e^{-iHt}\,, \tag{2.5}$$

where H is the generator of the translation corresponding to the timelike Killing vector. Thus, if the spectrum of H is bounded from below, the timelike variable t in $G_+(t;\mathbf{x},\mathbf{y})=\langle 0\,|\,\phi(x_0=0,\mathbf{x})e^{-iHt}\phi^\dagger(y_0=0,\mathbf{y})\,|\,0\rangle$ can be continued analytically to the lower half of a complex plane and so can t in $G_-(t;\mathbf{x},\mathbf{y})$ to the upper half of a plane. Because of Eq. (2.4), these analytic continuations give the same function $\mathcal{G}(z;\mathbf{x},\mathbf{y})$ satisfying

$$\mathcal{G}(z=t-i\epsilon;\mathbf{x},\mathbf{y})=G_+(t;\mathbf{x},\mathbf{y})\,, \tag{2.6a}$$

$$\mathcal{G}(z=t+i\epsilon;\mathbf{x},\mathbf{y})=\begin{cases}+G_-(t;\mathbf{x},\mathbf{y}) & (\text{boson})\,,\\ -G_-(t;\mathbf{x},\mathbf{y}) & (\text{fermion})\,,\end{cases} \tag{2.6b}$$

where ϵ is positive and infinitesimal.

Let us examine the analytical property of $\mathcal{G}(z;\mathbf{x},\mathbf{y})$ in z. The distance between x and y must be a monotonic function of $t=x_0-y_0$ for fixed $\mathbf{x}$ and $\mathbf{y}$. Thus there exists a function $t_{\text{light}}(\mathbf{x},\mathbf{y})$ such that, when $t=t_{\text{light}}(\mathbf{x},\mathbf{y})$, x is on the light cone of y, and x is in the spacelike (timelike) direction from y when $|\,t\,|<t_{\text{light}}(\,|\,t\,|>t_{\text{light}})$ (Fig. 1).

From Eq. (2.3) and Eqs. (2.6a) and (2.6b) we have

$$\mathcal{G}(z=t-i\epsilon;\mathbf{x},\mathbf{y})-\mathcal{G}(z=t+i\epsilon;\mathbf{x},\mathbf{y})=\begin{cases}G_C(t;\mathbf{x},\mathbf{y}) & (\text{boson})\,,\\ G_A(t;\mathbf{x},\mathbf{y}) & (\text{fermion})\,.\end{cases} \tag{2.7}$$

The right-hand side (RHS) of this equation vanishes for $|\,t\,|<t_{\text{light}}(\mathbf{x},\mathbf{y})$, and in general it is nonvanishing for $|\,t\,|\geq t_{\text{light}}(\mathbf{x},\mathbf{y})$. Therefore $\mathcal{G}(z;\mathbf{x},\mathbf{y})$ has branch cuts for $\text{Re}(z)>t_{\text{light}}$ and $\text{Re}(z)<-t_{\text{light}}$. The discontinuity across the cut is given by the (anti)commutator, G_C (G_A). Figure 2 shows the situation of these branch cuts.

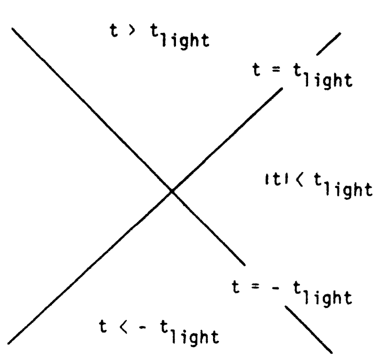

FIG. 1. The function $t_{\text{light}}(\mathbf{x},\mathbf{y})$ showing the light-cone structure of the spacetime.

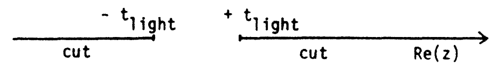

FIG. 2. The situation of the branch cuts of the function $\mathcal{G}(z;\mathbf{x},\mathbf{y})$ on the complex-z plane.

Propagators are given as the boundary values of $\mathcal{G}(z;\mathbf{x},\mathbf{y})$. For example G_+ and G_- are given by Eqs. (2.6a) and (2.6b), respectively, and Eq. (2.7) gives G_C or G_A. The Feynman propagator, $G_F(t;\mathbf{x},\mathbf{y})$, is also obtained from $\mathcal{G}$ as

$$\begin{aligned}G_F(t;\mathbf{x},\mathbf{y})&\\ &=\begin{cases}G_+(t;\mathbf{x},\mathbf{y})\theta(t)+G_-(t;\mathbf{x},\mathbf{y})\theta(-t) & (\text{boson})\,,\\ G_+(t;\mathbf{x},\mathbf{y})\theta(t)-G_-(t;\mathbf{x},\mathbf{y})\theta(-t) & (\text{fermion})\,,\end{cases}\\ &=\mathcal{G}(z=(1-i\epsilon)t;\mathbf{x},\mathbf{y})\end{aligned} \tag{2.8}$$

(Fig. 3).

It is straightforward to extend the definitions of G_+, G_C, G_A, and G_F to the case of finite temperature:

$$G_+^\beta(x,y)\equiv\frac{1}{Z}\text{Tr}[e^{-\beta H}\phi(x)\phi^\dagger(y)]\,, \tag{2.9a}$$

$$G_-^\beta(x,y)\equiv\frac{1}{Z}\text{Tr}[e^{-\beta H}\phi^\dagger(y)\phi(x)]\,, \tag{2.9b}$$

with $Z=\text{Tr}(e^{-\beta H})$ and

$$\left.\begin{matrix}G_C^\beta(x,y)\\ G_A^\beta(x,y)\end{matrix}\right\}=G_+^\beta(x,y)\mp G_-^\beta(x,y)\,. \tag{2.9c}$$

In the above, if $\phi(x)$ carries a spinor, the sum over the spinorial indices is not included in the trace, and $G_*^\beta(x,y)$ ($*=+,\ -,\ C,\ A$) carry spinorial indices. These functions can be continued analytically to complex t giving $\mathcal{G}^\beta(z;x,y)$ which satisfies

$$\mathcal{G}^\beta(z=t-i\epsilon;\mathbf{x},\mathbf{y})=G_+^\beta(t;\mathbf{x},\mathbf{y})\,, \tag{2.10a}$$

$$\mathcal{G}^\beta(z=t-i\epsilon;\mathbf{x},\mathbf{y})=\begin{cases}+G_-^\beta(t;\mathbf{x},\mathbf{y}) & (\text{boson})\,,\\ -G_-^\beta(t;\mathbf{x},\mathbf{y}) & (\text{fermion})\,,\end{cases} \tag{2.10b}$$

for positive and infinitesimal ϵ.

Let us show that the function $\mathcal{G}^\beta$ is (anti)periodic in the imaginary direction of z. Combining Eqs. (2.9a) and (2.9b) with Eq. (2.5) gives

$$\begin{aligned}G_-^\beta(t+i\beta;\mathbf{x},\mathbf{y})&=\frac{1}{Z}\text{Tr}[e^{-\beta H}\phi^\dagger(y)e^{-\beta H}\phi(x)e^{\beta H}]\\ &=\frac{1}{Z}\text{Tr}[e^{-\beta H}\phi(x)\phi^\dagger(y)]\\ &=G_+^\beta(t;\mathbf{x},\mathbf{y})\,.\end{aligned} \tag{2.11}$$

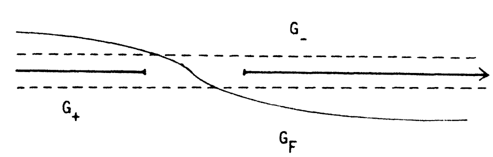

FIG. 3. Propagators given as the boundary values of $\mathcal{G}(z;\mathbf{x},\mathbf{y})$.

This equation, which is called the KMS condition[5] of the two-point function, holds whether ϕ is bosonic or fermionic. It sometimes happens that the operator $e^{-\beta H}$ does not belong to the trace class, i.e., the trace over $e^{-\beta H}$ does not converge. In such a case, the KMS condition is recognized as the criterion for the "thermality" of the state in consideration since its Fourier transform is nothing but the fluctuation-dissipation relation as given below and, thus, it assures that the response of the detector is detailed balancing. From the KMS condition, Eq. (2.11), and the definition of $\mathcal{G}^\beta$, Eqs. (2.10a) and (2.10b), the (anti)periodicity of $\mathcal{G}^\beta$ follows:

$$\mathcal{G}^\beta(z+i\beta;\mathbf{x},\mathbf{y})=\begin{cases}+\mathcal{G}^\beta(z;\mathbf{x},\mathbf{y}) & \text{(boson)}\,,\\ -\mathcal{G}^\beta(z;\mathbf{x},\mathbf{y}) & \text{(fermion)}\,.\end{cases} \tag{2.12}$$

In terms of these two-point functions, the response of the detector mounted at $\mathbf{x}$ in a thermal bath is given as follows. First we define the Fourier transforms of the propagators with respect to the timelike variable t as

$$F^\beta_*(\omega;\mathbf{x})\equiv\int_{-\infty}^{+\infty}dt\,e^{-i\omega t}G^\beta_*(t;\mathbf{x},\mathbf{x})\quad(*=+,-,C,A)\,. \tag{2.13}$$

The Fourier transform of the KMS condition, Eq. (2.11), is

$$F^\beta_+(\omega;\mathbf{x})=e^{-\beta\omega}F^\beta_-(\omega;\mathbf{x})\,. \tag{2.14}$$

This is the fluctuation-dissipation relation characteristic to thermal states.

In the following, we shall consider the response of the monopole-type detector coupled to the field $\phi(x)$ as

$$L_{\text{int}}=m(t)\frac{1}{\sqrt{2}}[\phi(t;\mathbf{x})+\phi^\dagger(t;\mathbf{x})] \tag{2.15a}$$

when $\phi(x)$ is a scalar and

$$L_{\text{int}}=m(t)\bar{\phi}(t;\mathbf{x})\phi(t;\mathbf{x}) \tag{2.15b}$$

with $\bar{\phi}=\phi^\dagger\gamma^0$ when $\phi(x)$ is a spinor. The probability $R(E\to E+\omega)$ per unit time for the detector to make a transition from $|E\rangle$ to $|E+\omega\rangle$ (E and $E+\omega$ denote the internal energies of the detector) is given by

$$R(E\to E+\omega)=|\langle E+\omega|m(0)|E\rangle|^2F^\beta_+(\omega;\mathbf{x}) \tag{2.16a}$$

when $\phi(x)$ is a scalar. On the other hand, when $\phi(x)$ is a spinor, the transition rate becomes

$$R(E\to E+\omega)=|\langle E+\omega|m(0)|E\rangle|^2\mathcal{F}^\beta(\omega;\mathbf{x})\,, \tag{2.16b}$$

where

$$\begin{aligned}\mathcal{F}^\beta(\omega;\mathbf{x})&\equiv\int_{-\infty}^{+\infty}dt\,e^{-i\omega t}\,\mathrm{tr}[G^\beta_+(t;\mathbf{x},\mathbf{x})\gamma^0G^\beta_-(-t;\mathbf{x},\mathbf{x})\gamma^0]\\ &=\int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\,\mathrm{tr}[F^\beta_+(\omega+\omega';\mathbf{x})\gamma^0F^\beta_-(\omega';\mathbf{x})\gamma^0]\,.\end{aligned} \tag{2.17}$$

Thus, except for the factor $\langle E+\omega|m(0)|E\rangle$ intrinsic to the property of the detector, the transition rate is characterized by $F^\beta_+(\omega,x)$ and $F^\beta_-(\omega,x)$.

From Eq. (2.14) and the definition of F^β_C and F^β_A,

$$\left.\begin{matrix}F^\beta_C(\omega;\mathbf{x})\\ F^\beta_A(\omega;\mathbf{x})\end{matrix}\right\}\equiv F^\beta_+(\omega;\mathbf{x})\mp F^\beta_-(\omega;\mathbf{x})\,, \tag{2.18}$$

we obtain

$$F^\beta_+(\omega;\mathbf{x})=\frac{1}{e^{\beta\omega}-1}[-F^\beta_C(\omega;\mathbf{x})] \tag{2.19a}$$

$$=\frac{1}{e^{\beta\omega}+1}F^\beta_A(\omega;\mathbf{x})\,. \tag{2.19b}$$

So far we have only made use of the KMS condition, Eq. (2.11). Thus Eqs. (2.19a) and (2.19b) hold irrespectively of whether ϕ is bosonic or fermionic. However, these equations alone are not enough to determine F^β_+ and F^β_- completely and, thus, the response of the detector, since the dependence of neither F^β_C nor F^β_A on the temperature β^{-1} is known.

It is at this point that the statistical property of the field ϕ becomes relevant. First we consider the case when the field is free. When the field ϕ is bosonic (fermionic), the (anti)commutator between ϕ's is a c number and G^β_C (G^β_A) is independent of the temperature,

$$\begin{aligned}G^\beta_C(x,y)&=\frac{1}{Z}\mathrm{Tr}\{e^{-\beta H}[\phi(x),\phi^\dagger(y)]_-\}\\ &=[\phi(x),\phi^\dagger(y)]_-\\ &=G_C(x,y)\quad\text{(boson)}\,,\end{aligned} \tag{2.20a}$$

$$G^\beta_A(x,y)=G_A(x,y)\quad\text{(fermion)}\,, \tag{2.20b}$$

and so are their Fourier transforms, $F^\beta_C(\omega;\mathbf{x})$ and $F^\beta_A(\omega;\mathbf{x})$. Thus we obtain

$$F^\beta_+(\omega;\mathbf{x})=\begin{cases}\dfrac{1}{e^{\beta\omega}-1}[-F_C(\omega;\mathbf{x})] & \text{(boson)}\,,\\[2ex] \dfrac{1}{e^{\beta\omega}+1}F_A(\omega;\mathbf{x}) & \text{(fermion)}\,.\end{cases} \tag{2.21}$$

When the field is interacting, Eqs. (2.20a) and (2.20b) do not hold since the (anti)commutator of ϕ's is no more a c number. This is natural since it is the response function of the Lehmann-Symanzik-Zimmermann (LSZ) asymptotic field that should satisfy Eq. (2.21).

III. SEWELL'S THEOREM

In this section the intuitive explanation of Sewell's theorem[4] is given by extending the argument in Ref. 8. It does not matter whether the particles are interacting or free. For simplicity the KMS condition of the two-point function is discussed, although extension of the proof to n-point functions is straightforward.

To describe the motion of a uniformly accelerating detector in the d-dimensional Minkowski space, it is convenient to employ the so-called Rindler coordinates $(\eta,\rho,x_2,x_3,\ldots,x_{d-1})$, satisfying

$$x_0=\rho\sinh a\eta,\quad x_1=\rho\cosh a\eta\,, \tag{3.1}$$

where $(x_0,x_1,\ldots,x_{d-1})$ are the usual Minkowski coordinates. In these coordinates the trajectory of the observer moving in the x_1 direction with a uniform acceleration a is given by $\rho=1/a$, $x_2=x_3=\cdots=x_{d-1}=0$. The Minkowski metric written in terms of the Rindler coordinates is

$$ds^2=a^2\rho^2 d\eta^2-d\rho^2-d\mathbf{x}^2 . \tag{3.2}$$

Thus η is the Killing coordinate.

Since we want to know the response of the detector to the Minkowski vacuum, the propagator to be considered is

$$G_+^R[(\eta,\rho,\mathbf{x});(\eta',\rho',\mathbf{x}')]\equiv{}_M\langle 0|\phi(x)\phi^\dagger(x')|0\rangle_M , \tag{3.3}$$

where $|0\rangle_M$ is the usual Minkowski vacuum and $\mathbf{x}$ represents the coordinates $(x_2,\ldots,x_{d-1})$ transverse to the detector motion. Propagators G_-^R, G_C^R, and G_A^R are defined in the same way.

Since the dependence of the propagators G_* on ρ and η is only through the Minkowski coordinates x_0, x_1, x_0', and x_1' as in Eq. (3.3), their analytic continuation $\mathcal{G}^R[z,z';(\rho,\mathbf{x}),(\rho',\mathbf{x}')]$ is periodic in z and z':

$$\begin{aligned}\mathcal{G}^R\left[z+\frac{2\pi}{a}i,z';(\rho,\mathbf{x}),(\rho',\mathbf{x}')\right]&=\mathcal{G}^R\left[z,z'+\frac{2\pi}{a}i;(\rho,\mathbf{x}),(\rho',\mathbf{x}')\right]\\&=\mathcal{G}^R[z,z';(\rho,\mathbf{x}),(\rho',\mathbf{x}')] .\end{aligned} \tag{3.4}$$

Since we are concerned only with the detection by the accelerating observer, the propagator will be evaluated on its trajectory, $\rho=\rho'=1/a$ and $\mathbf{x}=\mathbf{x}'=0$. In this case the local Lorentz frame natural to the observer is the one Fermi transported[9] along its trajectory. To take this local Lorentz rotation into account, we define propagators $\hat{G}_*^R(\eta,\eta';a)$ from $G_*^R(\eta,\eta';a)$ by pulling their spinorial indices with respect to the local frame at η and η' back along the trajectory to that at some reference point η_0 as

$$\hat{G}_*^R(\eta,\eta';a)=\Lambda(\eta,\eta_0;a)G_*^R(\eta,\eta';a)\Lambda^\dagger(\eta',\eta_0;a) , \tag{3.5}$$

where $\Lambda(\eta_1,\eta_2;a)$ is the matrix representing the rotation of the local Lorentz frame induced by the Fermi transportation along $\eta_1\to\eta_2$ on the trajectory.

The rotation matrix $\Lambda(\eta_1,\eta_2;a)$ can also be continued analytically to complex η_1 and η_2. As can be seen from Eq. (3.1) or Eq. (3.2), the translation $\eta\to\eta+(2\pi/a)i$ corresponds to 2π rotation in Euclidean (ix_0,x_1) plane. Therefore

$$\begin{aligned}\Lambda\left[\eta+\frac{2\pi}{a}i,\eta';a\right]&=\Lambda\left[\eta,\eta'+\frac{2\pi}{a}i;a\right]\\&=(-)^{2S}\Lambda(\eta,\eta';a) ,\end{aligned} \tag{3.6}$$

where S is the spin of the field ϕ.

Combining this equation with Eq. (3.4) gives

$$\begin{aligned}\hat{\mathcal{G}}^R\left[z+\frac{2\pi}{a}i,z';a\right]&=\hat{\mathcal{G}}^R\left[z,z'+\frac{2\pi}{a}i;a\right]\\&=\begin{cases}+\hat{\mathcal{G}}^R(z,z';a) & \text{(integral spin)} ,\\ -\hat{\mathcal{G}}^R(z,z';a) & \text{(half-integral spin)} ,\end{cases}\end{aligned} \tag{3.7}$$

where

$$\hat{\mathcal{G}}^R(z,z';a)\equiv\Lambda(z,z_0;a)\mathcal{G}^R(z,z';a)\Lambda^\dagger(z',z_0;a) . \tag{3.8}$$

Let us elaborate more on the η and η' dependence of $\hat{G}_*^R$ $(*=+,-,C,A)$ and the z and z' dependence of $\hat{\mathcal{G}}^R$. The global translation $\eta\to\eta+\delta\eta,\eta'\to\eta'+\delta\eta$ corresponds to the Lorentz boost in the rest frame since the observer is uniformly accelerating. The Minkowski vacuum, on which two-point functions are evaluated, is invariant under the Lorentz boost, and the rotation in spinor indices caused by this transformation is compensated by the matrix $\Lambda(\eta_1,\eta_2;a)$ in Eq. (3.5). Therefore $\hat{G}_*^R(\eta,\eta';a)$ depends on η and η' only through their difference, $\tau=\eta-\eta'$:

$$\begin{aligned}&\hat{G}_*^R=\hat{G}_*^R(\tau=\eta-\eta';a) ,\\&\hat{\mathcal{G}}^R=\hat{\mathcal{G}}^R(\xi=z-z';a) ,\end{aligned} \tag{3.9}$$

Now $\hat{\mathcal{G}}^R$ given by Eq. (3.8) is the analytic continuation of $\hat{G}_+^R$ in τ, and Eq. (3.7) becomes

$$\hat{\mathcal{G}}^R\left[\xi+\frac{2\pi}{a}i;a\right]=\begin{cases}+\hat{\mathcal{G}}^R(\xi;a) & \text{(integral spin)} ,\\ -\hat{\mathcal{G}}^R(\xi;a) & \text{(half-integral spin)} .\end{cases} \tag{3.10}$$

Provided the relation between spin and statistic holds, this equation just gives the KMS condition, Eq. (2.12). Extension of this result to n-point functions is straightforward. Thus the accelerating detector finds itself in the "thermal" state.

IV. PARTICLE SPECTRA AND THE HUYGENS PRINCIPLE

As we have seen in Sec. II, the KMS condition itself is not enough to determine the temperature dependence of the detector response function. The observed spectrum is not the direct consequence of the "thermality" of the state.

Following the derivation of Eqs. (2.19a) and (2.19b), the Fourier transforms $F_*^R(\omega;a,d)$ of the propagators $\hat{G}_*^R$ $(*=+,-,C,A)$ in d dimensions satisfy

$$F_+^R(\omega;a,d)=\frac{1}{\exp\left[\frac{2\pi}{a}\omega\right]-1}[-F_C^R(\omega;a,d)] \tag{4.1a}$$

$$=\frac{1}{\exp\left[\frac{2\pi}{a}\omega\right]+1}F_A^R(\omega;a,d) \tag{4.1b}$$

because of the KMS property of $\hat{G}^R_+$ and $\hat{G}^R_-$. Since the dependence of neither $F^R_C(\omega;a,d)$ nor $F^R_A(\omega;a,d)$ on the acceleration a is given so far, these equations alone tell nothing about the shape of the observed particle spectrum.

Before discussing the general case, let us consider free massless particles in d spacetime dimensions. Since the theory contains no dimensionful parameter, the two-point function G^R_+ should be proportional to $(L^{(2)})^{1-d/2}$ for a bosonic field, where $L^{(2)}$ is the square distance between the two points. Therefore G^R_+ analytically continued to complex τ has a branch cut at $L^{(2)}<0$ (timelike distance, i.e., $\tau\neq0$ along the observer's trajectory) when d is odd,

$$\mathcal{G}^R(\tau+i\epsilon;a)=-\mathcal{G}^R(\tau-i\epsilon;a) \quad (d\text{: odd, } \tau\neq0) \tag{4.2a}$$

and when d is even, this cut reduces to a pole at $L^{(2)}=0$ (lightlike distance, i.e., $\tau=0$ on the trajectory):

$$\mathcal{G}^R(\tau+i\epsilon;a)=\mathcal{G}^R(\tau-i\epsilon;a) \quad (d\text{: even, } \tau\neq0)\ . \tag{4.2b}$$

The two-point function of a spinor field must have the same analytical properties as that of a scalar field since the former is given by differentiating the latter.

Combining these equations with Eqs. (2.6a) and (2.6b), we find

$$\hat{G}^R_+(\tau;a)=\begin{cases}(-)^d\hat{G}^R_-(\tau;a) & \text{(scalar)}\ ,\\ (-)^{d+1}\hat{G}^R_-(\tau;a) & \text{(spinor)}\end{cases} \tag{4.3}$$

for $\tau\neq0$, or equivalently

$$\hat{G}^R_C(\tau;a)=0 \quad (d\text{: even, scalar; } d\text{: odd, spinor})\ , \tag{4.4a}$$

$$\hat{G}^R_A(\tau;a)=0 \quad (d\text{: odd, scalar; } d\text{: even, spinor}) \tag{4.4b}$$

for $\tau\neq0$ (Figs. 4 and 5).

In general, if a function $f(x)$ has its support only at

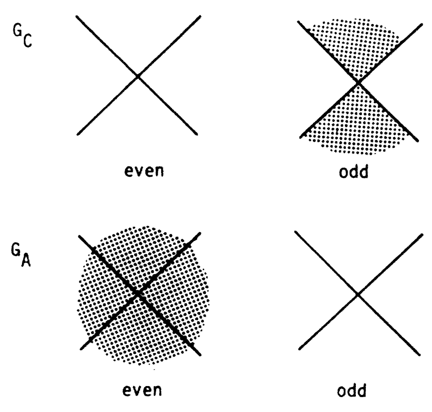

FIG. 4. Supports of the propagators of a massless scalar; G_C in even dimensions and G_A in odd dimensions are nonvanishing only on the light cones while the support of G_C in odd dimensions extends to the timelike regions and that of G_A in even dimensions to the whole Minkowski spacetime.

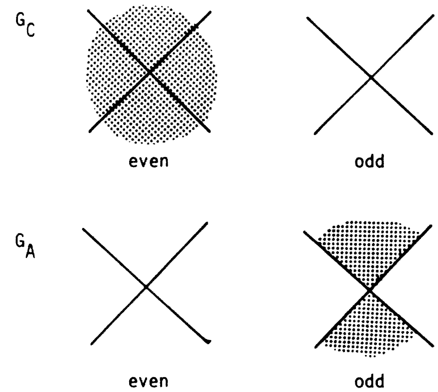

FIG. 5. Supports of propagators of a massless spinor; G_A in even dimensions and G_C in odd dimensions are nonvanishing only on the light cones while the support of G_A in odd dimensions extends to the timelike regions and that of G_C in even dimensions to the whole Minkowski spacetime.

$x=0$, $f(x)$ is given as a finite sum of derivatives of the delta function $\delta(x)$, and its Fourier transform $\int_{-\infty}^{+\infty}dx\, e^{-ikx}f(x)$ is a finite polynomial in k. Thus we can expect that F^R_C (F^R_A) for a massless scalar in even (odd) dimensions and for a massless spinor in odd (even) dimensions is a finite polynomial in ω. In fact, it is proven in the Appendix that, in the above-mentioned situations, F^R_C (F^R_A) for a scalar is a polynomial of degree $(d-3)$ in ω and, especially when $d=2$, 3, and 4, it is a monomial in ω and independent of the acceleration a. Therefore, in two or four dimensions, F^R_C is independent of a and F^R_+ shows the standard Planck spectrum with temperature $\beta^{-1}=a/2\pi$, whereas, in three dimensions, it is F^R_A that becomes independent of a and F^R_+ gives the Fermi-Dirac distribution reproducing the results in Refs. 6 and 7. For $d\geq5$, depending on whether d is even or odd, F^R_+ deviates from the Planck or the Fermi-Dirac spectrum by a factor of an even polynomial of degree $(d-4)$ or $(d-3)$ in a. It is also shown in the Appendix that, when d is even [odd], $F^R_A(\omega;a,d)$ $[F^R_C(\omega;a,d)]$ for a massless spinor is proportional to $F^R_A(\omega;a,d+1)$ $[F^R_C(\omega;a,d+1)]$ for a massless scalar. Combining this result with Eqs. (4.1a) and (4.1b), one can see that *F^R_+ for a massless spinor in d dimensions is proportional to that for a massless scalar in $(d+1)$ dimensions.* This means that, especially in two dimensions, F^R_+ for a massless spinor is the standard Fermi-Dirac distribution while, in three dimensions, it is the Planck type. Although F^R_+ deviates from both the Planck and the Fermi-Dirac spectra for $d\geq5$ when ϕ is a scalar and for $d\geq4$ when ϕ is a spinor, the above italicized statement suggests that the interchange of the two types of the spectra is the persisting feature in arbitrary odd dimensions.

These results do not contradict the general theorem by Sewell explained in the preceding section. On the contrary, as we have seen so far, they are derived from this theorem combined with the value of the commutator and the anticommutator of ϕ's in the timelike regions. The cause of the interchanging of the two types of spectra in odd dimensions is that the expectation values of the *anticommutator of a massless scalar* and the *commutator of a massless spinor* vanish if the two points are in the timelike distance. On the other hand, in even dimensions particles obey the standard thermal distributions: the expectation value of the commutator of a scalar (the anticommutator of a spinor) vanishes in the timelike regions.

The significance of the Huygens principle is the following. In ordinary quantum field theories at finite temperature, the response of the detector is characterized by the standard Planck or Fermi-Dirac distribution irrespectively of the Huygens principle as explained in Sec. II. The point is that G_C^β (G_A^β) is independent of β when the (anti)commutator between fields is a c number.

In the case of the accelerating detector, the situation is different since, even if the (anti)commutator is a c number, the propagator G_C^R (G_A^R), which has originally been a function of $(x_0,x_1,\mathbf{x})$ and $(x'_0,x'_1,\mathbf{x}')$, depends on the acceleration a through the coordinate transformation, Eq. (3.1), when expressed in terms of the Rindler coordinates. This a dependence of $\hat{G}_C^R$ $(\hat{G}_A^R)$ or equivalently that of F_C^R (F_A^R) in general changes the shape of the spectrum. When the Huygens principle holds, the dependence of F_C^R (F_A^R) on a is at most polynomial and, especially when $d \le 4-2S$, this dependence vanishes.

It is also pointed out in Ref. 6 that the response function of a massive field also deviates significantly from the standard thermal spectrum. This phenomenon can be explained in just the same way as the above. The reason for this deviation is that the propagator of a massive field has its support on timelike regions as well as on a light cone, and the Huygens principle fails to hold.

Thus it is concluded that, although the "thermality" of the state is guaranteed by the general theorem, this does not mean that the accelerating observer detects particles in their standard thermal distributions unless the Huygens principle holds.[10]

V. PHYSICAL INTERPRETATIONS

In the preceding section the response function of the accelerating detector is evaluated in the usual Minkowski vacuum. On the other hand, it is possible to consider the same detector embedded in the thermal bath with a temperature β^{-1}. In this case the thermal bath itself must be accelerated along with the detector. This means that the density matrix describing the system should be $e^{-\beta H_\eta}$, where H_η is the generator of the translation along the observer's trajectory, rather than $e^{-\beta H_t}$ with H_t being the ordinary Hamiltonian of the rest frame.

Following the arguments in Sec. II, the response function of the detector in the thermal bath, $F_+^{R,\beta}(\omega;a)$, is obtained as

$$F_+^{R,\beta}(\omega;a)=\begin{cases}\dfrac{1}{e^{\beta\omega}-1}[-F_C^R(\omega;a)] & \text{(boson)}\ ,\\[2ex] \dfrac{1}{e^{\beta\omega}+1}F_A^R(\omega;a) & \text{(fermion)}\ ,\end{cases} \tag{5.1}$$

since η is a timelike Killing coordinate as well as the proper time of the observer [see Eq. (2.21)]. Comparing this equation with Eqs. (4.1a) and (4.1b), we find

$$F_+^R(\omega;a)=F_+^{R,\beta=2\pi/a}(\omega;a)\ , \tag{5.2}$$

which means that the observer cannot tell whether it is in the Minkowski vacuum or in the thermal bath with $\beta=2\pi/a$. This relation holds irrespectively of dimension of the spacetime, spin and mass of the particle, and type of interactions. Of course this has been assured from the outset by the general theorem of Sewell.

The apparent discrepancy between the "thermality" of the state and the shape of the spectrum can be resolved if one carefully discriminates the two different components of the spectrum both of which depend on the acceleration a. The first one is the prefactor of the RHS of Eq. (4.1a) [Eq. (4.1b)], which results from the thermal nature of the state of a bosonic [fermionic] field, and the second one is the Fourier transform of the [anti]commutator, $F_C^R(\omega;a)$ $[F_A^R(\omega;a)]$, which gives the density of modes at the frequency.

As we have seen, the response of the accelerating detector is, in general, *not* that of the detector *at rest in a thermal bath*. This is because the density of modes at the frequency and at the position of the detector, $F_C^R(\omega;a)$ $[F_A^R(\omega;a)]$, is different from that of the system at rest, $F_C^M(\omega)$ $[F_A^M(\omega)]$. Under the uniform acceleration, the rate of the red-shift and the blue-shift are continuously increasing. This causes the shift of energy levels and the density of modes suffers a change.

Evaluation of the response function in de Sitter space and Schwarzschild space is parallel to that described in Sec. IV. When the field is massless and its coupling to the gravitational field is conformally invariant, these calculations become especially simple since the dimensional argument can be employed. The apparent interchanging of statistics is observed[11] in the maximally symmetric vacuum similarly to the case of the accelerating observer in the Minkowski vacuum.

In these cases the presence of the gravitational field affects the distribution of the thermalized particles. For example, in the case of Schwarzschild space, there is a strong potential barrier near the horizon. Particles with high energies can easily penetrate this barrier, while those with low energies are reflected there. Thus, the spectrum of particles radiated from the black hole is multiplied by the transmission coefficient depending on the energy of the observed particle. This factor causes the deviation of the spectrum from the standard thermal one.

This does not contradict the thermal property of the black hole. Since the transmission coefficient is reciprocal, the equilibrium would be attained if the black hole were embedded in the thermal bath.

We could have explained the phenomenon seen from the accelerating detector in the same way. This detector is equivalent to the one in the constant gravitational field. In this case the thermalized particles created near the apparent horizon lose their energies while climbing up the slope of the gravitational potential. This process, in general, changes the shape of the spectrum.

ACKNOWLEDGMENTS

I wish to thank T. Inami, M. Sasaki, H. Sato, and S. Takagi for valuable comments and careful reading of the manuscript. I have benefited from discussions with L. Susskind. I am also grateful for W. Israel for comments when revising this manuscript.

APPENDIX

Here we evaluate the Fourier transforms of propagators whose support is only on the light cones, i.e., G_C^R (G_A^R) when the field $\phi(x)$ is a massless scalar and d is even (odd) or when $\phi(x)$ is a massless spinor and d is odd (even). Let us first consider the case when $\phi(x)$ is a scalar. Since the square distance $L^{(2)}$ between the two points on the detector's trajectory is given by

$$L^{(2)}=\left[\frac{2}{a}\sinh(\tfrac{1}{2}a\tau)\right]^2 , \tag{A1}$$

it readily follows from the dimension counting that a scalar propagator with its support on the light cones is proportional to $\delta^{(d-3)}((2/a)\sinh(\frac{1}{2}a\tau))$ with the convention

$$\delta^{(-1)}(x)=\tfrac{1}{2}[\theta(x)-\theta(-x)] . \tag{A2}$$

Noticing that

$$\delta^{(n)}(f(x))=\left[\frac{1}{|f'(x)|}\frac{d}{dx}\right]^n\left[\frac{1}{|f'(x)|}\delta(x)\right] \tag{A3}$$

and $\sinh(x)$ is odd in x, one can see that $\delta^{(d-3)}((2/a)\sinh(\frac{1}{2}a\tau))$ can be expanded as

$$\delta^{(d-3)}\left[\frac{2}{a}\sinh(\tfrac{1}{2}a\tau)\right]=\begin{cases}\delta^{(-1)}(\tau)\ \ (d=2)\ ,\\ \displaystyle\sum_{n=0}^{(d-3)/2} C_{d,n}a^{d-3-2n}\delta^{(2n)}(\tau)\ \ (d\text{: odd }\geq 3)\ ,\\ \displaystyle\sum_{n=0}^{(d-4)/2} C_{d,n}a^{d-4-2n}\delta^{(2n+1)}(\tau)\ \ (d\text{: even }\geq 4)\ ,\end{cases} \tag{A4}$$

where the expansion coefficients $C_{d,n}$ are numerical constants independent of a and τ. Thus their Fourier transforms with respect to τ are

$$\int_{-\infty}^{+\infty} d\tau\, e^{-i\omega\tau}\delta^{(d-3)}\left[\frac{2}{a}\sinh(\tfrac{1}{2}a\tau)\right]=\begin{cases}\dfrac{1}{i\omega}\ \ (d=2)\ ,\\ \displaystyle\sum_{n=0}^{(d-3)/2} C_{d,n}a^{d-3-2n}(-)^n\omega^{2n}\ \ (d\text{: odd }\geq 3)\ ,\\ \displaystyle\sum_{n=0}^{(d-4)/2} C_{d,n}a^{d-4-2n}(-)^n i\omega^{2n+1}\ \ (d\text{: even }\geq 4)\ .\end{cases} \tag{A5}$$

They are polynomials of degree $(d-3)$ in ω and, especially when $d=2$, 3, and 4, they reduce to monomials $(i\omega)^{d-3}$ and independent of a.

In Minkowski spacetime, the propagators G_+ and G_- for a massless spinor is given by operating $i\gamma^\mu\partial_\mu$ on the corresponding scalar propagators. Taking into account the local Lorentz rotation along the trajectory, one can see that, for a massless spinor, G_A^R (G_C^R) in even (odd) dimensions are proportional to $\delta^{(d-2)}((2/a)\sinh(\frac{1}{2}a\tau))$ times a unit matrix for spinorial indices.

To summarize, we have obtained the following results concerning the Fourier transforms F_C^R and F_A^R of G_C^R and G_A^R in d dimensions.

(1) When d is even [odd], $F_A^R(\omega;a,d)$ [$F_C^R(\omega;a,d)$] for a massless spinor is proportional to $F_A^R(\omega;a,d+1)$ [$F_C^R(\omega;a,d+1)$] for a massless scalar.

(2) For a massless scalar, F_C^R (F_A^R) in even (odd) dimensions is a polynomial of degree $(d-3)$ in and, especially when $d\leq 4$, it is independent of a. Thus, for a massless spinor, F_A^R (F_C^R) in even (odd) dimension is independent of a for $d\leq 3$.

*Present address: Department of Physics, Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan.

[1]S. W. Hawking, Nature (London) **248**, 30 (1974).

[2]For reviews, see D. W. Sciama, P. Candelas, and D. Deutsch, Adv. Phys. **30**, 327 (1981); N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982); S. A. Fulling and S. N. M. Ruijsenaars, Phys. Rep. (to be published), and references therein.

[3]W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[4]G. L. Sewell, Ann. Phys. (N.Y.) **141**, 201 (1982); see also W. G. Unruh and N. Weiss, Phys. Rev. D **29**, 1656 (1984).

[5]O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics I, II* (Springer, New York, 1981).

[6]S. Takagi, Prog. Theor. Phys. **74**, 142 (1985).

[7]C. R. Stephens, University of Maryland report, 1985 (unpublished).

[8]G. W. Gibbons and M. J. Perry, Proc. R. Soc. London **A358**, 467 (1978); S. M. Christensen and M. J. Duff, Nucl. Phys. **B146**, 11 (1978).

[9]See, for example, S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, Cambridge, England, 1973).

[10]It would be worthwhile to note here that, in the original paper of the black-hole radiation, Hawking has made use of the optical approximation to derive the Planck spectrum of the emitted particle. This approximation, however, is not applicable in odd dimensions because of the lack of the Huygens principle.

[11]The author is informed by S. Takagi that he has also found this phenomenon in de Sitter space. See S. Takagi, Prog. Theor. Phys. **47**, 501 (1985); **47**, 1219 (1985).

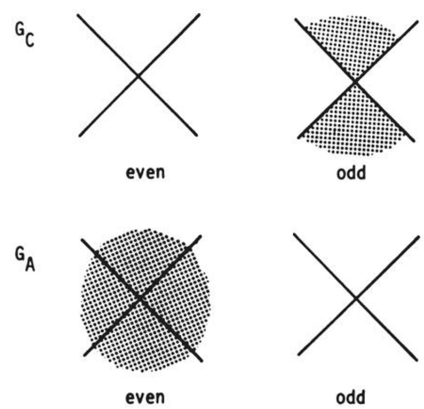

FIG. 4. Supports of the propagators of a massless scalar; G_C in even dimensions and G_A in odd dimensions are nonvanishing only on the light cones while the support of G_C in odd dimensions extends to the timelike regions and that of G_A in even dimensions to the whole Minkowski spacetime.

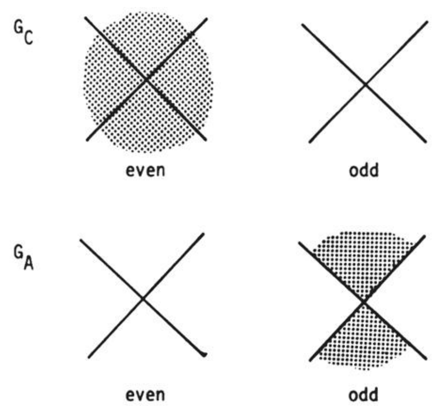

FIG. 5. Supports of propagators of a massless spinor; G_A in even dimensions and G_C in odd dimensions are nonvanishing only on the light cones while the support of G_A in odd dimensions extends to the timelike regions and that of G_C in even dimensions to the whole Minkowski spacetime.