Electric dipole moment of the electron in left-right-symmetric theories

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We calculate the electric dipole moment of the electron (d_e) in left-right-symmetric theories. Particular attention is given to clarify the way in which the various *CP*-violating phases enter in the calculation and on the distinction between the cases of Dirac and Majorana neutrinos. The calculated value of d_e can be comparable to the present experimental limit if the right-handed neutrino is sufficiently heavy.

It is well known that if the neutrinos are Majorana particles, then the leptonic Kobayashi-Maskawa matrix contains additional observable phases that are not present if they are Dirac particles.[1] Whether these additional phases can have large, observable effects depends on the particular model. In the $SU(2)\times U(1)$ model, the effects of these new phases are generally suppressed due to the absence of right-handed currents.[2] With this in mind, several authors have considered models with horizontal interactions,[3] with supersymmetry,[4] and with left-right symmetry.[5]

In this article we analyze in detail the calculation of the electric dipole moment of the electron (d_e) in left-right-symmetric theories based on the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, paying particular attention to the role of the various *CP*-violating phases in the theory and on distinguishing between the cases of Dirac and Majorana neutrinos. We find that a nonzero value of d_e can exist even in the case of Dirac neutrinos, contrary to the conclusions of Valle.[5] However, in this case, if only the known generations of leptons are considered, then the resulting value of d_e lies well below the present experimental limit. A value close to the experimental limit can arise if the electron has a small coupling to a heavy (either Dirac or Majorana) neutrino. In the simplest case the heavy neutrino can be the right-handed (electron) neutrino. A sizable value of d_e can arise in this case provided that $(\mu_D)_{ee}\simeq 1$ MeV, where μ_D is the Dirac mass term in the neutrino mass matrix.

Left-right-symmetric theories have been discussed extensively in the literature.[6] In such theories, the charged-current interaction of the leptons is given by

$$\mathscr{L}_{CC}=\frac{g}{\sqrt{2}}\sum_a (W_L^{\mu}\bar{l}_{aL}\gamma_\mu \nu_{aL}+W_R^{\mu}\bar{l}_{aR}\gamma_\mu N_{aR})+\text{H.c.}\ , \tag{1}$$

where the summation runs from 1 to n, n being the number of fermion families. We can always choose a representation in which the mass matrix of the charged leptons l_a is diagonal. In that basis, the gauge-boson mass terms will be given by

$$(W_L^* W_R^*)\begin{bmatrix} M_L{}^2 & \Delta^2 \\ \Delta^{*2} & M_R{}^2\end{bmatrix}\begin{bmatrix} W_L \\ W_R\end{bmatrix}, \tag{2}$$

and the neutrino mass terms by

$$(\bar{\nu}_R^c \bar{N}_R)\begin{bmatrix} \mu_\nu & \mu_D \\ \mu_D^T & \mu_N\end{bmatrix}\begin{bmatrix} \nu_L \\ N_L^c\end{bmatrix}+\text{H.c.} \tag{3}$$

In (3), ν_L and N_L^c denote column matrices with n neutral fields, one for each generation, and μ_ν, μ_D, and μ_N are $n\times n$ matrices.

In general, the mass parameters in Eq. (3) as well as Δ^2 in Eq. (2) are complex. However, by an appropriate choice of the phases of the various fields, some of them can be chosen to be real. For example, note that the charged currents in (1) are invariant under the phase redefinitions $W_R\to e^{i\alpha}W_R$, $N_R\to e^{-i\alpha}N_R$. Using this freedom, we can arrange Δ^2 to be real while the phase α will appear in μ_D and μ_N in Eq. (3). Thus, by this phase convention the *CP*-violating effects arise through the neutrino mass matrix. Alternatively, we can start by diagonalizing the gauge-boson mass matrix right away. This leads to

$$\begin{bmatrix} W_L \\ W_R\end{bmatrix}=U\begin{bmatrix} W_1 \\ W_2\end{bmatrix}, \tag{4}$$

where W_1 and W_2 are the mass eigenstates with masses M_1 and M_2, and U is a unitary matrix which can be written as

$$U=\begin{bmatrix} \cos\zeta & \sin\zeta \\ -e^{i\alpha}\sin\zeta & e^{i\alpha}\cos\zeta\end{bmatrix}. \tag{5}$$

The phase α can be removed from Eq. (1) by redefining N_R, so that we reproduce what was said above. The crucial point is that α in Eq. (5) and the phases in the neutrino mass matrix are *not* independent sources of *CP* violation contrary to the arguments of Ref. 5. Setting $\alpha=0$ in Eq. (4) merely fixes a convention and putting $\alpha\neq 0$ is just another, but equivalent, way of writing the same theory. In what follows we fix our convention by putting $\alpha=0$ in Eq. (5). In other words, we choose Δ^2 in Eq. (2) to be real

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so that U is an orthogonal matrix.

The neutrino mass matrix, though complex in this convention is symmetric nevertheless, so it can be diagonalized by using a $2n\times 2n$ unitary matrix V which gives

$$\begin{bmatrix}\nu_L\\ N_L^c\end{bmatrix}=V\chi_L\ ,\qquad (\bar{\nu}_R^c\bar{N}_R)=\bar{\chi}_R V^T\ , \tag{6}$$

where χ is a column matrix of $2n$ mass eigenstates. Equivalently, Eq. (6) can be written as

$$\nu_{aL}=\sum_{A=1}^{2n}P_{aA}\chi_{AL} \tag{7a}$$

$$\text{for } a=1,\ldots,n\ ,$$

$$N_{aL}^c=\sum_{A=1}^{2n}Q_{aA}\chi_{AL} \tag{7b}$$

where P and Q are $n\times 2n$ matrices, denoting the first and the last n rows of V, respectively, and are both complex. With these notations, we can rewrite Eq. (1) in terms of physical fields:

$$\mathcal{L}_{\mathrm{CC}}=\frac{g}{\sqrt{2}}\sum_{a=1}^{n}\sum_{A=1}^{2n}\sum_{i=1}^{2}W_i^{\mu}(U_{Li}P_{aA}\bar{l}_{aL}\gamma_\mu\gamma_\mu\chi_{AL}+U_{Ri}Q_{aA}^{*}\bar{l}_{aR}\gamma_\mu\chi_{AR})+\mathrm{H.c.} \tag{8}$$

The one-loop graph involving weak gauge bosons that contribute to the electromagnetic form factors is shown in Fig. 1. However, if we choose to calculate the form factors in a general ξ gauge,[6] extra diagrams appear where one or both of the W_i lines are replaced by the unphysical Higgs boson H_i which is absorbed by W_i in the unitary gauge. These diagrams are included in our calculation. It must be noted that calculation of these additional diagrams does not require a detailed knowledge of the Higgs sector of the theory. The fermion couplings to H_i can be determined[6,7] merely by looking at the scattering process $\chi_A+l_a\to\chi_A+l_a$ mediated by W_i and H_i and requiring the result to be independent of the gauge parameter ξ. Similarly, once we find out the WW_γ coupling, we can use the process $W_i+\chi_A\to\gamma+l_a$ mediated by W_i and H_i to obtain the WH_γ coupling and so on. The result of all these exercises is summarized in the Feynman rules given in Fig. 2.

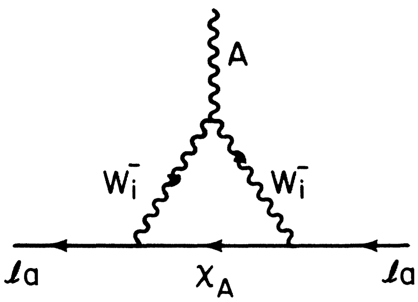
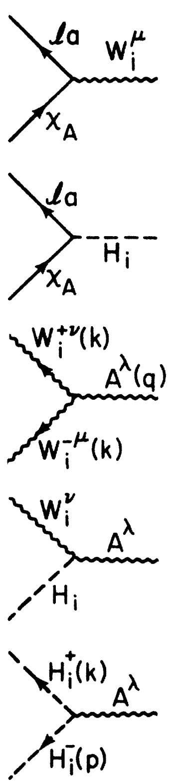


FIG. 1. Diagrams in the mass-eigenstate basis for the calculation of d_e. The diagrams with the unphysical Higgs bosons replacing the corresponding W_i are not shown but they have been calculated and included in Eq. (8).

Once we obtain these rules, we can choose the gauge pa-



$$\frac{ig}{\sqrt{2}}\gamma^\mu(U_{Li}P_{aA}L+U_{Ri}Q_{aA}^{*}R)$$

$$\frac{ig}{\sqrt{2}M_i}\left\{m_A(U_{Li}P_{aA}R+U_{Ri}Q_{aA}^{*}L)-\bar{m}_a(U_{Li}P_{aA}L+U_{Ri}Q_{aA}^{*}R)\right\}$$

$$ie\left\{(q-k)^\mu g^{\nu\lambda}+(k-p)^\lambda g^{\mu\nu}+(p-q)^\nu g^{\lambda\mu}\right\}$$

$$ie\, g^{\nu\lambda}M_i$$

$$ie(p-k)^\lambda$$

FIG. 2. Feynman rules.

rameter ξ to be unity since the diagrams in Fig. 1 are expected to be gauge invariant. The subtle problems of gauge invariance[7] that occur in the calculation of the K_L-K_S mass difference in left-right models are avoided here since the one-loop contributions of the flavor-changing neutral Higgs bosons to the imaginary part are expected to be dependent on the neutral-Higgs-boson mass. It clearly cannot be responsible for gauge cancellation.

The electric dipole moment d_a and the anomalous magnetic dipole moment G_{Ma} of the charged lepton l_a are defined by writing the amplitude of the electromagnetic vertex

$$M = i\bar{l}_a(p')\Gamma^\mu_a l_a(p)\epsilon^*_\mu \tag{9}$$

in the form

$$\Gamma^\mu_a = e\gamma^\mu + (G^M_a + id_a\gamma_5)i\sigma^{\mu\nu}(p-p')_\nu\,, \tag{10}$$

where ϵ^μ is the polarization vector of a real photon. A straightforward calculation of the diagrams in Fig. 1 now gives, for each χ_A and W_i running in the loops, the following contribution to d_a and G^M_a:

$$d_a^{(iA)} = -\frac{eg^2 m_A}{64\pi^2 M_i^2} U_{Li}U_{Ri}\mathrm{Im}(P_{aA}Q_{aA})\left[\frac{r_{Ai}^2 - 11r_{Ai} + 4}{(r_{Ai}-1)^2} + \frac{6r_{Ai}^2 \ln r_{Ai}}{(r_{Ai}-1)^3}\right], \tag{11}$$

$$\begin{aligned} G_a^{M(iA)} = \frac{eg^2}{64\pi^2 M_i^2}\Bigg[& m_A U_{Li}U_{Ri}\mathrm{Re}(P_{aA}Q_{aA})\left[\frac{r_{Ai}^2 - 11r_{Ai} + 4}{(r_{Ai}-1)^2} + \frac{6r_{Ai}^2 \ln r_{Ai}}{(r_{Ai}-1)^3}\right] \\ & + \bar{m}_a(|U_{Li}P_{aA}|^2 + |U_{Ri}Q_{aA}|^2) \\ & \times \left[\frac{7r_{Ai}^3 - 63r_{Ai}^2 + 54r_{Ai} - 16}{6(r_{Ai}-1)^3} - \frac{(r_{Ai}^4 - 7r_{Ai}^3 + 4r_{Ai}^2 - r_{Ai})\ln r_{Ai}}{(r_{Ai}-1)^4}\right]\Bigg], \end{aligned} \tag{12}$$

where we have used the shorthand

$$r_{Ai} \equiv m_A^2/M_i^2\,, \tag{13}$$

$\bar{m}_a$ is the mass of l_a, and we have dropped all terms of order $\bar{m}_a^2/M_i^2$ since all the known charged leptons are much lighter than the W_1 boson. The results for d_a and G_{M_a} can be found from Eqs. (11) and (12) by summing over the internal indices i and A. Equation (11) agrees with the results of Beall and Soni[8] and Eq. (12), to leading order in $\bar{m}_a^2/M_i^2$, agrees with the calculation by Marciano and Sanda.[9]

A few comments are in order. First, if the neutrino mass matrix in Eq. (3) were real, the diagonalizing matrices P and Q can be chosen to be real, so that we would have no dipole electric moment. By this procedure, m_A may turn out to be negative. We can then get positive eigenvalues by multiplying the components P_{aA} and Q_{aA} by a factor i for every a. This i is not CP violating.[10] Second, for d_a to be nonzero, we need $U_{Li}U_{Ri} \neq 0$, i.e., we need the W_L-W_R mixing in the mass matrix. Otherwise the mass eigenstates W_1 and W_2 become the same as the gauge eigenstates W_L and W_R. The diagrams with W_L and the corresponding unphysical Higgs boson running in the loop do not produce any CP violation at the one-loop level. This is well known from the calculations of electromagnetic form factors in the $SU(2)_L \times U(1)_Y$ models.[11] The same analysis applies for the W_R diagrams. Third, if all the neutrino masses are small compared to M_i, the expression in large parentheses in Eq. (11) becomes a constant, viz., 4, and we get

$$d_a = -\frac{eg^2}{16\pi^2}\sum_i \frac{U_{Li}U_{Ri}}{M_i^2}\sum_A m_A \mathrm{Im}(P_{aA}Q_{aA})\,. \tag{14}$$

But from Eqs. (3) and (7) it follows that

$$\sum_A m_A P^*_{aA}Q^*_{aA} = (\mu_D)_{aa} \tag{15}$$

so that from Eq. (14), we obtain

$$d_a \propto \mathrm{Im}(\mu_D)_{aa}\,. \tag{16}$$

This result is easy to understand from Fig. 3, which is drawn in terms of the gauge eigenstates. Since the dipole-moment operator connects l_{aL} with l_{aR}, the fermion line in the loop must be N_{aR} at one end and ν_{aL} at the other. The diagram can then go through only if there

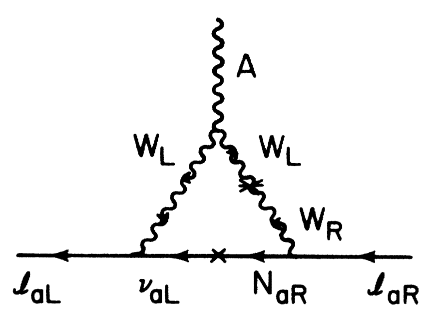


FIG. 3. A typical diagram contributing to the dipole-moment calculations in terms of the gauge eigenstates, showing different mass insertions.

is a mass insertion in the gauge-boson lines to convert W_R to W and a mass insertion of $(\mu_D)_{aa}$ which converts N_{aR} to ν_{aL} as seen in Fig. 3. For small $(\mu_D)_{aa}$, we can do perturbation in its powers, which gives the dependence in Eq. (16). For one thing this shows that even if the neutrinos are Dirac particles, i.e., if $\mu_\nu = \mu_N = 0$ in Eq. (3), the electric dipole moment of charged leptons does not vanish even in the absence of any intergenerational mixing. This is contrary to the conclusion of Ref. 5, where it is said that a nonzero d_a in left-right-symmetric models is a genuine marker of the Majorana nature of the neutrino. If, on the other hand, $\mu_D = 0$, then $\mathrm{Im}(P_{aA}Q_{aA}) = 0$ for all A. So the one-loop diagrams do not contribute to the electric dipole moment.

Let us now see what sort of magnitudes we can expect for the electric dipole moment of the electron, d_e. If $M_2 \gg M_1$, the dominant term in Eq. (11) is the term with $i = 1$. Using $g^2/(8M_1{}^2) = G_{\mathrm{Fermi}}/\sqrt{2}$ we can then write

$$d_e = (1.0\times 10^{-24}\ e\ \mathrm{cm}) \times \sin 2\zeta \sum_A \left[\frac{m_A}{1\ \mathrm{MeV}}\right] \mathrm{Im}(P_{1a}Q_{1a}) D_A\ , \qquad (17)$$

where D_A is the expression in large parentheses in Eq. (11) with $i = 1$, and ζ is defined in Eq. (5). This is to be compared with the present experimental limit:[12]

$$|d_e| \lesssim 10^{-24}\ e\ \mathrm{cm}\ . \qquad (18)$$

There are various available limits on the left-right mixing ζ. From current algebra analysis of the purely nonleptonic strange decays one obtains[13] $\zeta < 0.004$. From the success of the Adler-Weissberger relation, one can limit[14] $\zeta < 0.05$. In addition, if one assumes that the left-handed and right-handed quark currents have the same mixing matrix, one can obtain[14] $\zeta < 0.0055$ by using the bounds on the b-decay branching ratio $(b \to u)/(b \to c)$. Two limits of D_A are interesting:

$$\text{(I)}\ \ M_A \ll M_1,\ \ r_{A1} \ll 1,\ \ D_A \approx 4 - 3r_{Ai}\ , \qquad (19)$$

$$\text{(II)}\ \ M_A \gg M_1,\ \ r_{A1} \gg 1,\ \ D_A \approx 1 + 6\frac{\ln r_{A1}}{r_{A1}}\ . \qquad (20)$$

The second case is a little bit surprising, since it appears that d_e does not vanish even if we take the right-handed scale M_2 to infinity. However, this is deceptive. Decoupling is recovered because the left-right mixing parameter ζ is proportional to $1/M_2{}^2$. Taking the limit $\zeta < 0.005$, d_e becomes

$$d_e \lesssim 10^{-26}\frac{\mathrm{Im}(\mu_D)_{ee}}{1\ \mathrm{MeV}} D_A\ . \qquad (21)$$

Clearly d_e can be very close to the experimental bound if μ_D is roughly equal to 100 MeV. Apart from making artificial fine-tuning between different generations, there are two natural ways of accommodating this large value of μ_D without conflicting with the experimental bound on neutrino masses. One way is to have the right-handed neutrino mass μ_N large enough[15] so that $\mu_D{}^2/\mu_N$ is smaller than the experimental bound. In this case, a detectable value of d_e is a signature of the Majorana character of the neutrinos. Another way[16] is to introduce an additional $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$-singlet neutrino $\nu_{\sigma L}$ such that the neutrino spectrum of each generation consists of one massless two-component fermion and one Dirac particle. The mass matrix in this case is

$$\overline{N}_R(\mu_D \nu_{eL} + m_\sigma \nu_{\sigma L})\ . \qquad (22)$$

To be consistent with the neutrino mixing data, m_σ must be large enough. A natural value for m_σ in this model is M_R. Note that in this case all the massive neutrinos are Dirac particles.

It may be interesting to see what the constraint is due to the very accurately measured value of $g-2$ for the electron. The present bound on any unconventional contribution to $g-2$ is[17] $\delta g < 10^{-9}$. Assuming the term proportional to the neutrino mass dominates in Eq. (12), the contribution to δg is

$$\delta g = \frac{m_e}{M_1{}^2}\frac{g^2}{64\pi^2}\sin 2\zeta \mathrm{Re}(\mu_D)_{ee} D_A\ , \qquad (23)$$

where D_A is defined in Eq. (17). Using $\zeta < 0.005$ as before we obtain the limit

$$\delta g \le 4\times 10^{-13}\frac{\mathrm{Re}(\mu_D)_{ee}}{1\ \mathrm{MeV}}\ . \qquad (24)$$

Thus a value of d_e close to the experimental limit is not in conflict with the limits on δg.

In conclusion, a value of d_e close to the present experimental limit can be obtained in the context of left-right-symmetric models. However, as the example considered in the text illustrates, a nonzero value of d_e is in principle not necessarily an indication of the Majorana nature of the neutrinos. It is interesting that the value of d_e is essentially determined by ζ, the W_L-W_R mixing parameter, and by $(\mu_D)_{ee}$. In particular, the right-handed neutrino mass, which is usually assumed to be $\sim M_2$, could be large and a non-negligible value of d_e results if $\sin 2\zeta\, \mathrm{Im}(\mu_D)_{ee} \sim 1$ MeV.

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