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Two-fluid model of the Skyrmion

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We consider the field-theory description of the Skyrme model incorporating quantum fluctuations of a condensed phase. This viewpoint clarifies the usefulness of the model for applications at the physical value of F_π, rather than $F_\pi \to \infty$. By creating covariant field-theory states associated with a translational-dependent condensate, we emphasize the effects of fluctuations and coherence in a calculation of the q^2 dependence of the baryon electromagnetic form factor $F(q^2)$. Using the impulse approximation at large q^2, we find the model predicts $F(q^2) \sim \exp(-|q^2|/\lambda^2)/q^2$, where $\lambda \sim 1$ GeV is a calculable scale. Appending a pion form factor of $O(1/q^2)$, we interpret this in terms of higher-twist corrections, at moderate values of momentum transfer, to a leading term of $O(1/q^4)$. The dependence on a regularization scale μ^2 and $F_\pi{}^2$ is such that this result applies for a specified range of q^2, μ^2, and $F_\pi{}^2$. However, the detailed results of the calculation are such that the large-$F_\pi{}^2$ and large-μ^2 limits do not commute.

I. INTRODUCTION

The current interest in the Skyrme soliton has its roots in two recent discoveries.[1,2] One is the realization that the model, as a prototype effective field theory, is a prediction from QCD in the limit of an asymptotically large number of colors N_c. The soliton is very important in this limit, since the quantum action scales like $N_c/\hbar$ and the system approaches a quasiclassical limit. The second reason for interest is the remarkable phenomenology of the model. Using one or two adjustable parameters, many static properties are given to 30% accuracy, and some remarkable agreement[3,4] is achieved in reproducing many π-N phase shifts for energies up to a few GeV.

However, I consider these two successes to be, at least, mutually inconsistent and to present an important paradox. The paradox hinges on attempts to use the approximation of F_π (which scales like N_c) asymptotically large, while in nature $F_\pi = 186$ MeV is one of the smallest of hadronic scales. The model should not work well for real data, yet it does.

Given that F_π scales out of the Lagrangian, this constant plays the role of an inverse temperature β in the quantum generating functional or partition function. The puzzle involves arguing that $\beta \to \infty$ describes the system at finite β. But there are systems in nature which for finite β nonetheless act as if for some observables of the system $\beta \to \infty$. Historically, London[5] proposed that liquid helium could be modeled using a gas of zero-temperature, noninteracting and condensed bosons. The many-body physics of a condensed fraction makes this the correct starting point, in spite of the fact that neither is the temperature low enough nor are the interactions weak enough to generally motivate the picture as a straightforward approximation in a small parameter.

We propose that a condensed phase may be the explanation for the puzzle of the Skyrme model. For F_π larger than a critical value, marked by a phase transition, there may be a condensed fraction. The field-theoretic meaning of this, given long ago by Onsager and Penrose,[6] is that the contribution to the path integral for M degrees of freedom from a particular configuration (and very small fluctuations) is anomalously large; it is not of order $1/M$ as $M \to \infty$. If this overlap is instead of order 1, the condensed fraction makes its presence felt through a condensate expectation value or classical field. From this point of view, the value of the large-F_π limit is to put the system into the condensed phase which persists to some extent as small F_π is approached from above. The condensed fraction, but not all the variables of the field theory, could be smoothly related to $F_\pi = \infty$ even for F_π finite and not too small.

In this paper I will explore implications of a "two-fluid" Skyrme soliton for F_π finite. To implement the picture, we must create a field-theory description of the fields $\phi(x)$ and states of a condensed fraction. The soliton will be associated with states $|\eta_a\rangle$ have a position index a^μ, for which measurements of Φ give the classical field ϕ defined by

$$\langle \eta_a | \Phi(x) | \eta_a \rangle / \langle \eta_a | \eta_a \rangle = \phi(x-a) . \tag{1.1}$$

Here η is a shape parameter defined below. The states $|\eta_a\rangle$ do not have to span the density matrix of the system. However, if one separates their contribution by writing

$$\langle \Phi(x) \rangle = \int d^2[\eta] \rho_\eta \langle \eta | \Phi | \eta \rangle + \mathrm{Tr}[\rho_\sigma \Phi(x)] ,$$

where ρ_σ represents the terms left out by the quasiclassical approximations, the contribution of ρ_η to $\langle \Phi \rangle$ should be finite.

The decomposition above gives one a framework in which to understand the usefulness of the model. Even if the contribution to matrix elements by ρ_η is small, e.g., a 10% effect, it has a chance to dominate certain experimental observables. In this paper we will concentrate on exclusive matrix elements, and disregard the "normal" or noncondensed phase. In the exclusive case the contributions from the normal fraction might be too incoherent to give a large result, leading to relative enhancement of the condensate contribution. Moreover, the centrifugal problems[7] concerning the long-distance structure of the soliton are resolved within this viewpoint. This is because the condensed fraction needs to vanish as one approaches the edges of the soliton, in order to recover all the degrees of freedom of the complete field theory asymptotically.[8] But because the condensed fraction cannot be translationally invariant, the situation will be that it is difficult to separate the competition between the normalization of the condensed fraction as a function of x and the classical field value by merely studying a few static properties or minimizing a truncated Hamiltonian. Given the nonperturbative nature of such a problem, we hope that some progress can be made, as in the liquid-helium and superconductivity tradition, by writing down field-theory wave functions and comparing as many predictions as possible with data.

At finite F_π, quantum fluctuations and correlations are not negligible even if the condensed phase alone is treated. Furthermore, we can investigate the interesting question of what variable or quantity is small compared to F_π, in the large-F_π limit. The calculation will show that fluctuations are controlled by an infinite series in $\mu^2/F_\pi{}^2$, where μ^2 is a regularization scale. The sum of the series can be finite as $\mu^2\to\infty$ or as $F_\pi{}^2\to\infty$. To complete an analysis of fluctuations, we must also have a covariant formalism that is not limited to calculations of a few static quantities, but rather has the flexibility to describe arbitrary momentum transfer. These objectives, which we will satisfy below, have been blocked in the past by zero-mode problems caused by employing any operators breaking translational invariance, and by various static approximations.

The outline of the paper includes the following. In Sec. II we construct states and define the classical field. Here we must discuss regularization and fluctuations, which we model using a Fock-space basis. The basis is translationally covariant and we will be able to construct asymptotic momentum eigenstates that are superpositions of states of given position index. In Sec. III we present a trial calculation to demonstrate the potential of the method. We study the large-$(-q^2)$ limit of the electromagnetic form factor. The discussion motivating large $-q^2$ in this elastic process is also given in Sec. III. Conclusions are given in Sec. IV.

II. CONSTRUCTION OF STATES

A sufficiently general basis to construct states with a position index, $|\eta_a\rangle$, employs the Fock space of free-field theory. From a free-field point of view, the soliton is a multiparticle wave packet constrained to reproduce a certain classical field. For example, given a real free field $\Phi(x)$ described by

$$\Phi(x)=\int d\widetilde{k}(a_k e^{-ik\cdot x}+a_k^\dagger e^{ik\cdot x})\ , \tag{2.1}$$

where $d\widetilde{k}=d^3k/2\omega_k(2\pi)^3$ and

$$[a_k,a_k^\dagger]=(2\pi)^3 2\omega_k\delta^3(k-k')\ ,$$

we construct $|\eta_a\rangle$ associated with $a^\mu=(a^0,\mathbf{a})$ as follows:

$$|\eta_a\rangle=\exp\left[\int d\widetilde{k}\,\eta_k e^{ik\cdot a}a_k^\dagger-\tfrac{1}{2}|\eta_k|^2\right]|0\rangle\ . \tag{2.2}$$

This is a Bargmann (coherent) state[9,10] normalized by $\langle\eta_a|\eta_a\rangle=1$. We can require $|\eta_a\rangle$ to satisfy the classical field constraint (1.1) identically depending on the choice of the parametric function η_k:

$$\begin{aligned}\phi(\mathbf{x},0)&=\int d\widetilde{k}(\eta_k e^{i\mathbf{k}\cdot\mathbf{x}}+\eta_k^* e^{-i\mathbf{k}\cdot\mathbf{x}})\\&\equiv\langle\eta_0|\Phi(\mathbf{x},0)|\eta_0\rangle\ .\end{aligned} \tag{2.3}$$

For $t\neq0$ the time dependence of $\phi(x,t)$ is determined by that of (2.1). Note that one phase of η_k parametrically locates the position a^μ in (2.2).

In principle, the basis of $|\eta_a\rangle$ states defined by (2.2) is complete if one employs functional integrals over all complex shape parameters η_k using

$$1=\text{const}\times\int d^2[\eta]\,|\eta\rangle\langle\eta|\ ;$$

the index a^μ is included in this sum. Thus even the interacting states will have an advantageous description in this basis, which gives an immediate relation between the shape parameters and expected values. In what follows we will quote certain relations, as well as connections with the usual soliton quantization procedure, which have been developed in Ref. 11.

To both regulate products of operators and the amount of detail in matrix elements, we will regulate[10] the local operator $\Phi(x)$ by introducing a smearing function $f(x)$. We replace $\Phi(x)\to\Phi_f(x)$ by

$$\begin{aligned}&\Phi_f(x)=\int dx' f(x'-x)\Phi(x')\ ,\\&\int dx\, f(x)=1\ ,\\&\int dx\, e^{ik\cdot x}f(x)=f_k\ .\end{aligned} \tag{2.4}$$

The unregulated, naive local limit is $f(x)=\delta^4(x)$, $f_k=1$. The function f_k will introduce a scale $\mu^2=\int d\widetilde{k}\,|f_k|^2$ which determines the scale on which Φ_f is regularized. In terms of a regularized Φ_f, the free field distribution given by

$$\rho_{\eta'\eta}(\phi_0;x)=\langle\eta'|\delta(\Phi_f(x)-\phi_0)|\eta\rangle \tag{2.5}$$

is well defined. The interpretation of $\rho_{\eta'\eta}(\phi;x)$ is the distribution of values of field Φ_f between the states, evaluated at the c number ϕ_0, and at the point x. The off-diagonal quantity (2.5) can be evaluated explicitly for free fields. Here we just quote the result[11] between states of the same shape (η_k) but different positions a^μ:

$$\begin{aligned}\rho_{a'a}(\phi_0;x)&=\langle\eta_{a'}|\delta(\Phi_f(x)-\phi_0)|\eta_a\rangle\\&=\frac{\langle\eta_{a'}|\eta_a\rangle}{\sqrt{2\pi}\mu}\exp\{-\tfrac{1}{2}[\phi_0-\phi_f^+(x-a)\\&\qquad-\phi_f^-(x-a')]^2/\mu^2\}\ .\end{aligned} \tag{2.6}$$

The saddle-point functions $\phi_f^\pm$ are found to be

$$\phi_f^+(x)=\int d\tilde{k}\eta_k f_k^* e^{-ik\cdot x},\quad \phi_f^-(x)=[\phi_f^+(x)]^* . \tag{2.7}$$

The important difference between (2.6) and path-integral representations in the literature[12] is the presence of two saddle-point functions, associated with the off-diagonal character of $\rho_{a'a}$.

Since matrix elements of products of operators are divergent even for free fields, there will be divergences if one uses (2.6) to evaluate products of operators:

$$\langle \eta_{a'} | J(\Phi_f(0)) | \eta_a \rangle = \int d\phi_0 \rho_{a'a}(\phi_0;0) J(\phi_0) .$$

If $J(\Phi_f)$ is a polynomial, various divergent powers of μ^2 will occur as $\mu^2 \to \infty$; subtracting such terms defines a regularized matrix element. In this paper we will retain the μ^2 dependence.

The definition (2.6) depends on the overlap $\langle \eta_{a'} | \eta_a \rangle$ which can also be calculated:

$$\langle \eta_{a'} | \eta_a \rangle = \exp\left[\int d\tilde{k}\, |\eta_k|^2 (e^{ik\cdot(a-a')}-1)\right] . \tag{2.8}$$

The dependence on $(a-a')$ comes from our translationally covariant construction; the overlap is unity for $a=a'$ and exponentially damped[13] for large $a-a'$. For the discussion below we will need an estimate of the Fourier transform at large p. Using a type of saddle-point approximation in (2.8), we expand the exponent about $a=a'$. Then we obtain an estimate for

$$\begin{aligned} N(p) &= \int d^4a\, e^{-ip\cdot a} \langle \eta_0 | \eta_a \rangle , \\ &= \frac{(2\pi)^2}{\det T} \exp(-\bar{p}^{\alpha} T^{-1}{}_{\alpha\beta} \bar{p}^{\beta}/2) , \\ \bar{p}_\mu &= p_\mu - \int d\tilde{k}\, |\eta_k|^2 k_\mu , \end{aligned} \tag{2.9}$$

where $T^{\alpha\beta}=\int d\tilde{k}\, |\eta_k|^2 k^\alpha k^\beta$ is positive matrix. For a crude estimate we replace $T^{\alpha\beta}$ by $g^{\alpha\beta}\lambda^2$, where λ^2 is the largest eigenvalue of $T^{\alpha\beta}$. Then we obtain, for large p^μ,

$$N(p) \underset{p\to\infty}{\sim} \frac{(2\pi)^2}{\det T} \exp(-p^2/2\lambda^2) . \tag{2.10}$$

Although all of this has been in the context of free fields, it is a very useful format to incorporate information about the interacting soliton. Let us propose that the physical proton $|p\rangle$ with momentum p^μ has a finite overlap with states of fixed position $|\eta_a\rangle$ for some expectation value $\phi(x-a)$. Using completeness, we have a formal expression for the expectation value of a function $J(\Phi)$:

$$\begin{aligned} \langle p' | J(\Phi(0)) | p \rangle = \int d^2[\eta'] d[\eta] A_p(\eta) A_{p'}^*(\eta') \\ \times \int d\phi_0 \rho_{\eta'\eta}(\phi_0;0) J(\phi_0) . \end{aligned} \tag{2.11}$$

This applies to the complete theory trivially because all the complications of interactions can be absorbed into the shape coefficients $A_p(\eta)$. No one knows these, in general, but the rest of the calculation can be done as a functional of $A_p(\eta)$. An appropriate step within the soliton picture would be to single out the configurations in the sum over $d^2[\eta]d^2[\eta']$ which at least characterize the classical shape of the soliton, plus Gaussian fluctuations. For our discussion here we will not need to implement certain global quantum numbers such as spin and the total electric charge.

The time dependence of free-field theory is special, so that states will generally evolve away from whatever $\phi(\mathbf{x},0)$ we choose for them. Nevertheless, we can model the momentum eigenstates needed by (2.11) as follows. We create superpositions of different position and fixed shape with index p^μ:

$$| S_p \rangle = c_p \int d^4a\, e^{-ip\cdot a} | \eta_a \rangle , \tag{2.12}$$

where c_p is a normalization constant. One can check[11] that the $|S_p\rangle$ diagonalize the asymptotic energy-momentum operators $p^\mu=\int d\tilde{l}\, a_l^\dagger a_l l^\mu$, giving $P^\mu |S_p\rangle = p^\mu |S_p\rangle$. Expanding (2.2) and (2.12) one, in fact, obtains a Fock-space decomposition

$$| S_p \rangle = \exp\left[-\tfrac{1}{2}\int d\tilde{k}\, |\eta_k|^2\right] c_p (2\pi)^4 \sum_J \frac{1}{J!} \int \prod_{i=1}^{J} \left[d\tilde{k}_i \delta^4\left[p-\sum_i^J k_i\right] a_{k,i}^\dagger \eta_{k,i}\right] |0\rangle ;$$

there is one δ function of momentum as the result of superposing over all positions $\mathbf{a}$ and initial time locations a^0 of states in (2.12). Since the $|S_p\rangle$ are stationary asymptotically, the superposition wipes out whatever dispersion a single state $|\eta_a\rangle$ would suffer in the course of time.

The states $|S_p\rangle$ would presumably be the correct (in) states associated with an interacting soliton if the usual adiabatic switching of coupling constants can be invoked. That is, given the knowledge that there is a finite overlap of the physical proton with the $|S_p\rangle$ as couplings are turned on, one has only to adjust the expected value given by (2.3) and the functional averages (2.11) to obtain the real proton. For the discussion below, we will restrict the classical field $\phi(x)$ we choose to be in the same topological sector[14] as the classical solution. We will try to investigate a quantity which is least sensitive to the uncertain effects of functional averaging, for purposes of discussion.

Before entering the calculation, let us point out that there is more than one translationally covariant basis for creating coherent states. In Ref. 11 we show that the usual Hamiltonian quantization procedure, or background-field method, presupposes lowest-order states identical to ours with the replacement of general modes for the plane waves.

III. THE ELECTROMAGNETIC FORM FACTOR

To illustrate our ideas, let us consider the calculation of the elastic electromagnetic form factor $F(q^2)$ defined by

$$\langle p'\lambda' | J^{\mu}_{\text{em}}(0) | p\lambda \rangle = \bar{u}_{\lambda'}(p')\gamma^{\mu}u_{\lambda}(p)F(q^2) , \qquad (3.1)$$

where $q = p - p'$. We will make a model of the states $|p,\lambda\rangle$ ignoring spin, and then calculate (3.1). Conventional soliton approaches do not really define the states, and are limited to $q \rightarrow 0$, but the states $|S_p\rangle$ have a covariant construction. There are, however, calculational advantages if either $q^{\mu} \rightarrow 0$ or q^{μ} is large. A static limit involves no correlations and with extra assumptions reduces to the classical answer.[11] The large-momentum-transfer case can be justified using the impulse approximation, which we will now discuss in detail.

To use the impulse approximation for (3.1), a coherent interaction time of the system must be long compared to the collision time. For the "slow" modes we will argue that either the space-time dependence of free fields or interacting fields can be used. We can also assume that the extremely fast modes in the linearized interacting system have the same effect as in the free theory; this will affect the regularization dependence. More importantly, it may seem counterintuitive to apply large momentum transfer to an extended configuration and the soliton effective theory. Conventionally, such a regime is a short-distance one. For the physical proton, there are many arguments[15] that pointlike quark interactions would be dominant for $F(q^2)$ at large q^2.

However, such a viewpoint is vague and overlooks subtleties imposed by an exclusive process. In general, such processes are overwhelmingly constrained by the requirement of scattering of the entire state, a coherent proton, without excitation. Closer study reveals that even within perturbative QCD,[16] coherence is signaled by the buildup of large infrared logarithms that must be summed to all orders of divergences. Perhaps a better description of coherence is provided by the condensed fraction of the chiral (Skyrme) model.

To emphasize the applicability of the soliton picture, a physical analogy is useful. The field-theory proton has an infinite number of degrees of freedom with a hierarchy of structure, and so is analogous to a physical crystal. Indeed, a latticized version of the theory would be even more similar to a crystal. The fundamental degrees of freedom in a crystal, the nuclei, are analogous to quarks, but the extended crystal is sometimes described by an effective theory of, e.g., the phonons. Geometrically, a large-q^2 photon must interact with the pointlike constituent. But if one requires elastic scattering of an entire crystal, the picture is vastly different from inelastic incoherence. In the case of elastic photon-crystal scattering, a struck nucleus can be tied by collective phonons to the entire crystal; this is experimentally observed in the Mössbauer effect.[17] Similarly, a struck quark within a pion in the Skyrme model could be tied into collective modes we call the soliton. In either the Mössbauer effect or the quark-soliton picture, the ratio of the collective mode wavelength to the absorbed photon wavelength can be many orders of magnitude.

This analogy must be refined, because nuclei in crystals undergo small fluctuations of their momenta determined by kT. The fluctuations in the field theory depend strongly on $F_{\pi}{}^2$, of course, but also on the regularization scale μ^2. Therefore we must keep both the μ^2 and $F_{\pi}{}^2$ dependence.

From this analogy, we anticipate a two-step process involving the effective modes, described by $|S_p\rangle$, and a short-distance or fluctuation-dependent part. Keeping in mind that μ^2 is the cutoff for the effective theory, while $F_{\pi}{}^2$ regulates intrinsic fluctuations, we will want $-q^2$ large compared to some appropriate combination of μ^2 and $F_{\pi}{}^2$. The exact combination of μ^2 and $F_{\pi}{}^2$ we find below provides an example of the interplay of these scales. We will postpone discussing the nonpointlike character of the physical pion degree of freedom.

For medium q^2, the interplay of time scales would indicate that the time dependence of some model-dependent modes would be important. This is not an overwhelming complication, as we will learn from the Gaussian fluctuation workings in the large-q^2 case.

To make a calculation self-consistently at large q, we might also need to know high derivative terms from the Skyrme effective-action expansion. The model is written in terms of derivatives of $U = \exp(i2\pi\cdot\tau/F_{\pi})$, where π is the pion field. The electromagnetic current operator for the $\text{SU}(2)_L \times \text{SU}(2)_R$ case is written[2]

$$J^{\mu}_{\text{em}} = J^{\mu}_{v} + B^{\mu}/2 , \qquad (3.2)$$

where up to second-order or Skyrme terms one has

$$\begin{aligned} J^{\mu}_{v,2} = {} & \frac{iF\pi^2}{16}\text{Tr}[(\partial^{\mu}U)U^{\dagger}\tau_3] \\ & + \frac{i}{8e^2}\text{Tr}\{[(\partial^{\nu}U)U^{\dagger},\tau_3][(\partial^{\mu}U)U^{\dagger},(\partial^{\nu}U)U^{\dagger}]\} \\ & + U \leftrightarrow U^{\dagger} , \\ B^{\mu}_{2} = {} & \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\text{Tr}[(U^{\dagger}\partial_{\nu}U)U^{\dagger}\partial_{\alpha}U(U^{\dagger}\partial_{\beta}U)] . \end{aligned} \qquad (3.3)$$

As more derivative terms are added, J^{μ}_{em} will change by terms of order $q^2/F_{\pi}{}^2$. However the presence of the baryon number or topological current B^{μ}_2 is immune to correction. This is because the functional form of B^{μ}_2 is a winding number density geometrically fixed by topology.[12] Therefore we will use (3.3) to estimate a lower bound on the decrease with q^2 of $F(q^2)$ in the model.

Guided by the hedgehog ansatz $\pi = \hat{\mathbf{r}}F(r)$, we replace $\phi(\mathbf{x},0) \rightarrow F_{\pi}F(r)/2$, and we have

$$B^{0}_{2}(F) = -\frac{1}{2\pi^2}\frac{\sin^2 F(r)}{r^2}\frac{\partial F(r)}{\partial r} . \qquad (3.4)$$

The formalism will allow us to relate this operator "identity" to a matrix element. Before doing that, it is useful to have the classical result given by the Fourier transform of the charge density. One can show that B_2 is more singular than the vector current $J^{\mu}_{v,2}$ with the hedgehog ansatz. For large $|\mathbf{q}|$, one finds

$$\tilde{B}(q)=\int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}B_2^0[F(r)]$$
$$\sim\frac{4\pi}{q^2}\left[\cos(qr)rB(r)-\frac{\sin(qr)}{q}[rB(r)]' -\frac{1}{q^2}\cos(qr)[rB(r)]''+\cdots\right]\Bigg|_{r=0} . \quad (3.5)$$

The boundary conditions on $F(r)$ are $F(0)=\pi$, $F'=\partial F/\partial r$ and $F''=\partial^2F/\partial r^2$ finite as $r\to 0^+$. Consequently (3.5) gives a "classical" form factor of $\sim 1/q^4$ for large $|q|$. This appears to be a semiclassical, large-F_π property of the model, and it is a very interesting result. We will compare (3.5) with the matrix element calculated in the impulse approximation.

We evaluate $\langle S_{p'}|B_2^0(F(0))|S_p\rangle$ in the Breit frame $q^0=0$ using (2.6), (2.12), and (3.4). For this estimate we truncate the operators[18] $\pi(x,t)$ to $\pi\cdot\hat{\mathbf{r}}=F_\pi F(r,t)/2$, where the time dependence is given by the free theory. The substitutions give

$$\langle S_{p'}|B_2^0|S_p\rangle=c_{p'}^*c_p\int d^4a^-e^{-ip^+a^-}\langle\eta_{a'}|\eta_a\rangle \times\delta(0)I(q^2,a^-,\mu)\,, \quad (3.6)$$

$$\delta(0)I(q^2,a^-,\mu) =\int d^4a^+e^{i\mathbf{q}\cdot\mathbf{a}^+/\sqrt{2}} \times\int\frac{dg}{\sqrt{2\pi}\mu}B_2^0(g)\exp\{-[g-\bar{F}(a,a')]^2F_\pi{}^2/8\mu^2\}, \quad (3.7)$$

where $a^\pm=(a\pm a')/\sqrt{2}$, etc. We note the following regarding (3.6) and (3.7).

(a) The fluctuation integral over variable g is controlled by a scale $\mu^2/F_\pi{}^2$. The saddle-point function is regularized:

$$\bar{F}(a,a')=F_f^+(a)+F_f^-(a')\,. \quad (3.8)$$

(b) The factor $\delta(0)$ in (3.7) is associated with our choice of normalization $\langle S_{p'}|S_p\rangle=(2\pi)^3 2\omega_p\delta^3(p-p')$. With this definition, $c_{p'}^*c_p\delta(0)$ is finite, and

$$c_{p'}^*c_p\delta(0)\int d^4a^-e^{-ip^+a^-}\langle\eta_{a'}|\eta_a\rangle =\frac{(\omega_p\omega_{p'})^{1/2}}{\sqrt{2}}\frac{N((p+p')/2)}{N(m^2)}\,. \quad (3.9)$$

(c) The detail exhibited in (3.6) and (3.7) is a prototype of that expected from a complete calculation. One has explicit dependence on $\mu^2,q^2,F_\pi{}^2$, the overlap $\langle\eta_{a'}|\eta_a\rangle$, and the c-number function $\tilde{F}(r)$.

The following sequence of approximations to (3.6) and (3.7) is suggested. Since $\langle\eta_{a'}|\eta_a\rangle$ is rather sharply peaked at $a=a'$, and $p^+\to\infty$, we replace $I(q^2,a^-,\mu)$ by $I(q^2,0,\mu)$ in (3.6) to the leading power of p^+. Changing variables at each point (a) in (3.7) we have

$$I(q,0,\mu) =\frac{2}{F_\pi}\int d^3a\, e^{i\mathbf{q}\cdot\mathbf{a}}\int\frac{dg'}{\sqrt{2\pi}}B_2^0\left[\frac{2\mu g'}{F_\pi}+F_f(r)\right]e^{-g'^2/2}\,, \quad (3.10)$$

where $F_f(r)=\bar{F}(a,a)$ is just the original classical function $F(r)$, but regularized. A good choice is

$$F_f(|\mathbf{x}|)=(2\pi)^{-3/2}\mu^3 \times\int d^3x\, F(|\mathbf{x}'|)\exp[-(\mathbf{x}-\mathbf{x}')^2\mu^2/2]\,. \quad (3.11)$$

The Gaussian integral (3.10) can be evaluated analytically for B_2^0 as given by (3.4), and we find

$$I(q^2,0,\mu) =-\int d^3a\, e^{i\mathbf{q}\cdot\mathbf{a}}\frac{1}{F_\pi\pi^2}\left[\frac{\partial F_f}{\partial r}\frac{\sin^2F_f}{r^2} +\frac{\partial F_f}{\partial r}\frac{\cos(2F_f)}{2r^2} \times(1-e^{-8\mu^2/F_\pi{}^2})\right]. \quad (3.12)$$

Let us note the effect of fluctuations in (3.12). For μ^2 fixed, $F_\pi{}^2\to\infty$, we can drop the second term in (3.12), recovering the classical result (3.5). For $F_\pi{}^2$ fixed, $\mu^2\to\infty$, the second term remains. If we had expanded in powers of μ^2, we would have obtained an infinite series of divergences as $\mu^2\to\infty$:

$$-\sum_{J=1}^{\infty}(-8\mu^2/F_\pi{}^2)^J/J!=1-e^{-8\mu^2/F_\pi{}^2}\,.$$

The behavior as $\mu^2\to\infty$ is an ultraviolet characteristic that would not change in any linearization method. Such terms would be systematically removed in renormalizing the matrix element for F_π fixed, $\mu^2\to\infty$. For μ^2 fixed, $F_\pi{}^2\to\infty$; however, they would represent an essential singularity in powers of F_π. That would not be a problem for the large-F_π limit, which is an expansion in powers of $1/F_\pi$.

Another effect of fluctuations is the replacement of $F(r)$ by $F_f(r)$. For any finite μ, the discontinuity in the derivative of $F(r)$, given by $F'(0)$, will be removed by smearing with a symmetric function such as (3.11). That is, an expansion about $r=0$ of F_f will have the form $F_f(r)\sim F_f(0)+r^2F_f''(0)$, where $F_f''(0)$ depends on μ^2. This may affect the large-$|\mathbf{q}|$ behavior of the Fourier transform (3.12). To make sure we are measuring the baryon density and not just the smearing we will impose the condition $|q^2|\lesssim\mu^2$.

A third feature is that any function of the operator $F(r)$, when the fluctuations are integrated, will receive contributions from the distribution of values away from the classically prescribed expectation value. Thus, although we maintain $F(0)=\pi$ as a boundary condition that will ensure that the classical B^0 density (3.4) is finite at $r=0$, the fluctuations have built a divergent integrand in the second term of (3.12). The singularity is integrable in this case, however.

Recalling (3.1), we can extract $F(q^2)$ by squaring

$$\sum_{\lambda\lambda'}|\langle p'|J^0|p\rangle|^2=F^2(q^2)\mathrm{Tr}(\not{p}\gamma^0\not{p}'\gamma^0)/4m^2$$

so up to multiplicative constants

$$|\langle p'|J^0|p\rangle| \propto F(q^2)(\omega_p\omega_p')^{1/2} . \tag{3.13}$$

Here we have assumed a relativistic $\omega_p \sim |\mathbf{p}|$. Now consulting (3.6), (3.9), and (3.12), we only have to include the normalization factors to extract $F(q^2)$:

$$F(q^2) \propto \frac{N((p+p')/2)}{N(m^2)} I(q^2,0,\mu) , \tag{3.14}$$

where, according to (2.9),

$$N((p+p')/2) \sim e^{-(-q^2/8\lambda^2)} .$$

We can now examine the interplay of the scales $-q^2$, μ^2, and $F_\pi{}^2$ in (3.12). For large $|\mathbf{q}|$ an asymptotic series for (3.12) gives

$$I(q^2,0,\mu) \sim \frac{F_f''(0)}{F_\pi}\left[\frac{A}{q^6}+\frac{A'}{q^2}(1-e^{-8\mu^2/F_\pi{}^2})\right] , \tag{3.15}$$

where A and A' are constants; note that for finite μ, the factor $\bar{F}''(0)=\partial^2 F_f/\partial r^2|_0$ depends on μ. Recall that the fluctuations in (3.7) were controlled by $\mu^2/F_\pi{}^2$ while the energy of the soliton scales like F_π. The characteristic fluctuation time appears to be $[F_\pi(\mu^2/F_\pi{}^2)]^{-1}$ or a spatial scale of $O(F_\pi/\mu^2)$. To use our impulse approximation we need

$$\begin{aligned}&[(-q^2)^{1/2}]^{-1} \ll F_\pi/\mu^2 ,\\ &(-q^2)^{1/2} \gg \mu^2/F_\pi .\end{aligned} \tag{3.16}$$

For large $-q^2$ this can be satisfied in the physical limit of F_π small. The limit (3.16) is extremely easy to satisfy as $F_\pi \rightarrow \infty$, because the clamping of the field onto the classical value becomes exact. However, note that we cannot claim the impulse approximation is ever good in the unphysical limit $\mu^2 \rightarrow \infty$ with q^2 and $F_\pi{}^2$ fixed.

Consistency of the limits above requires $\mu^2/F_\pi{}^2 \gg 1$, so one cannot really choose to ignore the A' term in (3.15). Collecting the results, the fluctuations dominate the large-q^2 integrals.

We finally obtain

$$\begin{aligned}&F(q^2) \sim \frac{\text{const}}{q^2}\exp(-|q^2|/8\lambda^2) ,\\ &-q^2 \rightarrow \infty .\end{aligned} \tag{3.17}$$

Within the procedure the constant in (3.17) is calculable, although various detailed quantities contribute to it. For example, we used (2.9) to estimate $N((p+p')^2/4)$, which should be acceptable, but not to estimate the low-energy quantity $N(m^2)$.

The modulated power-law behavior shown in (3.17) is our main result. We interpret the power q^{-2} by recalling that the discontinuity in the derivative of $F(r)$ at the origin is a topological feature; it represents the fluctuating isospin knot at the center of the Skyrmion. Changing the boundary conditions while retaining the knot will change the value of the power.

The power law is modified by the exponential factor in (3.17). That factor comes from the finite size of the soliton, and represents a suppression from the field-theory overlap of the soliton scattering from p onto one at p'. The soliton size scales like $1/eF_\pi$, so one might expect $\lambda \simeq eF_\pi$. In fact a rough analytic estimate gives $\lambda \simeq F_\pi$. Using $F_\pi = 186$ MeV, the exponential in (3.17) gives $\exp[-(q/0.585\text{ GeV})^2]$. More detailed numerical work is required to evaluate the expressions near 1 GeV, although such a result is promising. It is evident that $\lambda \simeq 1$ GeV is within the capacity of models of the Skyrme function.

Combining this result and the discussion above, we deduce the following unified picture of $F(q^2)$. Within the Skyrme model, the large-q^2 dependence first falls like q^{-2} for $-q^2 \lesssim \lambda^2$, probing the isospin knot at the center. The exponential decrease takes over for $-q^2 \gtrsim \lambda^2$ due to finite size. Finally, power dependence resumes in the asymptotic regime, as the different picture of quark elastic scattering takes over.

The quark-elastic-scattering picture and the Skyrme picture appear to be complementary. To emphasize this, we should keep in mind that the pointlike pions of the Skyrme model are an idealization. At large enough q^2, it is reasonable to expect that one should discover a pion form factor $f_\pi(q^2)$ at some level of the scattering process. Such a form factor could either be manufactured self-consistently by higher derivative terms in the model, or be imposed beyond the model. We have argued that B^μ is immune to corrections, however. Furthermore, returning to the Mössbauer analogy, one would hardly expect the phonon Lagrangian to supply a nuclear form factor in that physical example. If one supplies the canonical prescription[15] $f_\pi(q^2) \sim 1/q^2$, the dependence of (3.17) predicts

$$F(q^2) \sim \frac{\text{const}}{q^4}\exp(q^2/8\lambda^2) . \tag{3.18}$$

The fact that (3.18) may be consistent with data at large $-q^2$ might be something of a coincidence. Within the quark-containing model,[19] the power of q^2 depends on the number of quarks; if $F(q^2) \sim 1/q^4$, there are three quarks. The power result (3.17) and (3.18) depends on the isospin SU(2) subgroup and winding number of the baryon; it is a result of the isospin mapping. The Wess-Zumino argument[20] does relate the number of colors and the baryon quantum numbers, but this may depend rather intimately on the choice of mapping.[21] The natural interpretation of the exponential factor, in any case, is that of an essential singularity in a higher-twist series.

The demonstration of a power dependence is more important here than the value of the power. Phenomenologically, the most reasonable claim would be that some condensed modes as we have outlined may be responsible for the unexpectedly rapid onset of power dependence in medium-q^2 data, and the problem[19] of apparent absence of large powers of $\alpha_s(q^2)$ in the same data. That is, λ^2 could be large enough (within the other approximations) to utterly suppress the domain of exponential dependence in data. Amazingly, this is guaranteed in the large-F_π limit. We are not sure of the reason for this, but it appears that fluctuations tending to decrease the overlap are unimportant as $\hbar \rightarrow 0$ or, for fixed $\hbar$, as the size of the field becomes very large.

IV. CONCLUSIONS

There are several points to discuss. Our impulse approximation procedure only represents a preliminary analysis of the concepts we wish to develop. We have seen that there exists a systematic way to reduce the calculations in the impulse approximation to quadratures.

A great deal of more detailed work can be done for arbitrary values of q^2. A side issue involves the interplay of the large-$F_\pi{}^2$ and large-μ^2 limits which can be studied in a variety of circumstances. The limits referred to generally cannot commute given the $\mu^2/F_\pi{}^2$ dependence we have found.

Let us emphasize that one approach to refining the approximations would include functional integrals over the shape parameters. But this can be done retaining the results of (3.6) and (3.7)—which themselves are legitimate matrix elements over all the field-theory degrees of freedom. A characteristic that we believe should not be erased by such an average is the finite expected value over the configurations at the origin represented by $F_f''(0)$. If this does not average to zero, then the q^{-2} dependence is an immediate result.

Before refining the analysis we have outlined here, a natural modification would be to recast the formalism into one defined in terms of more general quantized modes. This method, a complete discussion of zero modes, and a more detailed exposition of the extended states we have constructed, is given in Ref. 11.

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