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Dimensional reduction and theories with massive gauge fields

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We generalize the three-dimensional (Abelian) model of Deser, Jackiw, and Templeton with a gauge-invariant mass term to higher odd dimensions using antisymmetric tensor gauge fields. We show that this mass-generation mechanism for gauge fields is related through dimensional reduction to the gauge-mixing mechanism of Aurilia and Takahashi. We comment on possible scenarios for non-Abelian generalization.

I. INTRODUCTION

Recently, there has been a lot of interest in three-dimensional gauge theories.[1] These theories have interesting properties. One of these is the possibility of introducing a gauge-invariant mass term, which in the non-Abelian case can be identified with a topological quantity related to the Chern-Simons secondary class. In the Abelian theory, this additional term is responsible for the existence of a long-range potential whose field strength is short ranged, reminiscent of the Nielsen-Olesen string.[2] These investigations have led the authors of Ref. 1 to suggest that this might be an alternative to the Higgs mechanism for the generation of mass for gauge particles. An interesting question is whether it is possible to generalize such theories to higher odd dimensions. The next higher odd dimension, five, is of particular importance from the point of view of Kaluza-Klein type of theories.

There is another mechanism for the generation of mass for gauge particles in even dimensions, called the gauge-mixing mechanism[3] and has also been suggested as an alternative to the Higgs mechanism. In four dimensions, for example, the vector gauge field A_μ and a second-rank antisymmetric tensor gauge field $A_{\mu\nu}$ are included in the Lagrangian in a gauge-invariant way, leading to a massive particle, with spin 1. It has been pointed out[3] that mass arises out of the presence of a gauge-invariant current whose divergence contains an "anomaly" similar to the axial anomaly.

Can these two mechanisms for the mass of gauge particles in odd and even dimensions be related to each other?

In this paper we attempt to answer the two questions posed above. We obtain a generalization of the Deser-Jackiw-Templeton (DJT) model for gauge-invariant mass to odd dimensions higher than three by using antisymmetric tensor fields. We also obtain through dimensional reduction of these generalized DJT models the even-dimensional Aurilia-Takahashi models with gauge mixing, thus relating the two different mechanisms for gauge-invariant masses of gauge particles.

As mentioned above, we need antisymmetric tensor gauge fields to generalize the DJT model to higher odd dimensions. For $4n-1$ dimensions ($n=1,2,\ldots$) we need a tensor field of rank $2n-1$, whereas in $4n+1$ dimensions ($n=1,2,\ldots$) we need a *complex* tensor field of rank $2n$, as will be discussed below.

The technique of dimensional reduction was first used in dual models and later also exploited in supergravity.[4] Usually what is done is that the action in a higher-dimensional theory is reduced to lower dimensions by expanding the fields in normal modes corresponding to the compactified extra dimensions, and integrating out the extra dimensions.[4,5] This forms the basis of the modern Kaluza-Klein type of theories.[6] However, in the dimensional reduction used in this paper we consider the fields in higher dimensions to be independent of the extra dimensions to achieve our purpose.

At first sight it may not seem surprising, in view of the relation between Chern-Simons secondary invariants in odd dimensions and chiral anomalies in even dimensions,[7] that the mass term in generalized DJT models in odd dimensions is related to the mass term in gauge-mixing models in even dimensions. However, the Chern-Simons secondary invariant is related to the chiral anomaly in one dimension higher, whereas we related the gauge-invariant mass term in odd dimensions to an anomaly-like term in one dimension lower, through dimensional reduction.[8]

The contents of the rest of the paper are organized as follows. In Sec. II we generalize the DJT model to higher odd dimensions. In Sec. III we reduce the generalized DJT models to even dimensions and relate them to models using the gauge-mixing mechanism. We analyze in Sec. IV both the cases, and relate the "anomaly" in the gauge-mixing mechanism to the stringlike long-range potential of the generalized DJT models. In the last section, Sec. V, we present a discussion of our results and point out further interesting possibilities, especially in non-Abelian theories.

II. GENERALIZATION OF THE DJT MODEL

The three-dimensional, Abelian DJT model is

$$L=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{\mu}{4}\epsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha\ , \tag{1}$$

where the second term in L is the gauge-invariant (apart from surface terms) mass term.[9] This term, apart from the kinetic energy term, is the only one which is gauge invariant and quadratic in the field. If we wish to generalize this to the next odd dimension (five), we need to use an antisymmetric tensor gauge field $A_{\mu\nu}$. The corresponding mass term, by analogy with (1), is

$$L_M = \frac{\mu}{2\times 2!3!}\epsilon^{\lambda\mu\nu\rho\sigma}F_{\lambda\mu\nu}A_{\rho\sigma} , \tag{2}$$

where $F_{\lambda\mu\nu}=\partial_\lambda A_{\mu\nu}+\partial_\mu A_{\nu\lambda}+\partial_\nu A_{\lambda\mu}$. However there is a difficulty, since L_M in Eq. (2) is a five-divergence, and does not contribute. In seven dimensions, however, this generalization works, and we can write a mass term using a third-rank antisymmetric tensor $A_{\lambda\mu\nu}$:

$$L_M = \frac{\mu}{2\times 3!4!}\epsilon^{\kappa\lambda\mu\nu\rho\sigma\tau}F_{\kappa\lambda\mu\nu}A_{\rho\sigma\tau} , \tag{3}$$

where

$$F_{\kappa\lambda\mu\nu}=\partial_\kappa A_{\lambda\mu\nu}-\partial_\lambda A_{\mu\nu\kappa}+\partial_\mu A_{\nu\kappa\lambda}-\partial_\nu A_{\kappa\lambda\mu} .$$

The Lagrangian is invariant (up to surface terms) under the gauge transformation

$$A_{\lambda\mu\nu}\rightarrow\partial_\lambda\Lambda_{\mu\nu}+\partial_\mu\Lambda_{\nu\lambda}+\partial_\nu\Lambda_{\lambda\mu} .$$

The particle content of the model can be worked out easily. It describes a particle of mass μ in seven dimensions. It is thus possible to generalize the DJT model easily to $4n-1$ dimensions ($n=1,2,\ldots$) using an antisymmetric tensor of rank $2n-1$. The mass term is of the form

$$L_M = \frac{\mu}{2\times(2n)!(2n-1)!}\epsilon^{\mu_1\mu_2\cdots\mu_{4n-1}}F_{\mu_1\mu_2\cdots\mu_{2n}}A_{\mu_{2n+1}\cdots\mu_{4n-1}} , \tag{4}$$

where $F_{\mu_1\mu_2\cdots\mu_{2n}}=\partial_{\mu_1}A_{\mu_2\cdots\mu_{2n}}$ + cyclic permutations of $\mu_1\cdots\mu_{2n}$. L_M is invariant under the gauge transformation $A_{\mu_1\cdots\mu_{2n-1}}\rightarrow\partial_{\mu_1}\Lambda_{\mu_2\cdots\mu_{2n-1}}\mp$cyclic permutations of $\mu_1\cdots\mu_{2n-1}$.

A generalization to $4n+1$ dimensions is not possible using a real antisymmetric tensor of rank $2n$ since this leads to a term like (2), which is a divergence. We can avoid this problem by using complex fields. For example, in five dimensions, the Lagrangian

$$L=\frac{1}{3!}F^*_{\lambda\mu\nu}F^{\lambda\mu\nu}+\left[\frac{i\mu}{2\times 3!2!}\epsilon^{\lambda\mu\nu\rho\sigma}F_{\lambda\mu\nu}A^*_{\rho\sigma}+\text{H.c.}\right] \tag{5}$$

can be considered as a generalization of the DJT model. The equations of motion following from (5) are

$$\partial_\alpha F^{\alpha\beta\gamma}-\frac{i\mu}{6}\epsilon^{\lambda\mu\nu\beta\gamma}F_{\lambda\mu\nu}=0 , \tag{6}$$

together with the Bianchi identities

$$\partial_\alpha\tilde{F}^{\alpha\beta}=0 , \tag{7}$$

where

$$\tilde{F}_{\alpha\beta}=\tfrac{1}{6}\epsilon_{\lambda\mu\nu\alpha\beta}F^{\lambda\mu\nu} .$$

Using (6) and (7) one can arrive at

$$(\partial^2+\mu^2)\tilde{F}_{\alpha\beta}=0 \tag{8}$$

which establishes the mass of the particle as μ. However, since $A_{\mu\nu}$ is complex, the Lagrangian (5) would represent two particles, each of mass μ. It is not difficult to see that the same trick works in an arbitrary dimension $4n+1$, with complex antisymmetric tensor field of rank $2n$.

In three dimensions, DJT have also generalized their model to the non-Abelian case. It is natural to ask if this non-Abelian model can be generalized to higher dimensions. Unfortunately, we do not have Lagrangians for antisymmetric tensors with genuinely non-Abelian gauge invariance and attempts to construct non-Abelian theories have not been successful so far.[10]

Having described the generalization of the DJT model, we shall now consider the dimensional reduction of these models in the next section.

III. DIMENSIONAL REDUCTION OF THE DJT MODELS

As explained in Sec. I, dimensional reduction is usually done by considering the normal-mode expansion of the fields in extra dimensions. However, we consider a restricted class of reduction, viz., the one in which the fields are independent of the extra dimensions.

First, we shall consider the original DJT Lagrangian for the Abelian theory, viz., the Lagrangian in Eq. (1). This Lagrangian has the gauge invariance

$$A_\mu\rightarrow A_\mu+\partial_\mu\Lambda . \tag{9}$$

To perform a dimensional reduction to two dimensions we split A_μ as (A_a,ϕ) $(a=0,1)$, with $\phi=A_2$. We assume A_a and ϕ to be independent of x_2. Then

$$F_{a2}=-F_{2a}=\partial_a\phi .$$

The Lagrangian then reduces to

$$L=-\tfrac{1}{4}F_{ab}F^{ab}+\tfrac{1}{2}(\partial_a\phi)^2-\mu\tilde{F}\phi , \tag{10}$$

where $\tilde{F}=-\frac{1}{2}\epsilon^{ab}F_{ab}$, and where surface terms have been dropped. The latin indices go over 0 and 1. This Lagrangian is indeed the gauge-mixed model in two dimensions considered by Aurilia and Takahashi.[3] It is also the familiar Lagrangian corresponding to the bosonized version of the Schwinger model,[11] following the bosonization prescription of Coleman and Mandelstam.[12] Note that the bosonized Schwinger model incorporates the anomaly of the fermionic axial-vector current. We will consider this aspect in the next section.

Now let us take up the generalized DJT model in five dimensions whose Lagrangian is given by (5). The Lagrangian has the gauge invariance

$$A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \; . \tag{11}$$

To go over to four dimensions we treat the fields to be independent of the extra coordinate x_5. With the definition

$$A_{a5} = -A_{5a} = A_a \quad (a = 0,1,2,3)$$

we get the Lagrangian (on dropping surface terms)

$$L = \tfrac{1}{6} F^*_{abc} F^{abc} - \tfrac{1}{2} F^*_{ab} F^{ab} + \left[\frac{i\mu}{6} \epsilon_{abcd} F^{abc} A^{d*} + \text{H.c.} \right] , \tag{12}$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$, and the latin indices go over 0,1,2,3. If we rewrite in terms of real and imaginary parts of the various fields appearing in (12), we can obtain the gauge-mixing model,[3] mixing the real part of A_{ab} with the imaginary part of A_a, and the imaginary part of A_{ab} with the real part of A_a.

To see the particle content of the Lagrangian (12), we write the field equations and the Bianchi identities

$$\partial_a F^{abc} + i\mu \widetilde{F}^{bc} = 0 \; , \tag{13a}$$

$$\partial_a F^{ab} + i\mu \widetilde{F}^{b} = 0 \; , \tag{13b}$$

$$\partial_a \widetilde{F}^{ab} = 0 \; , \tag{13c}$$

$$\partial_a \widetilde{F}^{a} = 0 \; , \tag{13d}$$

where

$$\widetilde{F}^{ab} = \tfrac{1}{2} \epsilon^{abcd} F_{cd}, \quad \widetilde{F}^{a} = \tfrac{1}{6} \epsilon^{abcd} F_{bcd} \; .$$

These could of course, have been obtained by dimensional reduction of (6) and (7). Using the above equations, we can arrive at

$$(\partial^2 + \mu^2) \widetilde{F}_{ab} = 0 \; . \tag{14}$$

We also get a similar equation for $\widetilde{F}^{ab*}$. In contrast to the model of Ref. 3 which describes a self-conjugate particle, wet get a gauge-invariant description of a pair of conjugate massive particles.

Having obtained an interesting connection between the Abelian DJT model in three dimensions and the bosonized version of the two-dimensional Schwinger model by dimensional reduction, it is natural to wonder what one obtains on dimensional reduction of the non-Abelian DJT model. The Lagrangian for the non-Abelian DJT model in three dimensions is

$$L = \frac{1}{2g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{\mu}{2g^2} \epsilon^{\mu\nu\lambda} \text{Tr}(F_{\mu\nu} A_\lambda - \tfrac{2}{3} A_\mu A_\nu A_\lambda) \; , \tag{15}$$

where A,F are now matrices. On performing dimensional reduction as described above, we get in two dimensions

$$L = \frac{1}{2g^2} \text{Tr}(F_{ab} F^{ab}) - \frac{1}{g^2} \text{Tr}(D_a \varphi)(D^a \varphi) - \frac{\mu}{g^2} \text{Tr}(F_{ab} \varphi) \epsilon^{ab} \; , \tag{16}$$

where, as before, $\varphi = A_2$. The Lagrangian (16) coincides with the small-fluctuation limit of the Lagrangian of the bosonized version of QCD in two dimensions, as given by D'Adda, Davis, and Di Vecchia.[13] Our Lagrangian does, however, incorporate the correct non-Abelian anomaly, as seen from an infinitesimal non-Abelian transformation of the third term in L of Eq. (16) by $\varphi(x) \to \varphi(x) + \theta(x)$, $\theta(x)$ being an arbitrary matrix-valued function of space-time.[8]

IV. ANOMALIES AND LONG-RANGE POTENTIALS

It is known that the DJT (Abelian) model contains "stringlike" solutions[1] and the Aurilia-Takahashi gauge-mixed model has "anomalies."[3]

In the DJT model the existence of long-range potentials can be inferred from considering the equations of motion with external sources, viz.,

$$\partial_\mu F^{\mu\nu} + \mu \widetilde{F}^{\nu} = j^\nu \; . \tag{17}$$

For $\nu = 0$, we get

$$\nabla \cdot \mathbf{E} + \mu H = j^0 \; . \tag{17a}$$

Integrating over two-dimensional space one gets

$$\mu \int H \, d^2x = \mu \oint \mathbf{A} \cdot d\mathbf{x} \neq 0$$

showing thereby that A is a long-ranged potential, whose field strength is short ranged. As pointed out by DJT, this resembles the Nielsen-Olesen string. Now in the gauge-mixing mechanism, it is easy to check that the Lagrangian has a global invariance

$$\phi \to \phi + c$$

up to a two-divergence. The Noether current is

$$j_\mu = \partial_\mu \phi + \mu \epsilon_{\mu\nu} A^\nu \tag{18}$$

and is conserved. However this current is not gauge invariant because of the second term. If one defines

$$j^A_\mu = j_\mu - \mu \epsilon_{\mu\nu} A^\nu \equiv \partial_\mu \phi$$

then

$$\partial_\mu j^{\mu A} = -\mu \widetilde{F} \tag{19}$$

reminding us of the anomaly of the chiral current.[14] Obviously the topological mass which is responsible for the long-range potential in the DJT model is also responsible for the anomaly, in the dimensionally reduced model. Existence of long-range potentials can be shown in higher-dimensional DJT models also. However it is of interest to see the long-range potentials in the dimensionally reduced model itself. Now we shall take up the five-dimensional generalization of the DJT model and see the existence of anomaly and long-range potential in the lower-dimensional model itself.

The Lagrangian density

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}{}^2+\frac{1}{2\times 3!}F_{\mu\nu\rho}{}^2+\frac{\mu}{4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}A_{\alpha\beta} \tag{20}$$

has a global symmetry corresponding to the transformations

$$A_\mu \rightarrow A_\mu + C_\mu \ .$$

The corresponding "current"

$$j^{\mu\nu}=-F^{\mu\nu}+\frac{\mu}{2}\epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta} \tag{21}$$

is not gauge invariant. However, the current

$$j'^{\mu\nu}=j^{\mu\nu}-\frac{\mu}{2}\epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}\equiv -F^{\mu\nu}$$

is gauge invariant, but not conserved. This gives rise to the "anomaly equation"

$$\partial_\mu j'^{\mu\nu}=\mu\widetilde{F}^{\,\nu} \ . \tag{22}$$

We have the situation analogous to bosonized Schwinger model. Note that the same Lagrangian is invariant under $A_{\alpha\beta}\rightarrow A_{\alpha\beta}+C_{\alpha\beta}$, but the "current" $j^{\lambda\mu\nu}=F^{\lambda\mu\nu}+\mu\epsilon^{\alpha\lambda\mu\nu}A_\alpha$ is not gauge invariant. But we have a "current" $j'^{\lambda\mu\nu}=F^{\lambda\mu\nu}$ which is gauge invariant but not conserved.

Now, if we consider the equations of motion with external source, we have the equation

$$-\partial_i E^i-\frac{\mu}{2}\epsilon^{ijk}\partial_i A_{jk}=J_0 \ . \tag{23}$$

When integrated over volume V, we find the potential should be nonzero at infinity. The behavior resembles the DJT model though it is an even-dimensional theory.

V. CONCLUSIONS

In conclusion we present a discussion of our results and other interesting possibilities suggested by them.

(i) We have generalized the DJT model to higher dimensions and this automatically brings in higher-rank tensors in order for the additional term to give rise to mass. It is natural to wonder how a non-Abelian version of the generalized DJT model in higher dimensions would look like. We would then have to answer the question: How does one define the topological quantities analogous to the Chern-Simons secondary invariants or Pontryagin index using higher-rank tensor gauge fields? On the other hand, there does exist a gauge-mixing type of model in four dimensions with non-Abelian fields $A_{\mu\nu}$ and A_ν due to Freedman and Townsend.[10] Could this give a clue to a five-dimensional non-Abelian model of the DJT type? We have not found a satisfactory answer to these questions.

(ii) It is interesting to note that though the DJT theory is not invariant under parity (P) and time reversal (T), the bosonized Schwinger model obtained from it on dimensional reduction does respect P and T provided the intrinsic P and T of the scalar field are defined to be opposite to those of the vector field. This freedom exists in two dimensions because the three-dimensional Poincaré invariance is broken on dimensional reduction. A similar remark would be true of higher-dimensional, generalized DJT models.

(iii) We have shown that dimensional reduction of the Abelian DJT model in three dimensions leads to the bosonized Schwinger model in two dimensions. However, the non-Abelian DJT model does not lead to the complete bosonized version of two-dimensional QCD, but only a small-fluctuation limit of it. It would be interesting to know if a modification of the dimensional-reduction technique employed here could give a complete bosonized version of two-dimensional QCD. Work on this is in progress. Recently, different "prescriptions" for bosonization have been given in two-dimensional non-Abelian theories[15] leading to different symmetries for bosonic theories in the presence of interaction. If dimensional reduction could indeed be used as an approach to bosonization, it would be another way of understanding this problem.

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[1]S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. (N.Y.) **140**, 372 (1982); R. Jackiw, in *Asymptotic Realms of Physics*, edited by A. Guth, K. Huang, and R. Jaffe (MIT Press, Cambridge, MA, 1983); Comments Nucl. Part. Phys. **13**, 141 (1984); K. Ishikawa, Phys. Rev. Lett. **53**, 1615 (1984).

[2]H. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[3]A. Aurilia and Y. Takahashi, Prog. Theor. Phys. **66**, 693 (1981); Phys. Rev. D **23**, 1752 (1981). For other applications of tensor gauge fields, see P. K. Townsend, CERN Report No. TH 3067 (unpublished), and references therein.

[4]J. Scherk and J. H. Schwarz, Phys. Lett. **57B**, 463 (1975); E. Cremmer and J. Scherk, Nucl. Phys. **B103**, 399 (1976); E. Cremmer, Report No. LPTENS 81/18, 1981 (unpublished).

[5]It is of interest to point out that this technique was used to obtain a gauge-mixing model of Aurilia and Takahashi from a massless theory in five dimensions. See T. R. Govindarajan, J. Phys. G **8**, L17 (1982).

[6]See, for example, A. Salam and J. Strathdee, Ann. Phys. (N.Y.) **141**, 316 (1982).

[7]B. Zumino, Wu Yong-Shi, and A. Zee, Nucl. Phys. **B239**, 477 (1984).

[8]It can easily be checked that $\varphi(x)\rightarrow\varphi(x)+\theta(x)$ and $A_a(x)\rightarrow A_a(x)$, for $\theta(x)$ in the adjoint representation of the group, is the infinitesimal form of the usual axial-vector transformation for an infinitesimal chiral field $\varphi(x)$. Inciden-

tally, A. Niemi and G. Semenoff [Phys. Rev. Lett. **51**, 2077 (1983)] have shown the relation between the Chern-Simons secondary class and the non-Abelian anomaly in one dimension less. Their method of functional differentiation gives the correct non-Abelian anomaly, appropriate to a regularization-preserving vector gauge invariance, in agreement with our result for the anomaly.

[9]Our metric in all dimensions is $\mathrm{diag}(1,-1,-1,\ldots,-1)$.

[10]D. Z. Freedman and P. K. Townsend, Nucl. Phys. **B177**, 282 (1981); A. P. Balachandran *et al.*, Phys. Rev. D **26**, 1443 (1982); R. J. Nepomechie, Nucl. Phys. **B212**, 301 (1983); L. Smolin, Phys. Lett. **137B**, 379 (1984).

[11]J. Schwinger, Phys. Rev. **125**, 397 (1962); **128**, 2425 (1962).

[12]S. Coleman, Phys. Rev. D **11**, 2088 (1975); S. Mandelstam, *ibid.* **11**, 3026 (1975).

[13]A. D'Adda, A. C. Davis, and P. Di Vecchia, Phys. Lett. **121B**, 335 (1983).

[14]In the Schwinger model anomaly of the chiral current is critically responsible for the mass of the gauge boson. If the anomaly is absent, the vector current will be forced to be trivially zero.

[15]E. Witten, Commun. Math. Phys. **92**, 455 (1984); A. M. Polyakov and P. B. Wiegman, Phys. Lett. **131B**, 121 (1983); P. Di Vecchia, B. Durhuus, and J. L. Petersen, *ibid.* **144B**, 245 (1984); P. Di Vecchia and P. Rossi, *ibid.* **140B**, 344 (1984).