Effective Lagrangian for Skyrmion physics

Ian J. R. Aitchison,* Caroline M. Fraser,† and Paul J. Miron
Department of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, England
(Received 18 July 1985)

The effective chiral Lagrangian for low-energy pion-Skyrmion physics is considered as an expansion in powers of $(\partial\Phi)$, where Φ is the chiral field. The $O((\partial\Phi)^2)$ term is the nonlinear σ-model Lagrangian L_2. Two theoretical approaches to the higher-order terms are examined. The first involves integration over fermion degrees of freedom. The $O((\partial\Phi)^4)$ terms obtained this way, together with L_2, are found to be consistent with low-energy $\pi\pi$ phase shifts—in particular, in the dominant ρ and σ channels. In the second approach, an effective pion Lagrangian is generated by integrating some simple chiral meson-field Lagrangians over the heavy mesons. At $O((\partial\Phi)^4)$ two terms are found: a term $L_{4,\rho}$ of Skyrme form, with a model-independent coefficient determined by the ρ mass, and a non-Skyrme term $L_{4,\sigma}$, the coefficient of which is more model dependent and determined by the σ mass; both terms are compatible with the fermion integration results. It is concluded that the $O((\partial\pi)^4)$ terms are well established theoretically, and in good qualitative agreement with pion data. These terms do not, however, stabilize the soliton. Some discussion of the possible stabilizing effects of $O((\partial\pi)^6)$ terms is given.

I. INTRODUCTION

QCD is the underlying theory of strong interactions, but it is very difficult to give an account of low-energy hadronic processes starting *ab initio* from the QCD Lagrangian. In such a situation it may be valuable to try to develop an effective theory, framed in terms of observed hadronic degrees of freedom but incorporating as many relevant features of QCD as possible (for example, its flavor symmetries). Prior to QCD, effective Lagrangians for low-energy hadronic processes were introduced during the current-algebra era. In recent years, considerable progress has been made in further elaborating such Lagrangians, and in relating them to QCD. An important new development has been the recognition[1] that, as suggested by the large-N_c limit,[2,3] baryons can be regarded as solitons of a meson field theory, in the manner first proposed by Skyrme.[4] Thus a new constraint on effective meson Lagrangians is that they should be capable of yielding a physically reasonable soliton sector.

These ideas find a simple realistic expression in the two-flavor Lagrangian originally studied by Skyrme[4]

$$L_S(\boldsymbol{\pi})=L_2(\boldsymbol{\pi})+L_{4,S}(\boldsymbol{\pi}) , \tag{1.1}$$

where

$$L_2(\boldsymbol{\pi})=\tfrac{1}{2}(\partial_\mu\phi_a)^2 , \tag{1.2}$$

$$L_{4,S}(\boldsymbol{\pi})=\frac{1}{4e^2f^4}[(\partial_\mu\phi_a\partial_\nu\phi_a)^2-(\partial_\mu\phi_a)^4] , \tag{1.3}$$

and $\Phi_a=(\sigma,\boldsymbol{\pi})$ with $\Phi^2=\sigma^2+\boldsymbol{\pi}^2=f^2$. From today's perspective we interpret $L_2(\boldsymbol{\pi})$ as the Lagrangian of the nonlinear σ model,[5] which is known to incorporate, up to $O(\partial^2)$, the consequences of the spontaneously broken (two-flavor) chiral symmetry of QCD, realized via the Goldstone degrees of freedom. This term both describes aspects of low-energy π-π scattering and also, when regarded as a classical-field Lagrangian, has soliton solutions possessing a topologically conserved quantum number which can be identified with baryon number.[3,4]

These solitons are, however, not stable against collapse, and the further term $L_{4,S}$ in (1.1) was originally introduced "by hand" to stabilize them. Such a term will also lead, in the π-π sector, to $O(p^4)$ effective-range corrections to the $O(p^2)$ soft-pion results successfully predicted by L_2 (Ref. 6). Thus, on the one hand, we should like to understand the appearance of $L_{4,S}$—or alternative terms—as arising from some underlying theory and, on the other, we must subject all such extensions of L_2 to phenomenological testing in both the meson and soliton sectors.

We have investigated two theoretical approaches to the construction of an effective meson Lagrangian L_{eff}. In the first, fermionic degrees of freedom—quarks—are introduced. Terms like $L_{4,S}$ are then looked for in the derivative expansion of the effective action obtained by integrating (wholly or partially) over the quarks—or in a similar expansion of the Casimir energy of the quark sea (which, however, can also be evaluated numerically, without approximation[7]). According to this view, terms like $L_{4,S}$ are to be calculated directly from some initial Lagrangian involving quark dynamics. The terms of order $(\partial\phi)^4$ have been much discussed recently.[8] It turns out that they are not of Skyrme form $L_{4,S}$; rather, they are

$$L_4^q=\frac{N_c}{48\pi^2f^4}[(\partial_\mu\phi_a\partial_\nu\phi_a)^2-\tfrac{3}{2}(\partial_\mu\phi_a)^4+f^2(\Box\phi_a)^2] . \tag{1.4}$$

It appears quite significant to us that the several different derivations[8] all give the same result. However, it seems clear now that L_4^q does not, in fact, stabilize the soliton.[7,9]

At first sight, this seems disappointing. But there is

 ©1986 The American Physical Society

also the question of the π-π sector to consider. We shall argue in Sec. II that, indeed, the result (1.4) is actually a valuable one, since the L_4^q terms are in qualitative agreement with π-π data. The dominant features of the data beyond the threshold region are, of course, the ρ resonance and the strong $I=0$ s-wave π-π interaction (which we identify with the σ, the chiral partner of the π). Recently, Pham and Truong[10] have shown very clearly, using dispersion relations for π-π scattering, how the ρ is associated (at low energies) with a term of $L_{4,S}$ form, while the σ corresponds to a different, destabilizing, term of the form

$$L_{4,n}=\frac{\gamma}{2e^2f^4}(\partial_\mu\phi)^4 \; ; \tag{1.5}$$

e^2 in (1.3) and (1.5) is related to the ρ parameters, and γ to the σ parameters. In Sec. II we shall see how the parameters e^2 and γ are obtained from L_4^q.

Deferring for the moment further discussion of the stability problem, we want to emphasize here the importance of Pham and Truong's work in relating parameters of $O(\partial^4)$ terms in an effective Lagrangian to observed meson parameters. The first attempt to connect these parameters to π-π data was made by Donoghue, Golowich, and Holstein,[11] but these authors chose to fit the d-wave scattering lengths (see also the recent work of Blaizot, Chemtob, and Pasquier[12]). However, the d-wave amplitudes are far smaller than the much better determined s- and p-wave ones in the energy region of interest: a_2^2 in particular is very small and poorly determined. Further, the d wave is not directly related to a low-lying meson state, in the way that the s and p waves are.

The connection of $L_{4,S}$ and $L_{4,n}$ with the ρ and the σ, established by Pham and Truong,[10] allows us to make contact with the second theoretical approach to the problem of constructing an effective Lagrangian, namely, one based on the $1/N_c$ expansion. It is believed[2,3] that, assuming confinement, QCD based on the gauge group $SU(N_c)$ is equivalent to a theory of mesons and glueballs in which the quartic meson coupling constant is $1/N_c$. Baryons are then solitons in this meson theory. To the extent to which $1/N_c$ is small, the meson theory is weakly interacting and the solitons may be treated semiclassically. We are therefore led to seek an effective meson Lagrangian, which is to be used at the tree level only.

As Witten has remarked,[13] the $1/N_c$ expansion itself in no way requires us to produce the meson Lagrangian from an underlying dynamical theory—it can be taken from experiment and analyzed for solitons. (Nevertheless there are important theoretical requirements which such a Lagrangian should incorporate, for example, chiral symmetry and, ideally, anomaly-related structure.[14]) Although in principle an infinite number of mesons should presumably be present, this is clearly impractical, and one hopes that at low energies it should be sufficient to consider an effective Lagrangian involving only low-lying states with masses up to, say, the order of 1 GeV, i.e., the π, σ, ρ, ω, and A_1, in the two-flavor case.

Even if a plausible Lagrangian L_m involving these meson fields can be constructed, it would still be very complicated to work with, since the associated field equations are coupled and nonlinear (note, however, that Broniowski and Banerjee[15] have made a frontal attack on the problem along these lines). A considerable simplification occurs if we exploit an essential physical difference between the π fields on the one hand, and all the other meson fields on the other: in the chiral limit the π's are massless, but none of the other states are. We may therefore write the solutions of the classical-field equations for the massive states as an expansion in powers of (derivative/mass), expressing all the heavy fields in terms of the π fields and their derivatives. This procedure produces an effective action $\Gamma(\pi)$ which is one-particle irreducible with respect to the π field only, and is the result of summing up all tree graphs involving the heavy mesons. The associated density $L(\pi)$ is then an effective Lagrangian for the pion sector, expressed as a derivative expansion, and involving the meson parameters as coefficients of the various terms. We carry out such a calculation for the $\pi,\sigma,\rho,\omega,A_1$ system in Sec. III, and show how Pham and Truong's result for the coefficients of $L_{4,S}$ and $L_{4,n}$ (found by phenomenological analysis) can be obtained from the effective-meson-Lagrangian starting point.

The results of Secs. II and III can be directly compared, since each expresses the effective Lagrangian $L(\pi)$ as an expansion in powers of $(\partial\pi)$. Up to $O((\partial\pi)^4)$, the two approaches give the same (essentially unique) Skyrme term $L_{4,S}$, and qualitatively similar $L_{4,n}$ terms; these two terms are related to the ρ and σ, respectively, as first found by Pham and Truong. We conclude that the calculated $O((\partial\pi)^4)$ terms are well established theoretically (there being a remarkable consistency between the fermion integration and $1/N_c$ viewpoints), and in good qualitative agreement with the π-π data.

However, these terms do not stabilize the classical soliton, and consequently the origin of its stability must lie elsewhere. We offer some comments on this in Sec. IV.

II. L_4^q AND LOW-ENERGY π-π DATA

We wish to calculate the contribution of $L_2(\pi)+L_4^q$ to low-energy π-π scattering. To do this, we extract the $O(\pi^4)$ terms in these expressions.

The field Φ_a in (1.2) and (1.4) satisfies the constraint $\sigma=(f^2-\boldsymbol{\pi}^2)^{1/2}$ of the nonlinear σ model. Thus the $O(\boldsymbol{\pi}^4)$ term in L_2 is the well-known soft-pion result[5]

$$L_{2,\pi^4}=\frac{1}{2f^2}(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) \tag{2.1}$$

to which we shall also add the explicit pion-mass term[5]

$$-\frac{m_\pi{}^2}{8f^2}(\boldsymbol{\pi}^2)^2 \tag{2.2}$$

since it is a poor approximation to set $m_\pi=0$ near threshold. To $O(\boldsymbol{\pi}^4)$, the $\Box\Phi_a\Box\Phi_a$ term in (1.4) becomes

$$(\partial_\mu\boldsymbol{\pi})^4+(\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi})^2+2(\partial_\mu\boldsymbol{\pi})^2\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi} \tag{2.3}$$

so that the terms we want in L_4^q are

$$L^q_{\pi^4}=\frac{N_c}{48\pi^2 f^4}\{[(\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi})^2-(\partial_\mu\boldsymbol{\pi})^4]+\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^4+(\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi})^2+2(\partial_\mu\boldsymbol{\pi})^2\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi}\}\ . \tag{2.4}$$

Before proceeding with the π-π calculation, we first observe that (2.4) may be compared with the $O(\boldsymbol{\pi}^4)$ terms in the Skyrme form (1.3) and in the non-Skyrme form (1.5). Expanding each of the latter in powers of the π field, we may identify

$$\frac{1}{e^2}=\frac{N_c}{12\pi^2} \tag{2.5}$$

and

$$\gamma=\tfrac{1}{4}\ . \tag{2.6}$$

The remainder of (2.4) consists of parts which, on shell, are proportional to pion-mass terms. Bearing in mind the result of Pham and Truong (see also Sec. III), we shall want to see if the parameters (2.5) and (2.6) are consistent with the observed ρ and σ channels in π-π scattering.

It is a straightforward matter [see, for instance, Eqs. (3.1)–(3.3) of Ref. 16, and Eqs. (3.20)–(3.23) below] to calculate the contribution of $L_2(\pi)+L^q_{\pi^4}(\pi)$ to π-π amplitudes of definite I and l. We find

$$F^{I=0}=\frac{2s-m_\pi^2}{f^2}+\frac{N_c}{48\pi^2 f^4}[s^2+16sm_\pi^2-4m_\pi^4+(s-4m_\pi^2)^2P_2(\cos\theta)]\ , \tag{2.7}$$

$$F^{I=1}=\frac{t-u}{f^2}\left[1+\frac{N_c}{48\pi^2 f^4}(2s+8m_\pi^2)\right]\ , \tag{2.8}$$

$$F^{I=2}=\frac{2m_\pi^2-s}{f^2}+\frac{N_c}{48\pi^2 f^4}(2s^2-16sm_\pi^2+24m_\pi^4)\ , \tag{2.9}$$

where s,t,u are the Mandelstam variables with

$$t=-\frac{(s-4m_\pi^2)}{2}(1-\cos\theta)\ . \tag{2.10}$$

These amplitudes are, of course, all real. In order to make contact with phase shifts, we must face the old problem[6] of how to unitarize an effective Lagrangian (alternatively, one may use dispersion relations to extrapolate the data to threshold, as Pham and Truong[10] did). One simple prescription is the "K-matrix unitarization" scheme. Unitarity forces any partial-wave amplitude t_l^I for two-body scattering to have a branch point at threshold, i.e., at $s=4m_\pi^2$ in the present case; the discontinuity across the associated cut is determined by unitarity. In the elastic region, these unitarity constraints on t_l^I are fully satisfied by writing

$$(t_l^I)^{-1}=(k_l^I)^{-1}-i\rho_l\ , \tag{2.11}$$

where k_l is free of the branch point at $s=4m_\pi^2$, which is contained entirely in the phase-space factor ρ_l:

$$\rho_l=\frac{1}{16\pi}\frac{q^{2l+1}}{\sqrt{s}},\quad q^2=\tfrac{1}{4}(s-4m_\pi^2)\ . \tag{2.12}$$

In terms of the phase shift δ_l^I defined by

$$t_l^I=\frac{e^{i\delta_l^I}\sin\delta_l^I}{\rho_l}=(\rho_l\cot\delta_l^I-i\rho_l)^{-1}\ . \tag{2.13}$$

k_l^I is given by

$$(k_l^I)^{-1}=\rho_l\cot\delta_l^I \tag{2.14}$$

and is the partial-wave K-matrix element. We shall take the view that expressions (2.7)–(2.10), which are, of course, free of the normal threshold branch point, are to be interpreted as K-matrix elements via

$$F^I=\sum_l(2l+1)P_l(\cos\theta)q^{2l}k_l^I\ . \tag{2.15}$$

This yields

$$\delta_0^0=\arctan\left[\frac{q}{\sqrt{s}}\frac{1}{16\pi f^2}\left[2s-m_\pi^2+\frac{N_c}{48\pi^2 f^2}(s^2+16sm_\pi^2-4m_\pi^4)\right]\right]\ , \tag{2.16}$$

$$\delta_0^2=\arctan\left[\frac{q}{\sqrt{s}}\frac{1}{16\pi f^2}\left[2m_\pi^2-s+\frac{N_c}{48\pi^2 f^2}(2s^2-16sm_\pi^2+24m_\pi^4)\right]\right]\ , \tag{2.17}$$

$$\delta_1^2=\arctan\left[\frac{q^3}{12\pi f^2\sqrt{s}}\left[1+\frac{N_c}{48\pi^2 f^2}(2s+8m_\pi^2)\right]\right]\ , \tag{2.18}$$

$$\delta_2^0=\arctan\left[\frac{q^5 N_c}{240\pi^3 f^4\sqrt{s}}\right]\ . \tag{2.19}$$

These phase shifts are tabulated in Table I.

We now comment on these results.

The most significant $O(s^2)$ corrections are in the $I=l=0$ and $I=2$, $l=0$ channels. In the former, the correction contributes some 10° at 700 MeV, in the direction of increasing the $O(s)$ (soft-pion) phase shift, and thereby improving agreement with experiment.[6,17] This is equivalent to introducing a finite-mass parameter m_σ in this channel (the nonlinear limit being $m_\sigma\to\infty$); we interpret this as arising from the quark dynamics. In the $I=2$, $l=0$ channel, the $O(s^2)$ term contributes some +12° at 700 MeV, usefully correcting the tendency of the

TABLE I. π-π phase shifts calculated using Eqs. (2.16)–(2.19).

$E(=\sqrt{s})$ (MeV)	δ_0^0	δ_0^2	δ_1^1
360	11°	−3.6°	1°
440	21°	−7°	2.7°
520	31°	−10°	5°
600	41°	−12°	8.7°
680	50°	−12°	13°
760	58°	−11°	18.6°

$O(s)$ term to become too large in magnitude, relative to the (admittedly poorly determined) experimental data.

The p-wave phase shift is small, but rising. Of course, we see no ρ resonance explicitly in L_4^q—but this phase shift can certainly represent the "tail" of the ρ. We also note that the d-wave scattering lengths are given (in units of m_π^{-4}) by

$$a_2{}^0=(160\pi^3)^{-1}(m_\pi/f)^4\approx 9.9\times10^{-4},\quad a_2{}^2=0\ ,$$

which are to be compared with the quoted experimental values[17]

$$a_2{}^0=(17\pm3)\times10^{-4},\quad a_2{}^2=(1.3\pm3)\times10^{-4}\ .$$

In summary, integration over fermion degrees of freedom seems to provide, at $O(p^4)$, qualitatively correct additions to the $O(p^2)$ soft-pion results in the π-π sector. What about soliton stability? As remarked previously[18] the value (2.5) for the Skyrme coefficient is comparable to that required in pure-Skyrme models of the nucleon. (We note that stability is a delicate question of the balance between L_2 and higher-order derivative terms, so that the latter may play an important role for this purpose even if they are associated with rather small corrections to the soft-pion phase shifts calculated from L_2.) However, the addition of the non-Skyrme term with coefficient (2.6) removes stability.[7,9,19]

We shall come back to the stability problem in Sec. IV. We now turn to the alternative approach to generating an effective pion Lagrangian.

III. EFFECTIVE $L(\pi)$ FROM MESON FIELD LAGRANGIAN

In the preceding section, we have argued that an effective Lagrangian of the form [up to $O((\partial\Phi)^4)$]

$$L(\pi)=L_2+L_{4,S}+L_{4,n} \tag{3.1}$$

with

$$\frac{1}{e^2}=\frac{N_c}{12\pi^2} \tag{3.2}$$

and

$$\gamma=\tfrac{1}{4} \tag{3.3}$$

appears to be consistent with quark dynamics, and with low-energy pion phenomenology. The aim of this section is to see to what extent the form of (3.1) can be justified starting from an effective meson field Lagrangian $L_m=L(\pi,\sigma,\rho,\omega,\mathbf{A}_1)$ involving the indicated fields, and to relate the parameters of (3.1) to masses and coupling constants in L_m. These predictions will then be compared with (3.2) and (3.3).

The view we have been taking throughout this paper is that the nucleon is a soliton in some effective *pion* theory. Thus, while in principle we could imagine setting up, and solving numerically, the coupled equations of motion for all the fields in L_m, and then studying the pion-field solutions, we expect that it should be possible to obtain a satisfactory Lagrangian for soliton studies by "integrating out" the heavy-meson fields in L_m. More formally, if Γ_m is the tree-level action associated with L_m, we obtain $\Gamma_m(\pi)$, the generator of vertices which are one-particle irreducible with respect to the π field only, from the relation

$$\Gamma(\pi)=\Gamma_m(\pi,\bar\sigma,\bar{\rho},\bar\omega,\bar{\mathbf{A}},)\ , \tag{3.4}$$

where

$$\left.\frac{\delta\Gamma_m}{\delta\sigma}\right|_{\sigma=\bar\sigma}=0 \tag{3.5}$$

and similarly for the other fields. That is, the barred fields are expressed in terms of π via the classical equations of motion. Since we are never going beyond the tree level, the associated Lagrangian $L(\pi)$—which we wish to calculate—is just $L_m(\pi,\bar\sigma,\bar\rho,\bar\omega,\bar A_1)$. We approximate the equations of motion by an expansion in powers of derivatives, the scale being set by the meson masses.

An equivalent, and somewhat more convenient, procedure is to evaluate all relevant tree graphs which are not 1π irreducible, expand the propagators in powers of momenta, and convert back to an equivalent Lagrangian.

We now have to decide on the form of L_m, which is to be consistent with low-energy data and chiral symmetry. Such an effective Lagrangian is not necessarily unique. We shall study here a number of possibilities, all of which include (a) a linear σ sector (to accommodate the large $I=0=J$ π-π phase shift) and (b) the A_1 as the chiral partner of the ρ. We shall not include the ω, since [see Eq. (4.7) below] its leading contribution turns out to be $O((\partial\pi)^6)$, and in this section we are concerned with $O((\partial\pi)^4)$ terms only.

We begin with the well-known[5] local SU(2)×SU(2)-invariant theory, and introduce

$$\begin{aligned}
D'_\mu\sigma&=\partial_\mu\sigma+g\,\mathbf{A}'_\mu\cdot\boldsymbol{\pi}\ ,\\
D'_\mu\boldsymbol{\pi}&=\partial_\mu\boldsymbol{\pi}+g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi}-g\,\mathbf{A}'_\mu\sigma\ ,\\
\boldsymbol{\rho}'_{\mu\nu}&=\partial_\mu\boldsymbol{\rho}_\nu-\partial_\nu\boldsymbol{\rho}_\mu+g(\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu+\mathbf{A}'_\mu\times\mathbf{A}'_\nu)\ ,\\
\mathbf{A}'_{\mu\nu}&=\partial_\mu\mathbf{A}'_\nu-\partial_\nu\mathbf{A}'_\mu+g(\boldsymbol{\rho}_\mu\times\mathbf{A}'_\nu+\mathbf{A}'_\mu\times\boldsymbol{\rho}_\nu)\ .
\end{aligned} \tag{3.6}$$

The Lagrangian is

$$\begin{aligned}
L_m^L=\;&\frac{\beta}{2}[(D'_\mu\sigma)^2+(D'_\mu\boldsymbol{\pi})^2]-\frac{(m^2-m_\pi{}^2)}{8f^2}(\sigma^2+\boldsymbol{\pi}^2)^2\\
&+\frac{(m^2-3m_\pi{}^2)}{4}(\sigma^2+\boldsymbol{\pi}^2)+fm_\pi{}^2\sigma\\
&-\tfrac{1}{4}(\boldsymbol{\rho}'^{\,2}_{\mu\nu}+\mathbf{A}'^{\,2}_{\mu\nu})+\frac{m_\rho{}^2}{2}(\boldsymbol{\rho}'^{\,2}_\mu+\mathbf{A}'^{\,2}_\mu)\ .
\end{aligned} \tag{3.7}$$

Expanding about the asymmetric ground state by setting $\sigma=\sigma'+f$, we find that A'_μ and $D_\mu\pi$ are mixed, where $D_\mu=\partial_\mu+g\rho_\mu\times$. The physical (unmixed) A_1 field is defined by

$$\mathbf{A}_\mu=\mathbf{A}'_\mu-\frac{gf\beta}{\beta g^2f^2+m_\rho^2}D_\mu\pi \ . \tag{3.8}$$

Demanding that the coefficient of $(D_\mu\pi)^2$ is $\frac{1}{2}$ leads to

$$\beta=\frac{1}{1-\alpha} \ , \tag{3.9}$$

where $\alpha=g^2f^2/m_\rho^2$ and the A_1 mass (from the "partial Higgs mechanism") is

$$m_A{}^2=\frac{m_\rho{}^2}{1-\alpha} \ , \tag{3.10}$$

while (3.8) becomes

$$\mathbf{A}_\mu=\mathbf{A}'_\mu-\frac{gf}{m_\rho^2}D_\mu\pi \ . \tag{3.11}$$

We now look at terms which will generate $O(\pi^4)$ terms via tree graphs, in order to identify the parameters e^2 and γ in $L_{4,S}$ and $L_{4,n}$. The relevant bits are

$$\begin{aligned}\hat{L}^{L}_{m}=&\ \tfrac{1}{2}(\partial_\mu\pi)^2-\tfrac{1}{2}m_\pi{}^2\pi^2+\tfrac{1}{2}\beta(\partial_\mu\sigma')^2-\tfrac{1}{2}m^2\sigma'^2-\tfrac{1}{4}\rho'^2_{\mu\nu}+\tfrac{1}{2}m_\rho{}^2\rho_\mu{}^2+g\partial_\mu\pi\cdot\rho^\mu\times\pi+\frac{g^2f\beta}{m_\rho{}^2}\partial_\mu\sigma'\pi\cdot\partial^\mu\pi\\&+\frac{\beta g^4f^2}{2m_\rho{}^4}(\pi\cdot\partial_\mu\pi)^2-\frac{g^2f}{m_\rho{}^2}\sigma'(\partial_\mu\pi)^2-\frac{(m^2-m_\pi{}^2)}{2f}\sigma'\pi^2-\frac{(m^2-m_\pi{}^2)^2}{8f^2}\pi^4-\frac{g^3f^2}{m_\rho{}^4}\partial_\mu\rho_\nu\cdot(\partial^\mu\pi\times\partial^\nu\pi)\\&-\frac{g^2}{4}\left[\frac{gf}{m_\rho{}^2}\right]^4(\partial_\mu\pi\times\partial_\nu\pi)^2 \ .\end{aligned} \tag{3.12}$$

From this we obtain the $\sigma'\pi\pi$ vertex (Fig. 1)

$$-\frac{i}{f}[m^2-m_\pi{}^2+\alpha\beta q_1\cdot(q_2+q_3)-2\alpha q_2\cdot q_3] \tag{3.13}$$

and the $\rho\pi\pi$ vertex (Fig. 2)

$$-g\epsilon_{ijk}\left[(q_3-q_2)_\mu+\frac{\alpha}{m_\rho{}^2}(q_1\cdot q_2q_{3\mu}-q_1\cdot q_3q_{2\mu})\right] . \tag{3.14}$$

The on-shell $\rho\pi\pi$ coupling constant (related to the ρ width) is thus

$$g_{\rho\pi\pi}=g\left[1-\frac{\alpha}{2}\right] . \tag{3.15}$$

We calculate explicitly the tree graphs up to $O(p^4)$. Figure 1 contributes

$$\begin{aligned}i\delta_{ij}\delta_{kl}\frac{1}{f^2}\frac{1}{m^2-\beta s}[&(m^2-m_\pi{}^2)^2-2\alpha\beta s(m^2-m_\pi{}^2)+\beta^2\alpha^2s^2-2\alpha(m^2-m_\pi{}^2)(p_1\cdot p_2+p_3\cdot p_4)\\&+4\alpha^2p_1\cdot p_2p_3\cdot p_4+2\alpha^2\beta s(p_1\cdot p_2+p_3\cdot p_4)]\end{aligned} \tag{3.16}$$

and Fig. 2 contributes

$$-i(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk})\frac{1}{s-m_\rho{}^2}\left[g^2(t-u)+\frac{g^2}{m_\rho{}^2}(p_1{}^2-p_2{}^2)(p_3{}^2-p_4{}^2)+\frac{\alpha^2}{f^2}[s(u-t)-(p_1{}^2-p_2{}^2)(p_3{}^2-p_4{}^2)]\right] . \tag{3.17}$$

Expanding to order p^4, and crossing Fig. 1 appropriately, gives the following coefficients of $i\delta_{ij}\delta_{kl}$: for the σ channel

$$\begin{aligned}\frac{1}{f^2}\Bigg[&m^2(1-x)^2+s(1-\alpha\beta-2x+\beta x^2)-\alpha(1-x)(s-t-u)\\&+\frac{1}{m^2}[s^2(1-\alpha-x-\beta x+\beta^2x^2)+s(t+u)(\alpha-\alpha^2-\alpha\beta x)+\alpha^2(p_1{}^2+p_2{}^2)(p_3{}^2+p_4{}^2)]\Bigg] ,\end{aligned} \tag{3.18}$$

where $x=m_\pi{}^2/m^2$, and for the ρ

$$\frac{g^2}{m_\rho{}^2}(2s-t-u)+\frac{\alpha(1-\alpha)}{m_\rho{}^2f^2}[-2tu+s(t+u)-2(p_1{}^2p_2{}^2+p_3{}^2p_4{}^2)-(p_1{}^2+p_2{}^2)(p_3{}^2+p_4{}^2)] . \tag{3.19}$$

These expressions can be converted to effective vertices via

$$(\partial_\mu\pi)^4=-2st-2su+2(p_1{}^2+p_2{}^2)(p_3{}^2+p_4{}^2) , \tag{3.20}$$

$$(\partial_\mu \pi \cdot \partial_\nu \pi)^2 = -2ut - st - su + 2(p_1{}^2 p_2{}^2 + p_3{}^2 p_4{}^2) + (p_1{}^2 + p_2{}^2)(p_3{}^2 + p_4{}^2) \,, \tag{3.21}$$

$$(\pi \cdot \Box \pi)^2 = 2(p_1{}^2 + p_2{}^2)(p_3{}^2 + p_4{}^2) \,, \tag{3.22}$$

$$(\pi \cdot \Box \pi)(\partial_\mu \pi)^2 = s(s+t+u) - 2(p_1{}^2 + p_2{}^2)(p_3{}^2 + p_4{}^2) \,. \tag{3.23}$$

Including the contact terms from (3.12), we obtain finally

$$\begin{aligned} L_m^L(\pi) = {} & \tfrac{1}{2}(\partial_\mu \pi)^2 - \tfrac{1}{2} m_\pi{}^2 \pi^2 - \frac{x\alpha}{2f^2}\pi^2(\partial_\mu \pi)^2 + \frac{1}{2f^2}(\pi \cdot \partial_\mu \pi)^2(1 - 2x + \beta x^2) + \frac{m^2}{8f^2}(x^2 - x)\pi^4 \\ & + \frac{\alpha(1-\alpha/2)^2}{m_\rho{}^2 f^2}[(\partial_\mu \pi \cdot \partial_\nu \pi)^2 - (\partial_\mu \pi)^4] + \frac{1}{m^2 f^2}\left[\tfrac{1}{2}(1-\alpha)^2 - x + \frac{\beta^2 x^2}{2}\right](\partial_\mu \pi)^4 \\ & + \frac{1}{m^2 f^2}[1 - \alpha - (1+\beta)x + \beta^2 x^2](\partial_\mu \pi)^2(\pi \cdot \Box \pi) + \frac{1}{m^2 f^2}\left[\tfrac{1}{2} - \beta x + \frac{\beta^2 x^2}{2}\right](\pi \cdot \Box \pi)^2 + O(\pi^6) \,. \end{aligned} \tag{3.24}$$

The same results can also be obtained from the equations of motion. As expected, the $O(\partial^2)$ pieces reduce to $L_2(\pi)$ when $m_\pi{}^2 \to 0$.

The values of the "gauge" coupling constant g (and the related quantities α and β) are determined from the ρ width via Eq. (3.15). It is convenient to use the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[20]

$$m_\rho{}^2 = 2f^2 g_{\rho\pi\pi}{}^2 \tag{3.25}$$

which is well obeyed experimentally, and, as has recently been argued,[21] can be obtained as a consequence of regarding the ρ as a dynamical gauge boson of the hidden local symmetry of the nonlinear σ model. In terms of α, (3.25) reads

$$\alpha \left[1 - \frac{\alpha}{2}\right]^2 = \frac{1}{2} \,. \tag{3.26}$$

If this condition is satisfied, we obtain from (3.24) the coefficient of the Skyrme term (1.3) as

$$\frac{1}{e^2} = \frac{2f^2}{m_\rho{}^2} \,. \tag{3.27}$$

We have therefore confirmed Pham and Truong's result[10] that the Skyrme term is related to the ρ, and moreover the value (3.27) agrees with theirs.

However, (3.12) cannot be accepted as it stands, in the above model. Equation (3.26) has only one real root, at $\alpha \approx 2.84$, and for this value both the σ and the A_1 are tachyonic [cf. (3.10) and note that the σ pole in (3.16) is at $m/\sqrt{\beta}$]. The nearest approximate root of (3.26) is $\alpha = \frac{2}{3}$ (and $\beta = 3$). For this value of α, $m_A{}^2 = 3m_\rho{}^2$ and $g_{\rho\pi\pi}{}^2 \approx 0.3(m_\rho{}^2/f^2)$. These values are, respectively, some 50% too high and too low.

The parameter γ/e^2 in the $L_{4,n}$ term is

$$\frac{\gamma}{e^2} = \frac{f^2}{m^2}[(1-\alpha)^2 - 2x + \beta^2 x^2] \,, \tag{3.28}$$

where m is the σ mass parameter. Thus our effective Lagrangian has again confirmed the Pham and Truong result—that the $L_{4,n}$ term is related to the σ. However, their dispersion relation analysis yielded the numerical value

$$\frac{\gamma}{e^2} \simeq \frac{4f^2}{3m_\sigma{}^2} \tag{3.29}$$

with $m_\sigma \approx 700$ MeV. For $\alpha \approx \frac{2}{3}$, we can fit $m_\sigma \approx 700$ MeV with $m^2 = \beta m_\sigma{}^2 \approx 75 m_\pi{}^2$, but then the value of γ/e^2 predicted by (3.28) is much smaller than the value (3.29). Alternatively, we can choose the parameter m in (3.28) so as to reproduce the numerical value (3.29), but then the position of our σ pole is much too low ($\sim m_\pi$).

Thus we conclude that while this locally chiral model confirms the role of the ρ and σ in the physics of $L_{4,s}$ and $L_{4,n}$, the numerical values predicted for the coefficients are in poor agreement with the phenomenology.

Although the local chiral symmetry is to some extent appealing, for example, it embodies universal ρ/A_1 dominance,[22] it does not seem to be sacrosanct, and the above results appear to rule it out as an accurate theory in the soliton sector. Consequently we have also investigated a "minimal" globally chiral-symmetric model, defined by the Lagrangian

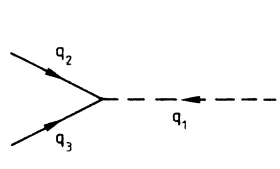

FIG. 1. $\sigma'\pi\pi$ vertex from Eq. (3.13).

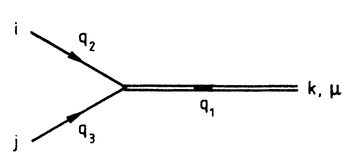

FIG. 2. $\rho\pi\pi$ vertex from Eq. (3.14).

$$L_m^G=\frac{\beta}{2}(\partial_\mu\phi_a)^2+\frac{(m^2-3m_\pi^2)}{4}\phi_a^{\,2}-\frac{(m^2-m_\pi^2)}{8f^2}(\phi_a^{\,2})^2+fm_\pi^{\,2}\sigma-\tfrac{1}{4}(\rho_{\mu\nu}^{\,2}+\mathbf{A}_{\mu\nu}'^{\,2})+\tfrac{1}{2}m_\rho^{\,2}(\rho_\mu^{\,2}+\mathbf{A}_\mu'^{\,2})$$

$$+g(\partial_\mu\sigma\mathbf{A}'^{\mu}\cdot\boldsymbol{\pi}+\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\rho}^\mu\times\boldsymbol{\pi}-\partial_\mu\boldsymbol{\pi}\cdot\mathbf{A}'^{\mu}\sigma)\ , \tag{3.30}$$

where now $\rho_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu$, $\mathbf{A}'_{\mu\nu}=\partial_\mu\mathbf{A}'_\nu-\partial_\nu\mathbf{A}'_\mu$. Shifting the σ field, and diagonalizing via

$$\mathbf{A}_\mu=\mathbf{A}'_\mu-\frac{gf}{m_\rho^{\,2}}\partial_\mu\boldsymbol{\pi} \tag{3.31}$$

we require

$$\beta=1+\frac{g^2f^2}{m_\rho^{\,2}}\ . \tag{3.32}$$

g in (3.31) is now the $\rho\pi\pi$ coupling constant, and the KSRF relation is $m_\rho^{\,2}=2g^2f^2$; hence we take $\beta=\frac{3}{2}$. Evaluating the σ and ρ tree graphs and expanding as before, one obtains the effective Lagrangian, up to $O(\pi^4)$:

$$L_m^G(\boldsymbol{\pi})=\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi^{\,2}\boldsymbol{\pi}^2-\frac{x}{4f^2}\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi})^2+\frac{1}{2f^2}(1-2x+\tfrac{3}{2}x^2)(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})^2+\frac{1}{2m_\rho^{\,2}f^2}[(\partial_\mu\boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi})^2-(\partial_\mu\boldsymbol{\pi})^4]$$

$$+\frac{1}{m^2f^2}(\tfrac{3}{8}-\tfrac{3}{2}x+\tfrac{9}{8}x^2)(\partial_\mu\boldsymbol{\pi})^4$$

$$+\frac{1}{m^2f^2}[(\tfrac{3}{4}-\tfrac{9}{4}x+\tfrac{9}{4}x^2)(\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi})^2+(\tfrac{1}{2}-\tfrac{3}{4}x+\tfrac{9}{8}x^2)(\boldsymbol{\pi}\cdot\Box\boldsymbol{\pi})^2]+\frac{m^2}{8f^2}(x^2-x)\boldsymbol{\pi}^4\ . \tag{3.33}$$

In this model, therefore,

$$\frac{1}{e^2}=\frac{2f^2}{m_\rho^{\,2}} \tag{3.34}$$

and

$$\frac{\gamma}{e^2}=\frac{3f^2}{4m^2}\ . \tag{3.35}$$

Here (3.34) has been obtained by choice of β, to fit $g_{\rho\pi\pi}$, and (3.35) can be made consistent with (3.29) by choosing $m^2\approx\frac{1}{2}m_\sigma^{\,2}$. Note, however, that $m_A^{\,2}=m_\rho^{\,2}$.

An "intermediate" possibility might be to drop the $\rho\times\rho$, $A\times\rho$, and $A\times A$ parts from (3.7)—since these induce the unusual $\partial_\mu\rho_\nu(\partial^\mu\pi\times\partial^\nu\pi)$ coupling in (3.12), which makes the KSRF relation hard to satisfy. In this case we again find

$$\frac{1}{e^2}=\frac{2f^2}{m_\rho^{\,2}} \tag{3.36}$$

by choosing $\alpha=\frac{1}{2}$ $(\beta=2)$ to fit the KSRF relation. Then $m_A^{\,2}=2m_\rho^{\,2}$ and γ/e^2 has the value (3.28) with $\alpha=\frac{1}{2}$, $\beta=2$. For $m_\sigma\sim700$ MeV, we have the same difficulty in reconciling (3.28) and (3.29) as previously.

It seems fair to conclude from this exercise that the relation (3.36) for the Skyrme coefficient is remarkably model independent (it is amusing to note that it can also be written as $e=g_{\rho\pi\pi}$), while the $(\partial\Phi)^4$ term associated with the σ channel is much more model dependent. Perhaps this is not surprising. Although σ and π are treated here as a chiral multiplet, they are very different in quark-model terms, as are the A_1 and the ρ. Neither the σ nor the A_1 looks like a clearly isolated resonance. Probably strong hadronic corrections (boson loop graphs) are important in the σ channel.[23] This suggests that we should accept (1.5), and use $\gamma/e^2=f^2/m_\sigma^{\,2}$ (equivalently $\gamma=m_\rho^{\,2}/2m_\sigma^{\,2}$) where m_σ is a mass parameter (related to σ dynamics) which is simply to be fitted from experiment.

We now compare these $O((\partial\pi)^4)$ results of the meson field Lagrangian with the corresponding terms in the fermion-loop calculation considered in the preceding section. There we found $e(=g_{\rho\pi\pi})=2\pi$. This implies $g_{\rho\pi\pi}^{\,2}/4\pi=\pi$, which is within 10% of the value obtained from the ρ width. For the non-Skyrme parameter γ, the fermion loop gave $\gamma(\simeq m_\rho^{\,2}/2m_\sigma^{\,2})=\frac{1}{4}$, which implies $m_\sigma\simeq\sqrt{2}m_\rho\simeq1200$ MeV, a not unreasonable value. Thus we believe that the meson field and quark loop approaches are in remarkably close correspondence, up to $O((\partial\pi)^4)$ terms. Furthermore, they are both in accord with the low-energy phase shifts, or—alternatively—with the values for e^2 and γ extracted by Pham and Truong.[10]

IV. SOLITON STABILITY

This desirable equivalence, at $O((\partial\pi)^4)$, between the two approaches to $L(\pi)$ leaves one major problem unsolved: the soliton is unstable for the indicated values of e and γ (Refs. 7, 9, and 16). Because both theory and experiment, as we have argued, support these values, we must assume that higher-order terms in $(\partial\pi)$ are responsible for stability.

Terms of order $(\partial\pi)^6$ and higher are, of course, present in both approaches to $L(\pi)$. The $O((\partial\pi)^6)$ terms predicted by the fermion-loop approach have been calculated by Zuk,[24] and are being studied to see if they lead to stability (see Ref. 37 below). An alternative possibility has been suggested by Adkins and Nappi,[25] who showed that the soliton of L_2 can be satisfactorily stabilized by omitting all L_4 terms, and coupling the π's to the ω meson via the interaction $\omega_\mu B^\mu$, where B^μ is the (topological) baryon current.[26] Such a coupling leads naturally, within the meson Lagrangian approach of Sec. III, to an $O((\partial\pi)^6)$ term in $L(\pi)$, as we can see as follows.

We introduce the ω field tensor

$$\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu \tag{4.1}$$

and consider the additional Lagrangian

$$L_\omega=-\tfrac{1}{4}\omega_{\mu\nu}{}^2+\frac{m_\omega{}^2}{2}\omega_\mu{}^2+g_\omega\omega_\mu B^\mu\ , \tag{4.2}$$

where the Goldstone-Wilczek[26] current B^μ is

$$B^\mu=-\frac{1}{12\pi^2\phi^4}\epsilon_{\mu\nu\lambda\rho}\epsilon_{abcd}\phi_a\partial^\nu\phi_b\partial^\lambda\phi_c\partial^\rho\phi_d\ . \tag{4.3}$$

The equation of motion for the ω field is

$$[(\Box+m_\omega{}^2)g_{\mu\nu}-\partial_\mu\partial_\nu]\bar\omega^\nu=-g_\omega B^\nu \tag{4.4}$$

which leads to the expansion

$$\bar\omega^\nu=-\frac{g_\omega}{m_\omega{}^2}B^\nu+O(\partial^5) \tag{4.5}$$

and the effective vertex

$$L_{6,\omega}=-\frac{g_\omega{}^2}{2m_\omega{}^2}B_\mu B^\mu\ . \tag{4.6}$$

A numerical check shows that the static approximation used in (4.5) gives very similar results to the exact solution of (4.4) as given by Adkins and Nappi.[25] The leading term in (4.6) is of order $(\partial\pi)^6$:

$$L_{6,\omega}(\pi^6)=\frac{g_\omega{}^2}{48m_\omega{}^2\pi^4f^6}[(\partial^\mu\pi\cdot\partial_\mu\pi)^3-3(\partial^\mu\pi\cdot\partial_\mu\pi)(\partial_\nu\pi\cdot\partial_\rho\pi)(\partial^\nu\pi\cdot\partial^\rho\pi)+2(\partial_\mu\pi\cdot\partial^\nu\pi)(\partial_\nu\pi\cdot\partial^\rho\pi)(\partial_\rho\pi\cdot\partial^\mu\pi)]\ . \tag{4.7}$$

It is interesting that a term of exactly the form (4.6) has been proposed by Jackson,[27] in connection with studies of the nucleon-nucleon potential from Skyrmion physics. The $\omega_\mu B^\mu$ which led to (4.6) may also appear natural from some points of view. In the fermion-loop approach, such a term would arise naturally from a coupling of the form $\omega_\mu\bar\psi\gamma^\mu\psi$, since the expectation value of $\langle\bar\psi\gamma^\mu\psi\rangle$ in the soliton background is precisely B^μ (Ref. 28). Alternatively, following Kaymakcalan *et al.*,[14,29] we may consider how to extend the vector-meson-dominance Lagrangians, of the type discussed above, to include a representation (in terms of spin 1, in addition to spin 0, mesons) of the non-Abelian anomaly structure of QCD (Refs. 30 and 31). Such "gauged Wess-Zumino actions"[14,28,32–35] presumably represent a more faithful representation of QCD in terms of mesonic degrees of freedom: they certainly include an $\omega_\mu B^\mu$ term.

However, they also include an $\omega\rho\pi$ coupling. Such a term will induce an $O((\partial\pi)^6)$ contribution via the tree graph of Fig. 3, which is of order

$$\left[\frac{F_\omega}{\mu}\right]^2\frac{(\partial\pi)^6}{m_\omega{}^2m_\rho{}^4}\ , \tag{4.8}$$

where the combination (F_ω/μ) is the $\omega\rho\pi$ coupling constant $f_{\omega\rho\pi}$ (Ref. 36). Whereas previously $f_{\omega\rho\pi}$ was considered to be an arbitrary parameter, the considerations of Kaymakcalan *et al.*[14] indicate that F_ω/μ is of order g^3/f. In this case, Fig. 3 is of order $(g^2/m_\omega{}^2f^6)(\partial\pi)^6$, using (3.25). This, of course, is of exactly the same form as the $\omega_\mu B^\mu$ term (4.7); indeed, Kaymakcalan *et al.* conclude that (4.8) dominates (4.7).

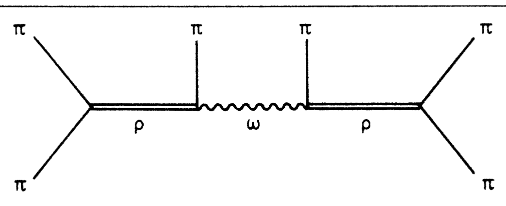

FIG. 3. Tree graph contributing to $O((\partial\Phi)^6)$.

For our present purpose, it is not necessary to investigate further the precise relative magnitudes of (4.7) and (4.8): their combined effect can be represented by a suitably interpreted g_ω. A more serious problem is that it does not seem possible to establish the dominance of a term of $L_{6,\omega}$ form, at $O((\partial\pi)^6)$. There are, of course, many other $(\partial\pi)^6$ terms generated by tree graphs of the L_m's we have considered in Sec. III. A simple dimensional argument indicates their order of magnitude. The expansion of the solutions of the equations of motion is conducted in powers of (∂/M) where M is the σ, ρ, or A_1 mass. The $O((\partial\pi)^4)$ terms have coefficients of order $(Mf)^{-2}$, and, hence, since each new π enters accompanied by f^{-1}, the coefficients of the $(\partial\pi)^6$ terms are $O((Mf)^{-4})$. Thus their ratio to (4.7) or (4.8) is $O(m_\omega{}^2f^2/M^4)$, which is certainly small. A detailed examination of tree graphs confirms this result (M^4 being replaced by products of the form $M_1{}^2M_2{}^2$ in appropriate cases), but unfortunately also reveals that most contributions are proportional to g^2, where g is the generic vector-meson coupling constant of the sort introduced in the previous section. The KSRF relation[20] then implies that the ratio of these contributions to (4.7) or (4.8) is $O(m_\omega{}^2m_\rho{}^2/M^4)$ which is no longer small.

Thus we are not able to justify any particular simple form for the $O((\partial\pi)^6)$ terms starting from the meson viewpoint L_m. Is the situation any different in the fermion-loop approach? We report on this in more detail elsewhere,[37] but the answer is no. The $O((\partial\phi)^6)$ terms in the fermion-loop expansion have been calculated by Zuk,[8] and when evaluated for the specific soliton ansatz used by Ripka and Kahana,[7] they do *not* match the exact Casimir energy as calculated by the latter, in the region of soliton size relevant for nucleon physics. Indeed, the exact energy does not appear to scale like any simple power in this region, indicating that many derivatives may be contributing. Although it is very interesting that the exact Casimir energy can provide stability of the chiral soliton at a sensible nucleon size,[7] the calculation—which is lengthy—has been performed only for a specific ansatz, rather than for a field shape dynamically determined by the action. Unfortunately this seems very difficult to remedy, due to the nonlocality of the action. That is why we hoped that a local truncation, at $O((\partial\phi)^6)$, might prove a reasonable approximation. In the meson field approach, by contrast, the field equations are local to begin with, and one is not obliged to make a derivative expansion.

While this work was being prepared for publication, we received a paper from Lai-Him Chan,[38] in which the $O((\partial\phi)^4)$ fermion-loop result is calculated and discussed in the Skyrmion context. The "non-Skyrme" contribution is shown to lead to a correct pion form factor, as given by ρ dominance. Thus this work supports our conclusion that the calculated $O((\partial\phi)^4)$ terms are in agreement with low-energy phenomenology. We thank Dr. Chan for sending us this paper.

Note added in proof. Professor T. N. Truong (private communication) agrees with us that the result (3.29) quoted from Ref. 10 is wrong by a factor of 2, so that (3.29) should read

$$\frac{\gamma}{e^2} \simeq \frac{2f^2}{3m_\sigma{}^2} \; .$$

However, this correction does not significantly alter the conclusions in the text, after (3.29) and (3.36).

ACKNOWLEDGMENTS

Two of us (I.J.R.A. and C.M.F.) are very grateful to Professor T. N. Truong, of the Ecole Polytechnique and Professor G. Ripka of CEN, Saclay, for the invitation to spend some days at their respective institutions, where we learned about the work described in Refs. 7 and 10; we thank Professor Ripka and Professor Truong for sending us these papers. We are also grateful to Professor Ripka for encouraging us to take the quark-loop result L_4^q more seriously, and for suggesting, in particular, that we calculate the associated π-π phase shifts. C.M.F. would like to thank Somerville College, Oxford, for financial support. P.J.M. acknowledges the Science and Engineering Research Council, United Kingdom for their financial support.

*Present address: TH Division, CERN, Geneva 23, Switzerland.

†Present address: Theoretical Physics Dept., Blackett Laboratory, Imperial College, London SW7 2BZ.

[1]E. Witten, Nucl. Phys. **B223**, 433 (1983).

[2]G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); **B75**, 461 (1974).

[3]E. Witten, Nucl. Phys. **B160**, 57 (1979).

[4]T. H. R. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. **31**, 556 (1962).

[5]See, for example, B. W. Lee, *Chiral Dynamics* (Gordon and Breach, New York, 1972).

[6]See, for example, B. R. Martin, D. Morgan, and G. Shaw *π-π Interactions in Particle Physics* (Academic, New York, 1976).

[7]G. Ripka and S. Kahana, Phys. Lett. **155B**, 327 (1985).

[8]R. I. Nepomechie, Ann. Phys. (N.Y.) **158**, 67 (1984); I. J. R. Aitchison and C. M. Fraser, Phys. Lett. **146B**, 63 (1984); R. Mackenzie, F. Wilczek, and A. Zee, Phys. Rev. Lett. **53**, 2203 (1984); J. Balog, Phys. Lett. **149B**, 197 (1984); P. Simic, Phys. Rev. Lett. **55**, 40 (1985); J. A. Zuk, Z. Phys. C **29**, 303 (1985); Ripka and Kahana (Ref. 7).

[9]R. Mackenzie, F. Wilczek, and A. Zee, Phys. Rev. Lett. **53**, 2203 (1984).

[10]T. N. Pham and T. N. Truong, Phys. Rev. D **31**, 3027 (1985).

[11]J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rev. Lett. **53**, 747 (1984).

[12]J. P. Blaizot, M. Chemtob, and V. Pasquier, Saclay Report No. PhT 85-038, 1985 (unpublished).

[13]E. Witten, in *Proceedings of the Lewes Workshop on Solitons in Nuclear and Elementary Particle Physics,* edited by A. Chodos, E. Hadjimichael, and C. Tze (World Scientific, Singapore, 1984).

[14]Ö. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984); H. Gomm, Ö. Kaymakcalan, and J. Schechter, *ibid.* **30**, 2345 (1984); Y. Brihaye, N. K. Pak, and P. Rossi, Nucl. Phys. **B254**, 71 (1985).

[15]W. Broniowski and M. K. Banerjee, Phys. Lett. **158B**, 335 (1985).

[16]I. J. R. Aitchison and C. M. Fraser, Phys. Rev. D **32**, 2190 (1985).

[17]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984); N. M. Cason *et al.*, Phys. Rev. D **28**, 1586 (1983); L. Rosselet *et al.*, *ibid.* **15**, 572 (1977).

[18]I. J. R. Aitchison and C. M. Fraser, Phys. Lett. **146B**, 63 (1984).

[19]K. Fujii, S. Otsuki, and F. Toyoda, Prog. Theor. Phys. **73**, 1287 (1985).

[20]K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16**, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[21]M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).

[22]For reviews, see S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969); J. J. Sakurai, *Currents and Mesons* (University of Chicago, Chicago, 1969).

[23]T. N. Truong, Acta Phys. Pol. **B15**, 633 (1984).

[24]Zuk (Ref. 8).

[25]G. S. Adkins and C. R. Nappi, Phys. Lett. **137B**, 251 (1984).

[26]J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 986 (1981).

[27]A. D. Jackson, invited talk at the Skyrmion Workshop, Cosener's House (RAL), Abingdon, U.K., 1984.

[28]I. J. R. Aitchison and C. M. Fraser, Phys. Rev. D **31**, 2605 (1985).

[29]Ö. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984).

[30]W. A. Bardeen, Phys. Rev. **184**, 1848 (1969).

[31]J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).

[32]E. Witten, Nucl. Phys. **B223**, 422 (1983).

[33]K. C. Chou, H. Y. Guo, and K. Wu, Phys. Lett. **134B**, 67 (1984).

[34]H. Kawai and S. Tye, Cornell report, 1984 (unpublished).

[35]N. K. Pak and P. Rossi, Nucl. Phys. **B250**, 279 (1985).

[36]M. Gell-Mann, D. Sharp, and W. E. Wagner, Phys. Rev. Lett. **8**, 261 (1962).

[37]I. J. R. Aitchison, C. M. Fraser, K. E. Tudor, and J. A. Zuk, Phys. Lett. **165B**, 162 (1985).

[38]L-H. Chan, Phys. Rev. Lett. **55**, 21 (1985).