Solitary waves in a Skyrmion-quark Lagrangian

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We investigate nontopological solitary wave solutions of a Skyrmion-quark Lagrangian. The quark wave functions are of the hedgehog type and the chiral angle corresponding to the classical pionic field goes to zero both at the origin and infinity. The Skyrme parameter is varied and nontopological solutions are found in a restricted range of values. The mass of the system is calculated and explicit solutions are shown for nodeless quark wave functions. The properties of the states are investigated.

I. INTRODUCTION

The Skyrme model[1–4] and its quantization[5] provides us with a tool to investigate baryon structure and reactions by identifying the topological charge of the classical solutions with baryon number and generating fermions by means of collective coordinate quantization.[5]

The Skyrmion-quark mixing in which fermions are part of the theory from the onset has been addressed by Nair and Rodgers.[6] They showed that the predictions of the theory improve the pure Skyrmion results when a sizable amount of mixing is allowed. Nevertheless their analysis is not fully consistent because the quark wave functions are not derived from the equations of motion, but are taken from bag-model predictions. In the limit when the stabilizing Skyrme parameter[4] e tends to infinity the Skyrmion-quark Lagrangian becomes the chiral-invariant [or partially conserved axial-vector current (PCAC)] nonlinear σ model.[7] This model has topological[8] as well as nontopological[9] solitary wave solutions. The question of the addition of a Skyrme-type term is then the issue. This is the subject under investigation in the present work.

It has been known since the late 1960s that Weinberg's Lagrangian[10] is a good phenomenological chiral model for pions and nucleons at the tree level. It is obtained from the linear σ model[7] by a chiral transformation and by taking the σ field to be infinitely heavy and therefore decoupling it from the theory completely without spoiling chiral invariance. The nucleon-pion coupling is a pseudovector and it yields good agreement with low-energy processes such as $\pi n \to \pi\pi n$[11] and s-wave $\pi\pi$ scattering. Nevertheless, d-wave $\pi\pi$ scattering needs the addition of Skyrme-type terms[12] to get agreement with the experimental data. These terms might be interpreted as originating from coupling to vector-meson degrees of freedom[13] or else they might arise in an effective theory in which fermions are integrated out.[14] It appears then natural to add them to the Lagrangian and investigate their influence on previously found solitary wave solutions.[15] In these solutions the baryonic number resides in the quarks only and the pion field acts as a confining agent. We will find that the Skyrme term does not preclude the existence of such solutions provided that the quark mass and the Skyrme parameter are kept within a certain range of values.

The states obtained are classical. The coupling to physical pions and other quantum excitations (like collective rotations) is obtained by expanding the Lagrangian around the classical solution.[16–18] The stability of this procedure against the quantum fluctuations is still an open question even for the topological Skyrmion.[19] In any case a baryon will be either a Skyrmion and its quantum fluctuations or a Skyrmion-quark composite and its quantum fluctuations or a mixture of both.[6] In the present work we focus on the latter, noting that a linear superposition of both the nontopological Skyrmion quark and the topological Skyrmion is not an eigenstate of the nonlinear classical equations of motion. Quantum effects may, however, admix them. Therefore, at the classical level each of them is a stable state.

Section II deals with the model Lagrangian, Sec. III deals with the states obtained, and Sec. IV examines some properties of the nontopological baryons.

II. THE MODEL LAGRANGIAN

Adding the Skyrme term[4] to Weinberg's Lagrangian[11] as applied to quarks and taking $g_v/g_A \sim 1$ to simplify the model we obtain

$$L = i\bar{q}\partial q - m_q\bar{q}q + (1+g^2\pi^2)^{-1}[g\bar{q}\,\gamma^\mu\gamma_5\tau\cdot\partial_\mu\pi q - g^2\gamma^\mu\bar{q}\tau\cdot(\pi\times\partial_\mu\pi)q] + \tfrac{1}{2}(1+g^2\pi^2)^{-2}\partial_\mu\pi\cdot\partial^\mu\pi$$
$$-\frac{4g^4}{e^2}[(\partial_\mu\pi\cdot\partial^\mu\pi)^2-(\partial_\mu\pi\cdot\partial^\nu\pi)^2](1+g^2\pi^2)^{-4}-\tfrac{1}{2}m_\pi{}^2\pi^2(1+g^2\pi^2)^{-1}\,, \quad (1)$$

where q is the quark field, π is the pion field, $g=(2f_\pi)^{-1}$, $f_\pi=93$ MeV, m_q is the quark mass, and m_π is the pion mass and the Skyrme term has been written in Weinberg's way.

The identification of the chiral angle θ for the classical solution is

$$\pi=2f_\pi \tan\left[\frac{\theta}{2}\right]\hat{\mathbf{r}} . \tag{2a}$$

Note that $\theta=(2N+1)\pi$ maps the pion field to infinity. If π is finite everywhere then no topological charge of the meson sector can arise for the present choice of pion fields. Equation (1) without the Skyrme term can be obtained from the linear σ model[10] or from the nonlinear σ model by chiral rotations on both the quark and pion fields.[11] The difference between both procedures appears in the symmetry-breaking pion-mass term. Phenomenologically symmetry-breaking effects seem to be better described using the former approach. We therefore chose that way in getting Eq. (1).

Solutions for the quark sector are obtained using the hedgehog ansatz[20]

$$|q\rangle=e^{-i\omega t}\psi(\mathbf{x})\,|\chi_h\rangle \tag{3}$$

obeying

$$(\sigma+\tau)\,|\chi_h\rangle=0 . \tag{4}$$

Another way of solving the equations of motion would be to integrate over the angular variables in the Lagrangian[21] obtaining only radial dependence before the variation of the Lagrangian is operated. We prefer to use the hedgehog ansatz to get an approximate picture of the nucleon and the $\Delta(1232)$ (Ref. 9). Calling

$$F(r)=\tan\left[\frac{\theta}{2}\right] \tag{5}$$

and for s-wave Direc fermions

$$\psi(\mathbf{x})=\begin{bmatrix} u(r) \\ i\sigma\cdot\hat{\mathbf{r}}v(r)\end{bmatrix} \tag{6}$$

the equations of motion become

$$(\omega-m_q)u-\dot{v}-\frac{2v}{r}=(1+F^2)^{-1}\left[u\left[\dot{F}+\frac{2F}{r}\right]-\frac{2v}{r}F^2\right], \tag{7}$$

$$(\omega+m_q)v+\dot{u}=(1+F^2)^{-1}\left[v\left[\dot{F}-\frac{2F}{r}\right]-\frac{2u}{r}F^2\right], \tag{8}$$

$$\begin{aligned}\ddot{F}\{1+32F^2g^2[e^2r^2(1+F^2)^2]^{-1}\}&+\dot{F}^2\{-2F(1+F^2)^{-1}+32g^2F(1-3F^2)[e^2r^2(1+F^2)^3]^{-1}\}\\ &+F^2\{4F[(1+F^2)r^2]^{-1}+32g^2F(F^2-1)[e^2r^4(1+F^2)^3]^{-1}\}+\frac{2\dot{F}}{r}-\frac{2F}{r^2}-m_\pi{}^2F=4n_qg^2m_quv(1+F^2) ,\end{aligned} \tag{9}$$

where n_q is the number of quarks.

Using Eq. (5) we can connect the Skyrme model[5] to obtain

$$(\omega-m_q)u-\dot{v}-\frac{2v}{r}=u\left[\frac{\dot{\theta}}{2}+\frac{\sin(\theta)}{r}\right]-\frac{2v}{r}\sin^2\left[\frac{\theta}{2}\right], \tag{7'}$$

$$(\omega+m_q)v+\dot{u}=v\left[\frac{\dot{\theta}}{2}-\frac{\sin\theta}{r}\right]-\frac{2u}{r}\sin^2\left[\frac{\theta}{2}\right], \tag{8'}$$

$$\begin{aligned}\left[\cos^2\left[\frac{\theta}{2}\right]\right]^{-1}&\left[\ddot{\theta}\left[\frac{1}{4}+g^2\frac{2\sin^2\theta}{e^2r^2}\right]+g^2\frac{\dot{\theta}^2\sin(2\theta)}{e^2r^2}-\frac{\sin(2\theta)}{4r^2}+\frac{\dot{\theta}}{2r}-g^2\frac{\sin^2(\theta)\sin(2\theta)}{e^2r^4}-m_\pi{}^2\sin\theta\right]\\ &=\left[\cos^2\left[\frac{\theta}{2}\right]\right]^{-1}2n_qg^2m_quv .\end{aligned} \tag{9'}$$

Equation (9′) is the Skyrmion equation[4] when $m_q=0$ if we multiply it by $\cos^2(\theta/2)$. In doing so we assume $\theta\neq(2n+1)\pi$.

Calling $\tilde{u},\tilde{v}$ the upper and lower components of the Dirac field corresponding to the solution of the pseudoscalar coupling of Nair and Rodgers[6] we can identify (ignoring symmetry-breaking effects)

$$u=-\left[\tilde{u}\cos\left[\frac{\theta}{2}\right]+\tilde{v}\sin\left[\frac{\theta}{2}\right]\right], \tag{10}$$

$$v=\tilde{u}\sin\left[\frac{\theta}{2}\right]-\tilde{v}\cos\left[\frac{\theta}{2}\right] \tag{11}$$

which shows the equivalence between both models in the nontopological regime up to a chiral rotation.

The baryon number is

$$B=B_\pi+B_q \ , \tag{12}$$

where B_π is the topological charge of the Skyrme model and

$$B_q=\sum_{q_i} B_{q_i} \int \bar{\psi}_{q_i}\gamma^0\psi_{q_i} d^3x \tag{13}$$

which is equal to one if the usual assignment of quark baryon number is taken, namely, $B_{q_i}=\frac{1}{3}$ and the wave functions are normalized:

$$\int \bar{\psi}\gamma^0\psi\, d^3x=1 \ . \tag{14}$$

Equation (14) serves as a constraint during the calculation.

We seek to find solutions for which $\theta\to 0$ both at the origin and infinity, i.e., $B_\pi=0$. Topological solutions of Eqs. (7)—(9) are not accessible; for then the pion field diverges either at the origin or infinity. The $B_\pi\neq 0$ sector can be reached through a chiral rotation of the meson field. In particular the pion field of the nonlinear σ model is obtained from the pion field of Eq. (1) by the nonlinear transformation

$$\boldsymbol{\pi}=\frac{2f_\pi\boldsymbol{\pi}'}{f_\pi\pm(f_\pi{}^2+\boldsymbol{\pi}'^2)^{1/2}} \ . \tag{2b}$$

This new pion field is now restricted to $0<|\boldsymbol{\pi}'|<f_\pi$ and has topological solutions with $B_\pi=2,3,\ldots$ (Refs. 8 and 22). The effect of this apparently harmless chiral rotation is to cut the chiral angle axis into the disconnected branches of $\tan(\theta/2)$. It is by this procedure that the nontopological regime becomes isolated.

We note also that although the pseudoscalar and pseudovector couplings give chirally equivalent equations the lowest-order phenomenology of both is different because they consider chirally rotated pion and quark fields.[11] The absolute sign of the pionic field in Eqs. (7)—(9) is no longer arbitrary[4] but is fixed to be negative in order to confine the quarks.

III. SOLUTIONS

At the origin the waves are given by

$$u(r)=\alpha-\frac{\beta r^2}{2} \ , \tag{15a}$$

$$v(r)=\gamma r \ , \tag{15b}$$

$$F=\delta r \ , \tag{15c}$$

where $\alpha,\beta,\gamma,\delta$ obey the coupled algebraic equations

$$\gamma=\left[\left[\frac{\omega-m_q}{3}\right]+\delta\right]\alpha \ , \tag{16a}$$

$$\beta=(\omega+m_q+\delta)\gamma+2\delta^2\alpha \ , \tag{16b}$$

$$\delta^3\left[1-\frac{64g^2\delta}{e^2}\right]-\frac{m_\pi{}^2\delta}{2}=2n_q g^2 m_q\gamma\alpha \ . \tag{16c}$$

At infinity a bounded quark obeys

$$\frac{v(r\to\infty)}{u(r\to\infty)}=\left[\frac{(m_q-\omega)}{(m_q+\omega)}\right]^{1/2} \ . \tag{16d}$$

Equations (15) and 16) are used to generate the initial conditions for the solutions. We use the method of Goldflam and Wilets[23] based upon the matching between solutions at a certain matching radius. The convergence of the numerical procedure is rather slow. The existence of a nontopological solution appears quite clearly. The parameter varied in the search is δ. A variation of δ to higher (lower) values must yield asymptotically a chiral angle of $\pm\pi$ ($\mp\pi$) in order to ensure the existence of a solution whose asymptotic angle is zero. Equations (7)—(9) were solved up to 7 fm in order to obtain the correct asymptotic behavior.

The energy of the system is

$$E=E_q+E_{\pi q}+E_\pi \tag{17a}$$

$$\quad =E_\pi+n_q\omega \ , \tag{17b}$$

where E_π is the pion kinetic and mass energy

$$E_\pi=\frac{4\pi}{g^2}\int dr((1+F^2)^{-2}\{F^2+\tfrac{1}{2}\dot{F}^2r^2 +8g^2(F^4+2\dot{F}^2F^2r^2)\times[er(1+F^2)]^{-2}\} +\tfrac{1}{2}m_\pi{}^2F^2(1+F^2)^{-1}) \ . \tag{18a}$$

E_q is the quark kinetic energy

$$E_q=4\pi n_q\int r^2dr\left[m_q(u^2-v^2)+u\dot{v}+\frac{2uv}{r}-\dot{u}v\right] \ . \tag{18b}$$

$E_{\pi q}$ is the pion-quark interaction energy

$$E_{\pi q}=4\pi n_q\int r^2dr(1+F^2)^{-1}\left[u^2\left[\frac{2F}{r}+\dot{F}\right]-\frac{4uv}{r}F^2 -v^2\left[\frac{2F}{r}-\dot{F}\right]\right] \ . \tag{18c}$$

Equation (17b) is obtained from Eq. (17a) using the equations of motion. The equality between both equations gives, therefore, a measure of the accuracy achieved. In all the cases the accuracy is better than 0.5%.

The number of quarks n_q is in principle unknown and we used $n_q=3$ as usual. We also made various search scans for $n_q=2$ and $n_q=1$. The result for these scans (although not very thorough) indicates the inexistence of nontopological solutions. It would be very daring to conclude anything about a possible dynamical reason for taking $n_q=3$ from this result. The reason why $n_q<3$ fails to yield solutions is because the strength of the source term in the right-hand side (RHS) of Eq. (9) is unable to "bend back" the pion field to $\theta=0$ (or $\pi=0$) and at the same time the resulting pion field is unable to confine the quarks.

An important characteristic of the model is that it lacks any scaling property that preserves the normalization of

TABLE I. Energy quantities (in MeV) of several solutions for different quark masses and Skyrme parameter.

m_q (MeV)	Skyrme parameter e	Quark eigenvalue ω (MeV)	E_q	E_π	$E_{\pi q}$	Total energy
442	20	350	1726	565	-674	1617
395	30	262	1307	745	-519	1534
355	300	280	1090	651	-248	1493

the quark waves. The scales in the problem are set by $f_\pi \sim m_\pi/2$, m_q, and ef_π.

We base the solutions in previous results,[15] found in the range $355 < m_q < 442$ MeV for $e \to \infty$. The pure Skyrmion[5] needs $e \sim 5$. We searched for solutions beginning with $e = 300 - \infty$ and relaxed the limits on the mass range. The nontopological character of the solutions was checked using Eqs. (7′)–(9′) to ensure $\theta(0) = \theta(\infty) = 0$. A simpler check consisted in noting that when $F < 1$ for all r then $\theta < \pi/2$ for all r. If $F > 1$ for some r then the direct check using Eqs. (7′)–(9′) must be carried on.

Table I shows the energies of three solutions. Solutions exist for

$$60 < e < \infty, \quad m_q = 355 \text{ MeV} ,$$

$$30 < e < \infty, \quad m_q = 395 \text{ MeV} ,$$

$$20 < e < \infty, \quad m_q = 442 \text{ MeV} ,$$

and $355 < m_q < 442$ MeV only. The range in e is in turn connected to the remark of Manohar and Georgi[24] that suggested that a pure Skyrmion should have an e value in a different range compared to the case in which both quarks and mesons are present. Although the lowest value for which we found solutions $e = 20$ is not much higher than $e \sim 5$ it affects the Lagrangian in its second power, i.e., a factor of 16. In any case the fact that solutions cease to appear for $e < 20$ is certainly meaningful.

Figures 1–3 show the upper and lower quark component and the pionic fields corresponding to Table I. By decreasing e, v increases and in some cases the scalar density $u^2 - v^2$ becomes negative. The pionic field slope at the origin increases by decreasing e and below the lower threshold the quarks are unbound. Quarks are confined to a region of around 0.6 fm and $u(0)$ is almost in all cases $u(0) \sim 1.5$ fm$^{-3/2}$. The energy of the state is approximately 1.5 GeV. The baryon obtained can be considered to be a mixture of a nucleon and Δ (Ref. 9) as implied by the quark hedgehog wave function [Eq. (4)], namely,

$$|H\rangle = \chi_h \chi_h \chi_h = \tfrac{1}{2} p_{-1/2} - \tfrac{1}{2} n_{1/2} + \frac{1}{2\sqrt{2}} (\Delta^0_{1/2} - \Delta^+_{-1/2} + \Delta^{++}_{-3/2} - \Delta^-_{3/2}) . \quad (19)$$

The hedgehog mass can be lowered by using an effective coupling constant g different from the one derived from the pion-decay constant. Such an approach proves

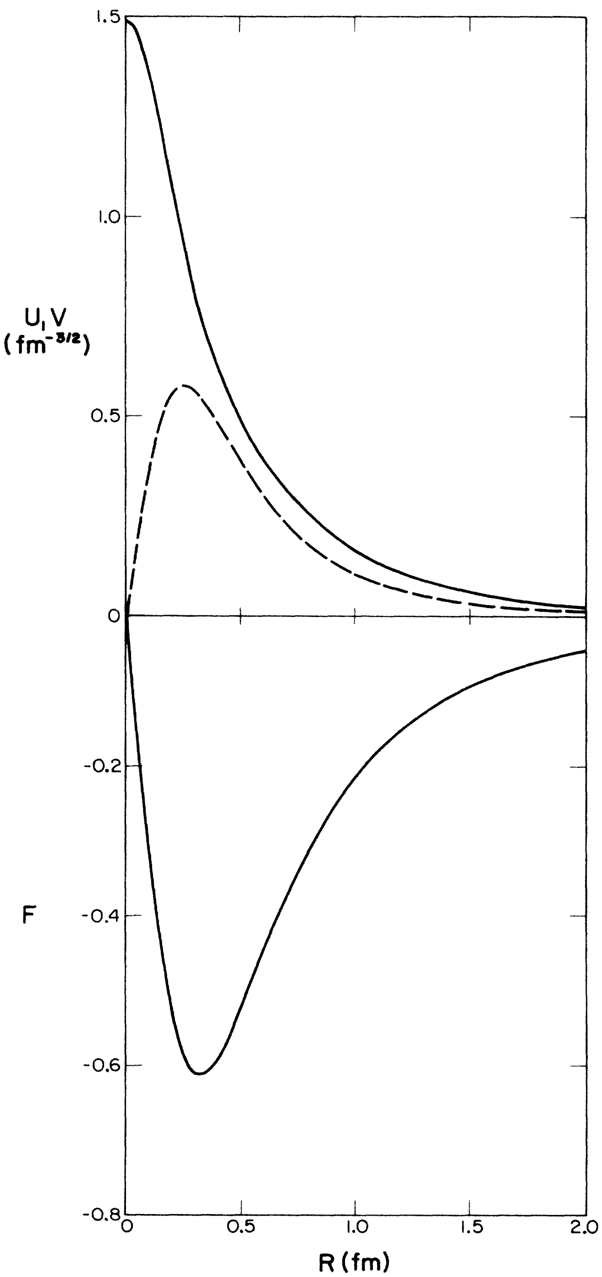

FIG. 1. The positive solid line corresponds to the upper component of the quark wave function u and the positive dashed line corresponds to the lower component of the quark wave function v [see Eq. (6)]. The negative solid line shows the pion field $F(r)$ obtained solving Eq. (9). The Skyrme parameter is $e = 20$ and $m_q = 442$ MeV.

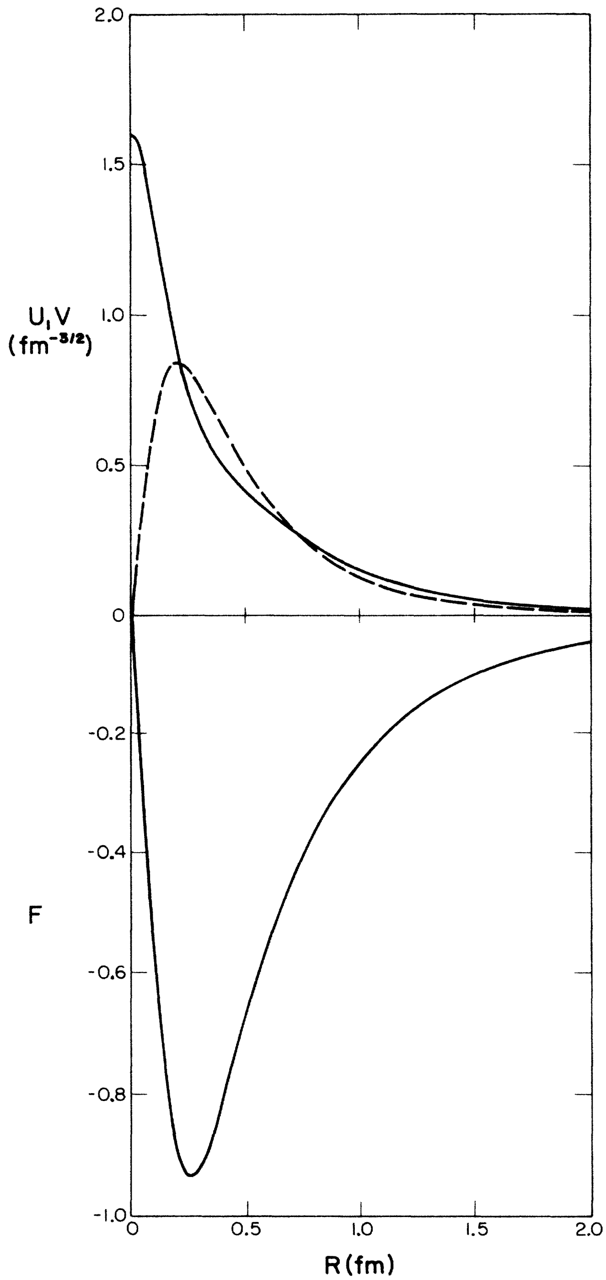

FIG. 2. Same as Fig. 1 for $e=30$ and $m_q=395$ MeV.

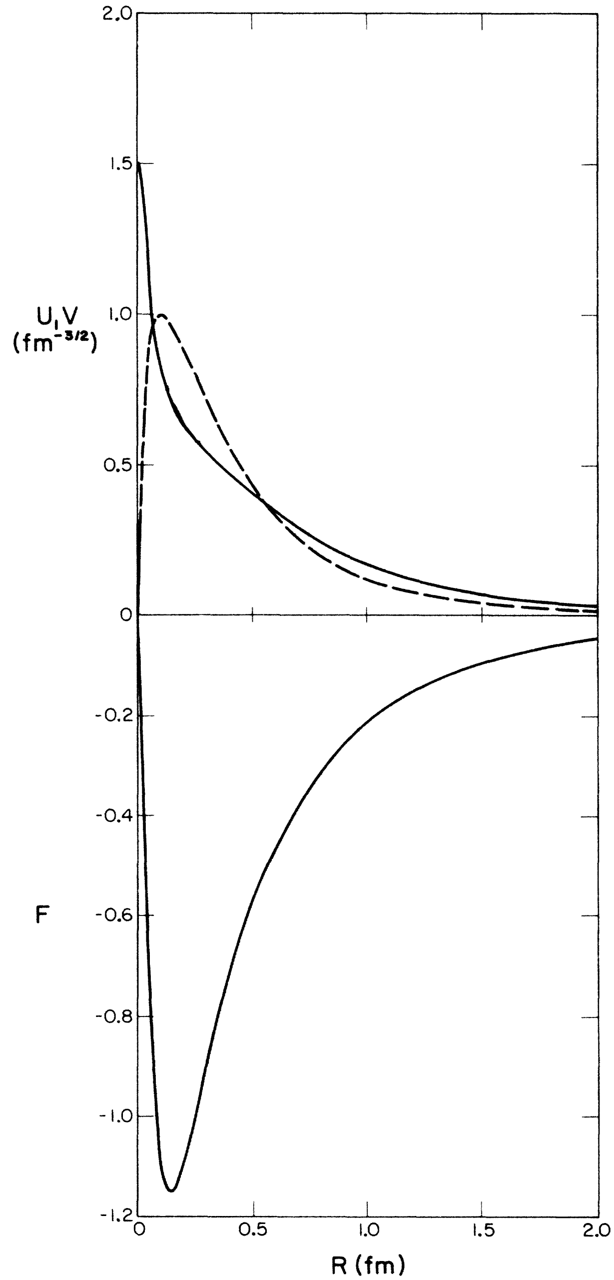

FIG. 3. Same as Fig. 1 for $e=300$ and $m_q=355$ MeV

to be useful in the topological Skyrmion[5] model in order to fit the nucleon and Δ masses. If the measured value of f_π is used the topological Skyrmion mass turns out to be around 1.4 GeV. A decrease in the value of f_π is also suggested by the tree-level calculation of the $\pi\pi$ elastic scattering amplitudes.[12] If instead of a nonlinear model a linear σ model is used,[9] the baryon mass turns out to be lower (around the average nucleon-Δ mass) without resorting to a phenomenological decrease in f_π. The reason can be traced to the fact that in the linear σ model the σ field is not confined to the chiral circle $\sigma^2+\pi^2=f_\pi{}^2$ as in all the nonlinear models.

Table II shows the results obtained by decreasing f_π for $e=40$ and $m_q=395$ MeV. For $f_\pi<75$ MeV the hedgehog state becomes stable against pion emission. Even for this favorable case the quantum tunneling of the quarks through the potential created by their interaction with the classical pion field is still possible, though it will need a high degree of correlation between the quarks for it to occur. A definite answer regarding the stability can be given only in the context of a systematic quantum expansion.

Looking at Eq. (8′) we note that an increase in g is equivalent (at the level of equations of motion) to an in-

TABLE II. Hedgehog mass for various values of f_π, $e=40$, and $m_q=395$ MeV.

Effective f_π (MeV)	Hedgehog mass (MeV)
93	1521
88	1470
84	1431
81	1392
78	1355
75	1320

crease in n_q. As Friedberg and Lee[25] have shown, it is a general property of nontopological soliton models that the soliton-baryon state becomes classically stable for sufficiently high n_q. The decrease in the hedgehog mass is therefore expected.

Further decrease of f_π below 75 MeV induces instabilities in the numerical procedure. Thus, we cannot ascertain whether classically stable solutions exist in the present model.

IV. PROPERTIES OF THE HEDGEHOG BARYON

We investigate the properties of the hedgehog baryon omitting the Skyrme term because its contribution is negligible. The axial charge g_A can be defined by

$$g_A\left\langle n_{1/2}\left|\frac{\sigma^i\tau^a}{2}\right|n_{1/2}\right\rangle=\int\langle n_{1/2}\,|\,A_i^a\,|\,n_{1/2}\rangle d^3x\ , \tag{20}$$

where A_i^a is the axial-vector current of isospin (spin) index $a\,(i)$. In the present model

$$\begin{aligned}A_\mu^a=&(1+g^2\pi^2)^{-1}\sum_i\bar{q}_i\left[g(\tau\times\pi)_a\gamma_5+g_2\tau\cdot\pi\pi_a+\frac{\tau_a}{2}(1-g^2\pi^2)\right]\gamma_\mu\gamma_5q_i\\&+(1+g^2\pi^2)^{-2}\left[g\pi\cdot\partial_\mu\pi\pi_a+\frac{1}{2g}(1-g^2\pi^2)\partial_\mu\pi_a\right].\end{aligned} \tag{21}$$

Equation (21) can be obtained from the linear model axial-vector current

$$A_\mu^a=\sum_i\bar{q}\,'_i\gamma^\mu\gamma_5\tau_aq_i'+\sigma\partial_\mu\pi_a'-\pi_a'\partial_\mu\sigma \tag{22}$$

applying the chiral transformations[11]

$$q'=Sq=\left[\frac{f_\pi+\sigma+i\tau\cdot\pi\gamma_5}{[2f_\pi(f_\pi+\sigma)]^{1/2}}\right]q\ , \tag{23a}$$

$$\pi'=\frac{2f_\pi\pi}{f_\pi+\sigma}\ , \tag{23b}$$

and using the constraint

$$\sigma=(f^2-\pi^2)^{1/2}\ . \tag{23c}$$

Using the Wigner-Eckart theorem for the quark sector of the vector-isovector operator A_i^a we find

$$\langle n_{1/2}\,|\,A_i^a\,|\,n_{1/2}\rangle_q=\tfrac{5}{3}\langle\chi_h\,|\,A_i^a\,|\,\chi_h\rangle\ . \tag{24}$$

The meson-sector contribution to g_A is calculated using the approach of Ref. 9, obtaining

$$\begin{aligned}g_A=\frac{20\pi}{3}\int r^2dr&\left\{u^2\left[\cos^2\left[\frac{\theta}{2}\right]-\tfrac{1}{3}\sin^2\left[\frac{\theta}{2}\right]\right]+v^2\left[\sin^2\left[\frac{\theta}{2}\right]-\tfrac{1}{3}\cos^2\left[\frac{\theta}{2}\right]\right]-\tfrac{4}{3}uv\sin\theta\right\}\\&+\tfrac{4}{9}\pi f_\pi{}^2\int r^2dr\left[\theta'+\frac{\sin(2\theta)}{r}\right].\end{aligned} \tag{25}$$

Equation (25) can be obtained also from Eq. (22) using the transformations of Eq. (23) or equivalently Eqs. (10) and (11) and Eq. (5).

Table III shows the results of the calculation of g_A for the hedgehog states of Table I. In all the cases investigated (including others not shown) we found $g_A\sim1.5$ which is 10% higher than the experimental value. Most of the contribution to g_A comes from the quark-meson part of Eq. (25). The pure meson contribution amounts to only 10%.

Another important quantity relevant to the comparison of the hedgehog state with the nucleon is the magnetic moment. The electromagnetic current is

$$\begin{aligned}j_\mu&=j_\mu(q)+j_\mu(\pi)\\&=\tilde{e}\left[\tfrac{1}{6}\bar{q}\gamma^\mu(1+3\tau_z)q+(\pi\times\partial_\mu\pi)_z\right]\ ,\end{aligned} \tag{26}$$

$\tilde{e}$ being the proton charge.

Proceeding in the way the calculation of g_A was carried out (separating isovector and isoscalar contributions) we obtain

$$\begin{aligned}\mu_p=&\,\tilde{e}\frac{4\pi}{3}\int\left[\frac{\sin(\theta)}{2}(u^2-v^2)+uv\cos(\theta)\right]r^3dr\\&+\frac{2\pi\tilde{e}}{9}f_\pi{}^2\int\sin^2(\theta)r^2dr\ .\end{aligned} \tag{27}$$

Table III shows the results of calculating $(2M/\tilde{e})\mu_p$ and $(2M/\tilde{e})\mu_n$, where M is the hedgehog mass for each case. The magnetic moments are sensitive to the value of e and f_π. For $e=20$ and $m_q=442$ MeV the values obtained are

TABLE III. Properties of the hedgehog baryon.

e	m_q (MeV)	g_A	$\frac{2M}{\tilde{e}}\mu_p$	$\frac{2M}{\tilde{e}}\mu_n$
20	442	1.56	2.68	−1.98
30	395	1.54	3.18	−2.35
300	355	1.54	2.905	−2.12

very close to the experimentally measured values of 2.79 and -1.91 for the proton and neutron magnetic moments. The properties investigated show that the soliton-quark nontopological state bears some relation to the physical nucleon if its hedgehog character is taken into account by means of the decomposition of Eq. (19).

In summary, the present model shows an alternative way of constructing baryons using classical chiral meson fields that have trivial topology.

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