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Dipole gravitational radiation in the nonsymmetric gravitational theory of Moffat

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The generation of gravitational radiation in the nonsymmetric gravitational theory (NGT) of Moffat is analyzed. It is shown that the theory predicts the emission of dipole gravitational radiation from a binary system. The source of the dipole radiation is a vector density S postulated to be proportional to the number density of fermion particles in the components of the system. This radiation is shown to result in a secular decrease in the orbital period of a binary system *in addition to that predicted by general relativity.* The size of the effect is proportional to the reduced mass of the system and to the square of the difference in l^2/[mass] between the two components of the system, where l is a parameter having units of [length] that is related to the number of fermion particles in each component. As part of the analysis, the stress-energy pseudotensor of the NGT, expanded to quadratic order in the gravitational fields, and the NGT gravitational-wave luminosity formula are derived for the first time. With a perfect-fluid model of matter, results are also given for the post-Newtonian expansions of the source densities of the gravitational fields. The results of this analysis are then applied to the binary pulsar system PSR 1913+16 which contains a pulsar orbiting an unobserved companion. With gravitational radiation attributed as the cause of the observed secular decrease in the orbital period, this system provides a test of the prediction by the NGT of dipole gravitational radiation. It is shown that the NGT can only fit the observations of this system provided the l parameter of the unseen companion is $\lesssim 350$ km.

I. INTRODUCTION AND SUMMARY

After presenting his theory of general relativity in its definitive form in 1915,[1] Einstein devoted much of the rest of his life to searching for a theory that would unify the gravitational and electromagnetic fields.[2] The basic approach of Einstein and many other authors who have pursued a "geometrical" unification of the two long-range forces of Nature has been to introduce a nonsymmetric metric with electromagnetism coupling to the antisymmetric part.[3] Einstein's unified field theory met with discouraging results,[4] and today such an approach is regarded as highly questionable in view of the successes of gauge theories in describing fundamental physical interactions.[5]

With a different physical interpretation, a nonsymmetric metric may be used as the basis of a *purely* gravitational theory. In 1979, Moffat proposed a new theory of gravitation called the nonsymmetric gravitational theory (NGT) in which the geometry of spacetime is non-Riemannian, being fully described by a nonsymmetric metric and a nonsymmetric affine connection.[6–8] In addition to a nonsymmetric stress-energy tensor, there occurs a new source in the form of a conserved "fermion-number current" that couples directly to the antisymmetric part of the metric. This current plays a major role in determining the geometry of spacetime.[9] The theory has been further developed and studied extensively by Moffat and collaborators.[10–40] A variety of theoretical and experimental consequences of the theory have been studied, including the post-Newtonian limit and observable solar-system effects,[22–26] neutron-star[27] and black-hole structure,[28–30] cosmological models,[7,31,32] and gravitational radiation.[28,33,34] In addition to a real field structure for the theory, complex,[35] hypercomplex,[36] quaternion,[37] and octonion[38] algebras have recently been investigated by using a technique of algebraic extension.[39] The theory has been reported to be in agreement with solar-system relativity experiments if a parameter l that is proportional to the number density of fermion particles in a body has the value for the Sun $l_\odot \lesssim 3\times 10^3$ km.[25] Most recently, the theory has been used to explain the observations of anomalously small apsidal motions in stellar binary systems.[40]

This paper is concerned with the predictions that the NGT makes for the generation of gravitational waves from physical systems. Mann and Moffat[33,34] have analyzed the way in which gravitational waves are generated by physical sources in the NGT and have determined the power output in gravitational waves in the linear approximation to the field equations. They obtained the result of general relativity, finding that only quadrupole and higher poles of the source distribution contribute to the power output in gravitational waves. We will show that when the field equations are expanded to second order in the fields, *NGT predicts the emission of dipole gravitational radiation from systems containing gravitationally bound objects.* The source of the dipole radiation is a vector density S postulated to be proportional to the number density of fermion particles in the objects. More precisely, $S=f^2nu$, where f is a fundamental coupling constant with units of [length] measuring how strongly a given type of fermion particle couples to the antisymmetric part of the metric, n is the number density of fermion particles, and u is their four-velocity. If we represent the gravitational fields of NGT that are generated by S generically by ψ, it can be shown that in the radiation zone the fields that contribute to the energy flux have the form

$$\psi(\mathbf{x},t) \propto R^{-1}(d^2/dt^2)\int S^0_{\text{ret}}x'd^3x' , \tag{1.1}$$

where the subscript "ret" means that S^0 is to be evaluated at the retarded spacetime point $(t-R,\mathbf{x}')$ with $R\equiv|\mathbf{x}|$. We say that NGT predicts the emission of "dipole" gravitational radiation, since the integral in Eq. (1.1) is the dipole moment of S^0.

For a binary system, the dipole contribution to the loss of energy from the system by its emission of gravitational waves is proportional to the square of the difference in l^2/[mass] between the two components of the system. Our result for the energy loss-rate formula is

$$(dE/dt)_{\text{NGT}}=(dE/dt)_{\text{GR}}+(dE/dt)_{\text{dipole}}+(\text{post-Newtonian quadrupole and higher-order corrections}) , \tag{1.2}$$

where $(dE/dt)_{\text{GR}}$ is the general-relativistic result that

$$(dE/dt)_{\text{GR}}=-\langle(\mu^2m^2/r^4)\tfrac{8}{15}(12v^2-11\dot r^2)\rangle , \tag{1.3}$$

where m and μ are, respectively, the total mass and the reduced mass of the system, $\mathbf{v}$ is the relative orbital velocity, $\dot r\equiv dr/dt$ for the orbital separation r, and the angular brackets denote an average over an orbital period. $(dE/dt)_{\text{dipole}}$ is the NGT dipole contribution,

$$(dE/dt)_{\text{dipole}}=-\langle(\mu^2m^2d^2/r^6)\tfrac{88}{3}(v^2+3\dot r^2)\rangle , \tag{1.4}$$

where the dipole parameter d is defined by

$$d\equiv l_1{}^2/m_1-l_2{}^2/m_2 . \tag{1.5}$$

with

$$l_A{}^2\equiv\int_{\text{body }A}S^0d^3x , \tag{1.6}$$

where S^0 is the time component of S.

In 1975, Hulse and Taylor[41] reported the discovery of a pulsar in a binary system: PSR 1913 + 16. Their observations showed that the pulsar is moving in a highly eccentric orbit $(e\sim 0.62)$ about some unseen companion. An analysis of the orbital parameters of the system and possible evolutionary histories indicate that most probably the companion is another neutron star of approximately the same mass as the pulsar, although the possibility of a white-dwarf or a black-hole companion is not excluded.[42] With an accurate clock of well-defined period moving in a precessing Keplerian orbit, the system has proved to be an ideal astrophysical laboratory for testing gravitational theory (see Ref. 43, Chap. 12). General relativity predicts that the orbital period of the system should decrease as the system loses energy by emitting gravitational radiation.[44] From their most recent observations of the system, Weisberg and Taylor[45] have shown that general relativity provides an excellent fit to the observed orbital period derivative.

Moffat[15] has considered the binary pulsar as a test of NGT, assuming that the energy loss-rate formula for gravitational-wave emission is the same for NGT as for general relativity (i.e., no dipole radiation). With this assumption, he argued that NGT is in agreement with the observations of the system. We find that *dipole gravitational radiation in NGT results in a secular decrease in the orbital period of a binary system in addition to that predicted by general relativity.*

The predicted decrease in the period of a binary system in NGT is given by

$$\begin{aligned}(\dot P/P)_{\text{NGT}}&=-\tfrac{3}{2}(\dot E/E)_{\text{NGT}}\\&=(\dot P/P)_{\text{GR}}+(\dot P/P)_{\text{dipole}} ,\end{aligned} \tag{1.7}$$

where $(\dot P/P)_{\text{GR}}$ is the general relativistic result that

$$(\dot P/P)_{\text{GR}}=-\tfrac{96}{5}\mu m^{2/3}(2\pi/P)^{8/3}f_{\text{GR}}(e) , \tag{1.8a}$$

$$f_{\text{GR}}(e)=(1-e^2)^{-7/2}(1+\tfrac{73}{24}e^2+\tfrac{37}{96}e^4) , \tag{1.8b}$$

and $(\dot P/P)_{\text{dipole}}$ is the dipole contribution given by

$$(\dot P/P)_{\text{dipole}}=-88\mu d^2(2\pi/P)^4f_{\text{dipole}}(e) , \tag{1.9a}$$

$$f_{\text{dipole}}(e)=(1-e^2)^{-11/2}(1+\tfrac{19}{2}e^2+\tfrac{69}{8}e^4+\tfrac{9}{16}e^6) , \tag{1.9b}$$

where e is the orbital eccentricity. To the necessary order, the Keplerian relations

$$(P/2\pi)^2=a^3/m,\quad E=-\tfrac{1}{2}(\mu m/a) , \tag{1.10}$$

where a is the semimajor axis of the orbit, were used, and an average was taken over one Keplerian orbit. Using Eq. (1.7), one finds that NGT can only fit the observations of the binary pulsar provided $l_c\lesssim 350$ km, where l_c is the l parameter of the unseen companion of the pulsar.

These results for the generation of dipole gravitational radiation in NGT were obtained by making use of the post-Newtonian gravitational-radiation methods developed within general relativity by Epstein and Wagoner,[46] Wagoner and Will,[47] and adapted by Will[48] to analyze the generation of gravitational radiation in alternative theories of gravitation. In the following sections, the details of the calculation are presented. The field equations of NGT are reviewed in the Appendix. A weak-field expansion of these equations is made in Sec. II, yielding gravitational-wave equations and the NGT stress-energy pseudotensor expanded to quadratic order in the fields. The NGT gravitational-wave luminosity formula is derived in Sec. III. From a post-Newtonian analysis, the energy loss-rate formula (total energy lost to gravitational radiation) for a binary system is derived in Sec. IV. In Sec. V, the binary pulsar is considered as a test of NGT. Concluding remarks are made in Sec. VI.

The notations and conventions used in this paper are as follows: greek indices run over the range 0,1,2,3; latin indices run over the range 1,2,3; parentheses and square brackets around indices denote symmetrization and antisymmetrization, respectively; commas denote ordinary partial derivatives. In keeping with the usage by Moffat and his collaborators, the signature $(+---)$ is adopted for the metric. "Geometrized units" with $G=c=1$ will be used throughout.

II. WEAK-FIELD EXPANSION OF THE NGT FIELD EQUATIONS

In this section, wave equations for the gravitational fields of NGT are obtained. These equations will result from certain gauge conditions. These gauge conditions will also furnish the conserved stress-energy pseudotensor of NGT.

We begin by expanding $g_{\mu\nu}$ about a flat background spacetime,

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\ , \tag{2.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric diag$(+1,-1,-1,-1)$ and where $|h_{\mu\nu}|\ll 1$. The symmetric and antisymmetric parts of $g_{\mu\nu}$ are then, respectively, given by

$$g_{(\mu\nu)}=\eta_{\mu\nu}+h_{(\mu\nu)}\ , \tag{2.2}$$

$$g_{[\mu\nu]}=h_{[\mu\nu]}\ . \tag{2.3}$$

Using the raising and lowering convention established in Eq. (A1), it follows that

$$g^{(\mu\nu)}=\eta^{\mu\nu}-h^{(\mu\nu)}+h_\alpha{}^{(\mu}h^{\nu)\alpha}+O(h^3)\ , \tag{2.4}$$

$$g^{[\mu\nu]}=h^{[\mu\nu]}+h_\alpha{}^{[\mu}h^{\nu]\alpha}+O(h^3)\ , \tag{2.5}$$

and

$$g=-1-h-\tfrac{1}{2}(h^2-h_{\mu\nu}h^{\nu\mu})+O(h^3)\ . \tag{2.6}$$

where indices on $h_{\mu\nu}$ are raised and lowered using $\eta^{\mu\nu}$, i.e., $h^\alpha{}_\nu\equiv\eta^{\alpha\mu}h_{\mu\nu}$ and where $h\equiv h^\alpha{}_\alpha$.

Permuting indices three times in the metric compatibility equation (A37) and subtracting the resulting equations yields

$$g_{(\rho\sigma)}\Lambda^\rho_{\mu\nu}=\tfrac{1}{2}(g_{\sigma\nu,\mu}+g_{\mu\sigma,\nu}-g_{\nu\mu,\sigma})-g_{[\mu\rho]}\Lambda^\rho_{\nu\sigma}-g_{[\rho\nu]}\Lambda^\rho_{\sigma\mu}\ . \tag{2.7}$$

By iteration, one obtains from Eq. (2.7)

$$\Lambda^\rho_{(\mu\nu)}={}^1\Lambda^\rho_{(\mu\nu)}-\eta^{\rho\sigma}(h_{(\lambda\sigma)}{}^1\Lambda^\lambda_{(\nu\mu)}+h_{[\mu\lambda]}{}^1\Lambda^\lambda_{[\nu\sigma]}+h_{[\lambda\nu]}{}^1\Lambda^\lambda_{[\sigma\mu]})+O(h^3) \tag{2.8}$$

and

$$\Lambda^\rho_{[\mu\nu]}={}^1\Lambda^\rho_{[\mu\nu]}-\eta^{\rho\sigma}(h_{(\lambda\sigma)}{}^1\Lambda^\lambda_{[\mu\nu]}+h_{[\mu\lambda]}{}^1\Lambda^\lambda_{(\nu\sigma)}+h_{[\lambda\nu]}{}^1\Lambda^\lambda_{(\sigma\mu)})+O(h^3)\ , \tag{2.9}$$

where

$${}^1\Lambda^\rho_{(\mu\nu)}=\tfrac{1}{2}\eta^{\rho\alpha}(h_{(\mu\alpha),\nu}+h_{(\alpha\nu),\mu}-h_{(\nu\mu),\alpha})\ , \tag{2.10}$$

$${}^1\Lambda^\rho_{[\mu\nu]}=\tfrac{1}{2}\eta^{\rho\alpha}(h_{[\alpha\nu],\mu}+h_{[\mu\alpha],\nu}-h_{[\nu\mu],\alpha})\ . \tag{2.11}$$

A similar procedure applied to Eq. (A16) yields

$$D^\rho_{(\mu\nu)}=-\frac{4\pi}{3}S^\lambda(\tfrac{1}{2}\delta^\rho_\mu h_{[\nu\lambda]}+\tfrac{1}{2}\delta^\rho_\nu h_{[\mu\lambda]}-\tfrac{3}{2}\eta_{\mu\nu}\eta^{\rho\sigma}h_{[\sigma\lambda]})+O(Sh^2)\ , \tag{2.12}$$

$$D^\rho_{[\mu\nu]}=-\frac{4\pi}{3}S^\lambda(\delta^\rho_\mu\eta_{\lambda\nu}-\delta^\rho_\nu\eta_{\mu\lambda}+\delta^\rho_\mu h_{(\lambda\nu)}+\delta^\rho_\nu h_{(\lambda\mu)})+O(Sh^2)\ . \tag{2.13}$$

With the previous results, the NGT field equations can be expanded to quadratic order in the fields. Using Eqs. (2.8)–(2.13) in Eqs. (A23) and (A25) gives

$$\begin{aligned}R_{(\mu\nu)}(\Lambda)=&-\tfrac{1}{2}\Box h_{(\mu\nu)}+\tfrac{1}{2}[(h_{(\mu\alpha)}{}^{,\alpha}-\tfrac{1}{2}h_{,\mu})_{,\nu}+(h_{(\nu\alpha)}{}^{,\alpha}-\tfrac{1}{2}h_{,\nu})_{,\mu}]-\tfrac{1}{2}(h^{(\alpha\beta)}{}_{,\beta}-\tfrac{1}{2}h^{,\alpha})(h_{(\mu\alpha),\nu}+h_{(\alpha\nu),\mu}-h_{(\nu\mu),\alpha})\\&-\tfrac{1}{2}h^{(\alpha\beta)}(h_{(\mu\alpha),\nu\beta}+h_{(\alpha\nu),\mu\beta}-h_{(\nu\mu),\alpha\beta}-h_{(\alpha\beta),\mu\nu})-\tfrac{1}{2}\eta^{\alpha\beta}h_{(\mu\alpha)}{}^{,\lambda}(h_{(\lambda\nu),\beta}-h_{(\nu\beta),\lambda})+\tfrac{1}{4}h^{(\alpha\beta)}{}_{,\mu}h_{(\alpha\beta),\nu}\\&-\tfrac{1}{2}\eta^{\alpha\beta}[h_{[\mu\alpha]}(\Box h_{[\nu\beta]}+h_{[\beta\lambda]}{}^{,\lambda}{}_{\nu}+h_{[\nu\lambda]}{}^{,\lambda}{}_{\beta})+h_{[\nu\alpha]}(\Box h_{[\mu\beta]}+h_{[\beta\lambda]}{}^{,\lambda}{}_{\mu}+h_{[\mu\lambda]}{}^{,\lambda}{}_{\beta})\\&\qquad+h_{[\mu\alpha]}{}^{,\lambda}(h_{[\nu\beta],\lambda}-h_{[\lambda\nu],\beta})+(h_{[\mu\alpha]}{}^{,\lambda}h_{[\beta\lambda],\nu}+h_{[\nu\alpha]}{}^{,\lambda}h_{[\beta\lambda],\mu})]\\&-\tfrac{3}{4}h^{[\alpha\beta]}{}_{,\mu}h_{[\alpha\beta],\nu}-\tfrac{1}{2}h^{[\alpha\beta]}h_{[\alpha\beta],\mu\nu}+O(h^3)\ ,\end{aligned} \tag{2.14a}$$

$$\begin{aligned}R_{[\mu\nu]}(\Lambda)=&\tfrac{1}{2}\Box h_{[\mu\nu]}+\tfrac{1}{2}(h_{[\mu\alpha]}{}^{,\alpha}{}_{\nu}-h_{[\nu\alpha]}{}^{,\alpha}{}_{\mu})-\tfrac{1}{2}(h^{(\alpha\beta)}{}_{,\beta}-\tfrac{1}{2}h^{,\alpha})(h_{[\alpha\nu],\mu}+h_{[\mu\alpha],\nu}-h_{[\nu\mu],\alpha})\\&-\tfrac{1}{2}h^{(\alpha\beta)}(h_{[\alpha\nu],\mu}+h_{[\mu\alpha],\nu}-h_{[\nu\mu],\alpha})_{\beta}\\&-\tfrac{1}{2}\eta^{\alpha\beta}[h_{[\mu\alpha]}(\Box h_{(\nu\beta)}+h_{(\beta\lambda)}{}^{,\lambda}{}_{\nu}-h_{(\nu\lambda)}{}^{,\lambda}{}_{\beta})-h_{[\nu\alpha]}(\Box h_{(\mu\beta)}+h_{(\beta\lambda)}{}^{,\lambda}{}_{\mu}-h_{(\mu\lambda)}{}^{,\lambda}{}_{\beta})\\&\qquad+h_{[\mu\alpha]}{}^{,\lambda}(h_{(\nu\beta),\lambda}+h_{(\beta\lambda),\nu}-h_{(\lambda\nu),\beta})-h_{[\nu\alpha]}{}^{,\lambda}(h_{(\mu\beta),\lambda}+h_{(\beta\lambda),\mu}-h_{(\lambda\mu),\beta})\\&\qquad+(h_{(\mu\alpha)}{}^{,\lambda}h_{[\beta\lambda],\nu}+h_{(\alpha\nu)}{}^{,\lambda}h_{[\lambda\beta],\mu}+h_{[\mu\alpha]}{}^{,\lambda}h_{(\beta\lambda),\nu}+h_{[\alpha\nu]}{}^{,\lambda}h_{(\lambda\beta),\mu})]+O(h^3)\ ,\end{aligned} \tag{2.14b}$$

$$R_{(\mu\nu)}(D)=-2\pi\eta_{\mu\nu}(S^{\lambda}h_{[\lambda\beta]})^{,\beta}-\frac{16\pi^2}{3}S_{\mu}S_{\nu}+O(Sh^2)\ , \tag{2.14c}$$

$$R_{[\mu\nu]}(D)=\frac{4\pi}{3}[2S_{[\mu,\nu]}-S^{\lambda}{}_{,\mu}h_{(\lambda\nu)}+S^{\lambda}{}_{,\nu}h_{(\lambda\mu)}-S^{\lambda}(h_{(\lambda\nu),\mu}-h_{(\lambda\mu),\nu})]+O(Sh^2)\ , \tag{2.14d}$$

$$R_{(\mu\nu)}(\Lambda D)=\frac{4\pi}{3}(S_{\mu}h_{[\nu\beta]}{}^{,\beta}+S_{\nu}h_{[\mu\beta]}{}^{,\beta})+O(Sh^2)\ , \tag{2.14e}$$

$$R_{[\mu\nu]}(\Lambda D)=O(Sh^2)\ , \tag{2.14f}$$

where indices have been raised and lowered with $\eta_{\mu\nu}$.

In order to obtain wave equations for the fields in NGT, we will exploit the coordinate freedom embodied in general covariance and specify the condition that

$$\theta^{\mu\nu}{}_{,\nu}\equiv 0\ , \tag{2.15}$$

where we have introduced the symmetric "gravitational potential" $\theta^{\mu\nu}$ defined by

$$\theta^{\mu\nu}\equiv h^{(\mu\nu)}-\tfrac{1}{2}\eta^{\mu\nu}h\ . \tag{2.16}$$

Let us also define

$$\phi^{\mu\nu}\equiv h^{[\mu\nu]} \tag{2.17}$$

and

$$A_{\mu}\equiv W_{\mu}+8\pi\eta_{\mu\nu}S^{\nu}\ . \tag{2.18}$$

Using Eqs. (2.4)—(2.6) and (2.15)—(2.17) in Eq. (A36c), one then finds that the divergence of $\phi^{\mu\nu}$ satisfies

$$\phi^{\mu\nu}{}_{,\nu}=4\pi(S^{\mu}+\theta^{\mu}{}_{\nu}S^{\nu}-\theta S^{\mu})+\phi^{\mu\nu}{}_{,\beta}\theta^{\beta}{}_{\nu}+\phi^{\beta\nu}\theta^{\mu}{}_{\beta,\nu}-\tfrac{1}{2}\phi^{\mu\nu}\theta_{,\nu}+O(\theta^2\phi,\theta\phi^2)\ , \tag{2.19}$$

where $\theta\equiv\theta^{\alpha}{}_{\alpha}\equiv\eta^{\alpha\beta}\theta_{\alpha\beta}$.

From the symmetric field equation (A36a) follows the resulting wave equation for $\theta^{\mu\nu}$:

$$\Box\theta^{\mu\nu}=-16\pi\tau^{(\mu\nu)}\ , \tag{2.20}$$

where $\Box\equiv\eta^{\mu\nu}\partial^2/\partial x^{\mu}\partial x^{\nu}$, and where terms that are nonlinear in the fields have been transferred to the right-hand side of Eq. (2.20). To quadratic order in the fields, $\tau^{(\mu\nu)}$ is given by

$$\tau^{(\mu\nu)}=T^{(\mu\nu)}+\Sigma^{(\mu\nu)}+t^{(\mu\nu)}\ , \tag{2.21a}$$

$$\Sigma^{(\mu\nu)}=-\tfrac{1}{3}(3\phi^{\alpha(\mu}S^{\nu)}{}_{,\alpha}-2\pi S^{\mu}S^{\nu}-2\pi\eta^{\mu\nu}S^{\alpha}S_{\alpha})\ , \tag{2.21b}$$

$$\begin{aligned}t^{(\mu\nu)}=\frac{1}{16\pi}[&-\tfrac{1}{2}\theta_{\alpha\beta}{}^{,\mu}\theta^{\alpha\beta,\nu}-\theta_{\alpha\beta}(\theta^{\alpha\beta,\mu\nu}+\theta^{\mu\nu,\alpha\beta}-\theta^{\alpha\mu,\nu\beta}-\theta^{\alpha\nu,\mu\beta})+\tfrac{1}{2}\theta\theta^{,\mu\nu}+\tfrac{1}{4}\theta^{,\mu}\theta^{,\nu}\\&+\tfrac{3}{2}\theta\Box\theta^{\mu\nu}+\theta_{,\alpha}\theta^{\mu\nu,\alpha}-\theta_{,\alpha}\theta^{\alpha(\mu,\nu)}-\theta^{\nu}{}_{\alpha,\beta}\theta^{\mu\alpha,\beta}-\theta^{\nu}{}_{\alpha,\beta}\theta^{\mu\beta,\alpha}-\theta_{\beta}{}^{(\mu}\theta^{,\nu)\beta}\\&+\tfrac{1}{2}\theta^{\mu\nu}\Box\theta-\theta^{\mu}{}_{\alpha}\Box\theta^{\alpha\nu}-\theta^{\nu}{}_{\alpha}\Box\theta^{\alpha\mu}+\tfrac{3}{2}\phi_{\alpha\beta}{}^{,\mu}\phi^{\alpha\beta,\nu}+\phi_{\alpha\beta}\phi^{\alpha\beta,\mu\nu}+\phi^{\nu}{}_{\alpha,\beta}\phi^{\mu\alpha,\beta}+\phi^{\nu}{}_{\alpha,\beta}\phi^{\mu\beta,\alpha}\\&+2\phi_{\alpha\beta}{}^{,(\mu}\phi^{\nu)\alpha,\beta}+\tfrac{4}{3}\phi^{\mu}{}_{\alpha}A^{[\alpha,\nu]}+\tfrac{4}{3}\phi_{\alpha}{}^{\nu}A^{[\mu,\alpha]}\\&+\tfrac{1}{2}\eta^{\mu\nu}(\tfrac{3}{2}\theta_{\alpha\beta,\sigma}\theta^{\alpha\beta,\sigma}-\tfrac{3}{4}\theta_{,\sigma}\theta^{,\sigma}+2\theta_{\alpha\beta}\Box\theta^{\alpha\beta}-\theta_{\alpha\beta,\sigma}\theta^{\alpha\sigma,\beta}-\theta\Box\theta+\theta^{\alpha\beta}\theta_{,\alpha\beta}-\tfrac{5}{2}\phi_{\alpha\beta,\sigma}\phi^{\alpha\beta,\sigma}\\&\qquad-\phi_{\alpha\beta}\Box\phi^{\alpha\beta}+\phi_{\alpha\beta,\sigma}\phi^{\alpha\sigma,\beta}+\tfrac{4}{3}\phi^{\alpha\beta}A_{[\alpha,\beta]})]\ ,\end{aligned} \tag{2.21c}$$

From the antisymmetric field equation (A36b) follows the wave equation for $\phi^{\mu\nu}$:

$$\Box\phi^{\mu\nu}=-16\pi\tau^{[\mu\nu]}\ , \tag{2.22}$$

where, to quadratic order in the fields,

$$\tau^{[\mu\nu]}=-\tfrac{1}{2}S^{[\mu,\nu]}+T^{[\mu\nu]}+\frac{1}{12\pi}A^{[\mu,\nu]}+\Sigma^{[\mu\nu]}+t^{[\mu\nu]}\ , \tag{2.23a}$$

$$\Sigma^{[\mu\nu]}=-\tfrac{1}{3}(\tfrac{3}{2}\theta^{\alpha[\mu}S^{\nu]}{}_{,\alpha}+2S^{\alpha,[\mu}\theta^{\nu]}{}_{\alpha}+\theta S^{[\mu,\nu]}+\tfrac{1}{2}\theta^{,[\mu}S^{\nu]}-\tfrac{1}{2}S^{\alpha}\theta_{\alpha}{}^{[\mu,\nu]})\ , \tag{2.23b}$$

$$t^{[\mu\nu]}=\frac{1}{16\pi}[(2\theta^{\alpha\beta,[\mu}\phi^{\nu]}{}_{\alpha,\beta}-\theta_{\alpha\beta}\phi^{\mu\nu,\alpha\beta}+\tfrac{3}{2}\theta\Box\phi^{\mu\nu}+\tfrac{1}{2}\phi^{\mu\nu}\Box\theta+\theta^{,\alpha}\phi^{\mu\nu}{}_{,\alpha}-2\phi^{[\mu}{}_{\alpha,\beta}\theta^{\nu]\alpha,\beta}$$
$$+2\phi^{[\mu}{}_{\alpha,\beta}\theta^{\nu]\beta,\alpha}+2\phi_{\alpha\beta}\theta^{\alpha[\mu,\nu]\beta}-\theta^{\mu\alpha}\Box\phi_{\alpha}{}^{\nu}+\theta^{\nu\alpha}\Box\phi_{\alpha}{}^{\mu})-\tfrac{4}{3}(\eta^{\mu\alpha}\theta^{\beta\nu}+\eta^{\beta\nu}\theta^{\mu\alpha}-\eta^{\mu\alpha}\eta^{\beta\nu}\theta)A_{[\alpha,\beta]}] . \quad (2.23c)$$

At this point, we use the additional gauge freedom of W_μ, and specify the condition that

$$W_{\mu}{}^{,\mu}\equiv 0 \quad (2.24)$$

which from the definition of A_μ, Eq. (2.18), means that

$$\mathscr{A}_{\mu}{}^{,\mu}=8\pi S^{\mu}{}_{,\mu} . \quad (2.25)$$

Taking the divergence of Eq. (2.22) and using Eqs. (2.15), (2.19), and (2.25) then yields the following wave equation for A_μ:

$$\Box A_\mu=-16\pi\tau_\mu , \quad (2.26)$$

where, to quadratic order in the fields,

$$\tau_\mu=\tfrac{3}{2}\eta_{\mu\alpha}T^{[\alpha\beta]}{}_{,\beta}+\Sigma_\mu+t_\mu , \quad (2.27a)$$

$$\Sigma_\mu=\theta_{\mu\alpha}{}^{,\beta}S^{\alpha}{}_{,\beta}-\tfrac{1}{2}\theta_{\alpha}{}^{\beta}{}_{,\mu}S^{\alpha}{}_{,\beta}-\tfrac{1}{2}\theta^{,\alpha}S_{\mu,\alpha}+\tfrac{3}{16}\theta_{,\alpha}S^{\alpha}{}_{,\mu}-\tfrac{3}{16}\theta_{,\mu\alpha}S^{\alpha}-\tfrac{1}{2}\theta^{\alpha}{}_{\beta}S^{\beta}{}_{,\mu\alpha}+\tfrac{1}{2}\theta_{\mu\alpha}\Box S^{\alpha}$$
$$+\tfrac{1}{2}S^{\alpha}\Box\theta_{\mu\alpha}-\tfrac{1}{16}\theta\Box S_\mu-\tfrac{1}{16}S_\mu\Box\theta , \quad (2.27b)$$

$$t_\mu=\frac{1}{16\pi}[3(\theta_{\mu\alpha,\beta}{}^{\gamma}\phi^{\alpha\beta}{}_{,\gamma}+\theta^{\alpha\beta}{}_{,\mu\gamma}\phi^{\gamma}{}_{\alpha,\beta}+\tfrac{1}{2}\theta^{,\alpha}\Box\phi_{\mu\alpha}+\phi^{\alpha\beta}\Box\theta_{\mu\alpha,\beta})$$
$$-2(\theta^{\alpha\beta}A_{[\mu,\alpha]\beta}+\theta_{\mu}{}^{\alpha,\beta}A_{[\alpha,\beta]}-\theta^{,\alpha}A_{[\mu,\alpha]}+\tfrac{1}{2}\theta_{\mu}{}^{\alpha}\Box A_\alpha-\tfrac{1}{2}\theta\Box A_\mu)] . \quad (2.27c)$$

To appropriate order, the matter response equation (A38) expanded in terms of $\theta^{\mu\nu}$, $\phi^{\mu\nu}$, and A_μ is given by

$$T^{(\mu\nu)}{}_{,\nu}=(\tfrac{1}{2}\theta_{\alpha\beta}{}^{,\mu}-\theta^{\mu}{}_{\alpha,\beta})T^{(\alpha\beta)}+\theta_{,\alpha}T^{(\alpha\mu)}-\tfrac{1}{4}\theta^{,\mu}T+\phi^{\mu}{}_{\alpha}T^{[\alpha\beta]}{}_{,\beta}+(\tfrac{1}{2}\phi_{\alpha\beta}{}^{,\mu}+\phi^{\mu}{}_{\alpha,\beta})T^{[\alpha\beta]}+\frac{8\pi}{3}S^{[\mu,\alpha]}S_\alpha-\tfrac{1}{3}A^{[\mu,\alpha]}S_\alpha . \quad (2.28)$$

From Eq. (A6), the expansions of the symmetric and antisymmetric components of $T^{\mu\nu}$ are, respectively, given by

$$T^{(\mu\nu)}=\rho_0[(1+\Pi)+p/\rho_0]u^{\mu}u^{\nu}-p\eta^{\mu\nu}(1+\tfrac{1}{2}\theta)+p\theta^{\mu\nu}+O(p\theta\phi) , \quad (2.29)$$

$$T^{[\mu\nu]}=-p\phi^{\mu\nu}+O(p\theta\phi) . \quad (2.30)$$

The gauge conditions (2.15) and (2.24) automatically supply the conservation laws

$$\tau^{(\mu\nu)}{}_{,\nu}=0 , \quad (2.31)$$

where $\tau^{(\mu\nu)}$ is given by Eq. (2.21). Equation (2.31) can be verified directly. The conserved quantities

$$P^{\mu}\equiv\int_{\Sigma}\tau^{(\mu\nu)}d^3\Sigma_\nu , \quad (2.32)$$

$$J^{\mu\nu}\equiv\int_{\Sigma}[x^{\mu}\tau^{(\nu\lambda)}-x^{\nu}\tau^{(\mu\lambda)}]d^3\Sigma_\lambda , \quad (2.33)$$

where Σ is a three-dimensional spacelike hypersurface, can then be defined. In a coordinate system $(t,\mathbf{x})$ in which Σ is a constant-time hypersurface that extends infinitely far in all spatial directions, these integrals become

$$P^{\mu}=\int\tau^{(\mu0)}d^3x , \quad (2.34)$$

$$J^{\mu\nu}=\int[x^{\mu}\tau^{(\nu0)}-x^{\nu}\tau^{(\mu0)}]d^3x . \quad (2.35)$$

From the post-Newtonian expansion of $\tau^{(\mu\nu)}$ (see Sec. V), it can be shown that the $\tau^{(\mu0)}$ are energy-momentum densities modulo perfect divergence terms. Therefore, the components of P^μ and $J^{\mu\nu}$ can be interpreted in the following way: P^0 is the total energy, P^j is the total momentum, J^{ij} is the angular momentum, and J^{0j} determines the motion of the center of mass. With the components of the center of mass $\mathbf{X}$ defined by

$$X^i\equiv(P^it+J^{0i})/P^0 , \quad (2.36)$$

it follows from the constancy of J^{0i} that

$$dX^i/dt=P^i/P^0 . \quad (2.37)$$

Therefore, constancy of P^0 and P^i ensures that the center of mass of a system in NGT is unaccelerated.

III. THE NGT LUMINOSITY FORMULA

In Sec. II, we found that the stress-energy pseudotensor $\tau^{(\mu\nu)}$ defines, among other quantities, the total energy P^0 of a system in NGT, where

$$P^0=\int\tau^{(00)}d^3x . \quad (3.1)$$

Using Eq. (2.31), one finds that for localized sources, P^0 is conserved up to a flux of energy carried to infinity by gravitational waves. In other words,

$$\frac{dP^0}{dt}=-R^2\oint t^{(0j)}\hat{\mathbf{n}}_j d\Omega \ , \tag{3.2}$$

where the integration is over a sphere of radius $R \equiv |\mathbf{x}|$ in the wave zone far from the source, $\hat{\mathbf{n}}_j \equiv x_j/R$ is the unit normal vector, and $t^{(0j)}$ is given by Eq. (2.21c).

With the fields $\theta^{\mu\nu}$, $\phi^{\mu\nu}$, and A_μ represented generically by ψ, their wave equations are seen to be of the form

$$\Box\psi=-16\pi\tau \ , \tag{3.3}$$

where the appropriate sources τ are, respectively, given by Eqs. (2.21), (2.23), and (2.27). With an outgoing-wave boundary condition, the solution to Eq. (3.3) is

$$\psi(\mathbf{x},t)=-4\int\frac{\tau(t-|\mathbf{x}-\mathbf{x}'|,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x' \ . \tag{3.4}$$

For field points far from the source ($R \gg r\equiv$"size" of source, $|\psi| \ll 1$),

$$\psi(\mathbf{x},t)=-\frac{4}{R}\int\tau(t-R+\hat{\mathbf{n}}\cdot\mathbf{x}',\mathbf{x}')d^3x'+O((r/R)^2) \ . \tag{3.5}$$

If the motions of the source are sufficiently slow (source within wave zone, $r<\lambda/2\pi\equiv$wavelength$/2\pi \ll R$), then Eq. (3.5) may be expanded in the form

$$\psi(\mathbf{x},t)=-\frac{4}{R}\sum_{m=0}^{\infty}\frac{1}{m!}\left[\frac{\partial}{\partial t}\right]^m\int\tau(t-R,\mathbf{x}')(\hat{\mathbf{n}}\cdot\mathbf{x}')^m d^3x' +O((r/R)^2) \ . \tag{3.6}$$

For further use, note that

$$\psi_{,j}=\hat{\mathbf{n}}_j\psi_{,0}+O((r/R)^2) \ , \tag{3.7}$$

where, because of the signature of the metric being used, $R=(-x_jx^j)^{1/2}$, so $R_{,j}=-\hat{\mathbf{n}}_j$.

Carrying out the summation over indices in Eq. (2.21c) for $t^{(0j)}$ and simplifying, using

$$\psi_{,jj}=\psi_{,00}+(\text{material sources})+O(\psi^2) \ , \tag{3.8}$$

and discarding the terms involving source densities (that vanish in the radiation zone if the sources are localized), one finds that Eq. (3.2) becomes

$$\frac{d\widetilde{P}^0}{dt}=-R^2\oint\widetilde{t}^{(0j)}\hat{\mathbf{n}}_j d\Omega \ , \tag{3.9}$$

where $\widetilde{P}^0=P^0$ (modulo boundary terms vanishing as R^{-1} for $R\to\infty$), and $\widetilde{t}^{(0j)}$ is given by

$$\begin{aligned}\widetilde{t}^{(0j)}=\frac{1}{16\pi}[&\tfrac{3}{4}\theta^{00}{}_{,0}\theta^{00}{}_{,j}-\theta^{0k}{}_{,0}\theta^{0k}{}_{,j}+\tfrac{1}{2}\theta^{ik}{}_{,0}\theta^{ik}{}_{,j}+\tfrac{1}{4}\theta^{00}{}_{,0}\theta^{ii}{}_{,j}-\tfrac{1}{4}\theta^{ii}{}_{,0}\theta^{00}{}_{,j}-\tfrac{1}{4}\theta^{ii}{}_{,0}\theta^{ii}{}_{,j}\\&+2\theta^{00}{}_{,0}\theta^{0j}{}_{,0}+\tfrac{1}{2}\theta^{0j}{}_{,k}\theta^{ii}{}_{,k}+\tfrac{1}{2}\theta^{00}{}_{,k}\theta^{0k}{}_{,j}-\tfrac{1}{2}\theta^{ii}{}_{,k}\theta^{0k}{}_{,j}-\tfrac{1}{2}\theta^{00}{}_{,i}\theta^{ij}{}_{,0}\\&+\tfrac{1}{2}\theta^{ii}{}_{,k}\theta^{jk}{}_{,0}+2\theta^{0j}{}_{,k}\theta^{0k}{}_{,0}-\tfrac{1}{2}\theta^{0j}{}_{,k}\theta^{00}{}_{,k}-\tfrac{1}{2}\phi^{ik}{}_{,0}\phi^{ik}{}_{,j}-\phi^{ij}{}_{,k}\phi^{0i}{}_{,k}-\phi^{ij}{}_{,k}\phi^{0k}{}_{,i}\\&-2\phi^{0k}{}_{,0}\phi^{0j}{}_{,k}+\phi^{ik}{}_{,0}\phi^{ij}{}_{,k}+\phi^{ik}{}_{,j}\phi^{0i}{}_{,k}+\tfrac{2}{3}(\phi^{ij}{}_{,0}A_i+\phi^{0j}{}_{,i}A_i-\phi^{0i}A_{i,j})] \ .\end{aligned} \tag{3.10}$$

In the far zone, the gauge conditions on $\theta^{\mu\nu}$, Eq. (2.15), and the divergence of $\phi^{\mu\nu}$, Eq. (2.19), assume the respective forms

$$\theta^{k0}{}_{,0}=-\hat{\mathbf{n}}_j\theta^{jk}{}_{,0} \ , \quad \theta^{00}{}_{,0}=\hat{\mathbf{n}}_j\hat{\mathbf{n}}_k\theta^{jk}{}_{,0} \ , \tag{3.11}$$

$$\phi^{k0}{}_{,0}=\hat{\mathbf{n}}_j\phi^{jk}{}_{,0} \ , \quad \phi^{0j}{}_{,j}=0 \ , \tag{3.12}$$

where Eq. (3.7) has been used. With these equations, along with Eq. (3.7), $\widetilde{t}^{(0j)}$ can be written in terms of only single time derivatives of the purely spatial components of the fields:

$$\begin{aligned}\widetilde{t}^{(0j)}=\frac{1}{16\pi}[&\tfrac{1}{4}\hat{\mathbf{n}}_j\hat{\mathbf{n}}_k\hat{\mathbf{n}}_l\hat{\mathbf{n}}_m\hat{\mathbf{n}}_n\theta^{kl}{}_{,0}\theta^{mn}{}_{,0}-\hat{\mathbf{n}}_j\hat{\mathbf{n}}_k\hat{\mathbf{n}}_l\theta^{km}{}_{,0}\theta^{lm}{}_{,0}+\tfrac{1}{2}\hat{\mathbf{n}}_j\theta^{ik}{}_{,0}\theta^{ik}{}_{,0}+\tfrac{1}{2}\hat{\mathbf{n}}_j\hat{\mathbf{n}}_k\hat{\mathbf{n}}_l\theta^{ii}{}_{,0}\theta^{kl}{}_{,0}\\&-\tfrac{1}{4}\hat{\mathbf{n}}_j\theta^{ii}{}_{,0}\theta^{ii}{}_{,0}-\tfrac{1}{2}\hat{\mathbf{n}}_j\phi^{ik}{}_{,0}\phi^{ik}{}_{,0}+2\hat{\mathbf{n}}_k\phi^{ik}{}_{,0}\phi^{ij}{}_{,0}-\hat{\mathbf{n}}_j\hat{\mathbf{n}}_k\hat{\mathbf{n}}_l\phi^{ik}{}_{,0}\phi^{il}{}_{,0}+\tfrac{2}{3}(\phi^{ij}{}_{,0}A_i+\phi^{jk}{}_{,0}A_i\hat{\mathbf{n}}_i\hat{\mathbf{n}}_k-\phi^{0i}A_{i,0}\hat{\mathbf{n}}_j)] \ .\end{aligned} \tag{3.13}$$

For any Euclidean tensor Q^{ij}, the "transverse-traceless" part, Q^{ij}_{TT}, is given by

$$Q^{ij}_{\mathrm{TT}}=P^i_kP^j_lQ^{kl}-\tfrac{1}{2}P^{ij}P_{kl}Q^{kl} \ , \tag{3.14}$$

where P_{ij} is the projection operator defined by

$$P_{ij}\equiv\delta_{ij}-\hat{\mathbf{n}}_i\hat{\mathbf{n}}_j \ . \tag{3.15}$$

When Eq. (3.13) for $\widetilde{t}^{(0j)}$ is inserted into Eq. (3.9), it then follows that

$$\frac{dP^0}{dt}=-\frac{R^2}{32\pi}\oint(\theta^{ij}_{\mathrm{TT},0}\theta^{ij}_{\mathrm{TT},0}-\phi^{ij}_{\mathrm{TT},0}\phi^{ij}_{\mathrm{TT},0}+\tfrac{4}{3}\phi^{0i}{}_{,0}A_i-\tfrac{4}{3}\phi^{0i}A_{i,0})d\Omega \ . \tag{3.16}$$

Since $\theta^{ij}_{\mathrm{TT}}=h^{(ij)}_{\mathrm{TT}}$ and $\phi^{ij}_{\mathrm{TT}}=h^{[ij]}_{\mathrm{TT}}$, we have the result that the NGT gravitational-wave "luminosity distribution" (energy radiated per unit time per unit solid angle) is given by

$$\frac{dL}{d\Omega}=\frac{R^2}{32}\langle h^{(ij)}_{\mathrm{TT},0}h^{(ij)}_{\mathrm{TT},0}-h^{[ij]}_{\mathrm{TT},0}h^{[ij]}_{\mathrm{TT},0}+\tfrac{4}{3}h^{[0i]}{}_{,0}A_i-\tfrac{4}{3}h^{[0i]}A_{i,0}\rangle\ , \tag{3.17}$$

where the brackets denote an average over several characteristic wavelengths.

IV. GRAVITATIONAL WAVES FROM BINARY SYSTEMS

To determine the total energy lost by a binary system to gravitational waves, solutions for the fields generated by the system must be obtained for insertion into Eq. (3.17). A post-Newtonian system of two stars that are small compared to their separation will be assumed, thus allowing all tidal interactions between them to be ignored. Each body's structure may then be treated as static and spherical in its own rest frame.

In order to obtain solutions for $\theta^{\mu\nu}$, $\phi^{\mu\nu}$, and A_μ, a post-Newtonian expansion of their respective source densities is made. (See Ref. 43 for background information on the post-Newtonian approximation.) In the post-Newtonian approximation, the level of accuracy is measured by the expansion parameter ϵ defined as

$$\epsilon^2\equiv(\text{maximum value of } U)\gtrsim U,v^2,\Pi,p/\rho_0\ , \tag{4.1}$$

where U is the Newtonian potential produced by rest-mass density ρ_0 according to

$$\nabla^2U=-4\pi\rho_0\ ,\quad U(\mathbf{x},t)=\int\frac{\rho_0(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d^3x'\ , \tag{4.2}$$

where $\nabla^2\equiv\partial^2/\partial x^i\partial x^i$. The sources of the fields in NGT, $T^{(\mu\nu)}$, $T^{[\mu\nu]}$, and S^μ are expected to have the following post-Newtonian orders:

$$T^{00}=O(\rho_0\epsilon^{2s})\ ,\quad T^{(0j)}=O(\rho_0\epsilon^{2s+1})\ ,\quad T^{(ij)}=O(\rho_0\epsilon^{2s+2})\ , \tag{4.3a}$$

$$T^{[0j]}=O(f^2|\nabla n|\epsilon^{2s+4})\ ,\quad T^{[ij]}=O(f^2|\nabla n|\epsilon^{2s+5})\ , \tag{4.3b}$$

$$S^0=O(f^2n\epsilon^{2s})\ ,\quad S^j=O(f^2n\epsilon^{2s+1})\ , \tag{4.3c}$$

where $s=0,1,2,3,\ldots$ (positive integer) determines the order. Then from Eqs. (2.20)—(2.21), (2.22)—(2.23), and (2.26)—(2.27), the fields $\theta^{\mu\nu}$, $\phi^{\mu\nu}$, and A_μ are, respectively, expected to have post-Newtonian solutions whose orders are

$$\theta^{00}=O(\epsilon^{2s+2})\ ,\quad \theta^{0j}=O(\epsilon^{2s+3})\ ,\quad \theta^{ij}=O(\epsilon^{2s+4})\ , \tag{4.4a}$$

$$\phi^{0j}=O(\kappa|\nabla\epsilon^{2s+2}|)\ ,\quad \phi^{ij}=O(\kappa|\nabla\epsilon^{2s+3}|)\ , \tag{4.4b}$$

$$A_0=O(\kappa\nabla^2\epsilon^{2s+2})\ ,\quad A_j=O(\kappa\nabla^2\epsilon^{2s+3})\ , \tag{4.4c}$$

where $\kappa\equiv f^2n/\rho_0$.

For $T^{(\mu\nu)}$ given by Eq. (2.29),

$$T^{00}=\rho_0(1+\Pi+v^2+2U)+O(\rho_0\epsilon^4)\ , \tag{4.5a}$$

$$T^{(0j)}=\rho_0(1+\Pi+v^2+2U)v^j+pv^j+O(\rho_0\epsilon^5)\ , \tag{4.5b}$$

$$T^{(ij)}=\rho_0(1+\Pi+v^2+2U+p/\rho_0)v^iv^j+p\delta^{ij}(1-2U)+O(\rho_0\epsilon^6)\ , \tag{4.5c}$$

while for S^μ given by Eq. (A7),

$$S^0=f^2n(1+U+\tfrac{1}{2}v^2)+O(f^2n\epsilon^4)\ , \tag{4.6a}$$

$$S^j=f^2n(1+U+\tfrac{1}{2}v^2)v^j+O(f^2n\epsilon^5)\ . \tag{4.6b}$$

Using Eqs. (4.5a) and (4.5b) in Eq. (2.20), one finds that the lowest-order solutions for θ^{00} and θ^{0j} are, in the near zone ($r\ll R\ll\lambda/2\pi$),

$$\theta^{00}=-4U\ , \tag{4.7}$$

$$\theta^{0j}=-4V^j\ , \tag{4.8}$$

where

$$\nabla^2V^j=-4\pi\rho_0v^j\ ,\quad V^j(\mathbf{x},t)=\int\frac{\rho_0(\mathbf{x}'t)v'^j}{|\mathbf{x}-\mathbf{x}'|}d^3x'\ . \tag{4.9}$$

It can be shown that

$$V^j{}_{,j}=-U_{,0}\ . \tag{4.10}$$

Using Eq. (4.6) in Eq. (2.22), we find that the lowest-order solutions for ϕ^{0j} and ϕ^{ij} are in the near zone, respectively, given by

$$\phi^{0j}=-\lambda_{,j}\ , \tag{4.11}$$

where

$$\nabla^2\lambda=-4\pi S^0,\quad \lambda(\mathbf{x},t)=\int\frac{S^0(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d^3x'\ ,\quad S^0=f^2n\ , \tag{4.12}$$

and

$$\phi^{ij}=-2\alpha_{[i,j]}\ , \tag{4.13}$$

where

$$\nabla^2\alpha_i=-4\pi S^i,\quad \alpha_i(\mathbf{x},t)=\int\frac{S^i(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}d^3x'\ ,\quad S^i=f^2nv^i\ . \tag{4.14}$$

From Eqs. (4.12) and (4.14), it follows that

$$\alpha_{j,j}=-\lambda_{,0}\ . \tag{4.15}$$

With the previous results, the source densities τ can be expanded to post-Newtonian order. With $\tau^{(\mu\nu)}$, $\tau^{[\mu\nu]}$, and τ_μ, respectively, given by Eqs. (2.21), (2.23), and (2.27), their post-Newtonian expansions are

$$\tau^{00}=\rho_0(1+\Pi+v^2+4U)-\frac{3}{8\pi}U_{,j}U_{,j}-\tfrac{1}{2}\lambda_{,j}S^0{}_{,j}+\frac{4\pi}{3}S^0S^0\,, \tag{4.16a}$$

$$\tau^{(0j)}=\rho_0(1+\Pi+v^2+4U)v^j+\tilde{p}v^j+\frac{3}{4\pi}U_{,0}U_{,j}-\frac{1}{2\pi}U_{,k}V_{k,j}+\frac{1}{2\pi}V_kU_{,kj}+2\rho_0V_j$$
$$+\frac{1}{8\pi}(\lambda_{,k}\lambda_{,k0j}-2\alpha_{[i,j]k}\lambda_{,ik})+\tfrac{1}{2}\lambda_{,j}S^0{}_{,0}-\tfrac{1}{2}\lambda_{,i}S^j{}_{,i}+\alpha_{[i,j]}S^0{}_{,i}+\frac{4\pi}{3}S^0S^j\,, \tag{4.16b}$$

$$\tau^{(ij)}=\rho_0v^iv^j-\frac{1}{4\pi}U_{,i}U_{,j}-\frac{1}{2\pi}UU_{,ij}-\frac{1}{8\pi}(\lambda_{,ik}\lambda_{,jk}+\lambda_{,k}\lambda_{,kij})+\delta_{ij}\left[\tilde{p}+\frac{3}{8\pi}U_{,k}U_{,k}-2\rho_0U+\frac{1}{8\pi}\lambda_{,kl}\lambda_{,kl}-\tfrac{1}{2}\lambda_{,k}S^0{}_{,k}\right], \tag{4.16c}$$

where $\tilde{p}\equiv p-(2\pi/3)S^0S^0$,

$$\tau^{[0i]}=-\tfrac{1}{2}S^{[0,i]}-\frac{1}{24\pi}A_{0,i}+\tfrac{2}{3}(S^0U)_{,i}+\tfrac{1}{2}US^0{}_{,i}+\tfrac{1}{2}\rho_0\lambda_{,i}-\frac{1}{4\pi}\lambda_{,j}U_{,ij}\,, \tag{4.17a}$$

$$\tau^{[ij]}=-\tfrac{1}{2}S^{[i,j]}+\frac{1}{12\pi}A^{[i,j]}+\rho_0\alpha_{[i,j]}-\tfrac{5}{3}US^{[i,j]}+\tfrac{2}{3}U^{,[i}S^{j]}+\tfrac{2}{3}S^{0,[i}V^{j]}-\tfrac{2}{3}S^0V^{[i,j]}$$
$$+\frac{1}{2\pi}U_{,[i}\lambda_{,j]0}+\frac{1}{2\pi}V^k{}_{,[i}\lambda_{,j]k}-\frac{1}{2\pi}U_{,k}\alpha_{[i,j]k}+\frac{1}{2\pi}\lambda_{,k[i}V_{j],k}+\frac{1}{2\pi}\lambda_{,k}V_{[i,j]k}\,, \tag{4.17b}$$

$$\tau_0=3\rho_{0,j}\lambda_{,j}-7\pi\rho_0S^0+\tfrac{7}{2}S^0{}_{,j}U_{,j}+\tfrac{7}{4}U\nabla^2S^0-\frac{3}{4\pi}\lambda_{,jk}U_{,jk}\,, \tag{4.18a}$$

$$\tau_i=-4V^i{}_{,j}S^0{}_{,j}+2U_{,i}S^0{}_{,0}+2V^j{}_{,i}S^0{}_{,j}+\tfrac{1}{2}U_{,j}S^i{}_{,j}+\tfrac{3}{4}U_{,0}S^0{}_{,i}+\tfrac{3}{4}U_{,j}S^j{}_{,j}+\tfrac{3}{4}S^0U_{,0i}$$
$$+\tfrac{3}{4}S^jU_{,ij}+2US^0{}_{,i0}+2V^jS^0{}_{,ij}-2V^i\nabla^2S^0+7\pi\rho_0S^i+\tfrac{1}{4}U\nabla^2S^i-3(\rho_0v^i)_{,j}\lambda_{,j}-\frac{3}{4\pi}U_{,ij}\lambda_{,j0}$$
$$+\frac{3}{4\pi}V^i{}_{,jk}\lambda_{,jk}-\frac{3}{4\pi}V^j{}_{,ik}\lambda_{,jk}\,. \tag{4.18b}$$

From the components of $\tau^{(\mu\nu)}$, given by Eq. (4.16), follows the symmetric post-Newtonian energy-momentum complex $\Theta^{\mu\nu}$, whose components are

$$\Theta^{00}=\rho^*(1+\tfrac{1}{2}v^2-\tfrac{1}{2}U+\Pi)-\frac{2\pi}{3}S^0S^0\,, \tag{4.19a}$$

$$\Theta^{0j}=\rho^*\left[v^j\left[1+\Pi+\tfrac{1}{2}v^2+3U+\frac{\tilde{p}}{\rho_0}\right]-\tfrac{7}{2}V^j-\tfrac{1}{2}W^j\right]$$
$$-\frac{2\pi}{3}S^0S^j\,, \tag{4.19b}$$

$$\Theta^{ij}=\rho^*v^iv^j+\tilde{p}\delta^{ij}+\frac{1}{4\pi}(U^{,i}U^{,j}-\tfrac{1}{2}\delta^{ij}|\nabla U|^2)\,, \tag{4.19c}$$

where

$$W_j\equiv\chi_{,0j}+V_j \tag{4.20}$$

with the "superpotential" χ defined according to

$$\nabla^2\chi=-2U\,,\quad \chi=-\int\rho_0|\mathbf{x}-\mathbf{x}'|\,d^3x'\,, \tag{4.21}$$

and from Eq. (A46),

$$\rho^*=\rho_0(1+\tfrac{1}{2}v^2+3U) \tag{4.22}$$

to post-Newtonian order. The $\Theta^{\mu 0}$ define conserved energy-momentum and angular momentum to post-Newtonian order:

$$P^\mu=\int\Theta^{\mu0}d^3x\,,\quad J^{\mu\nu}=2\int x^{[\mu}\Theta^{\nu]0}d^3x \tag{4.23}$$

(i.e., $\Theta^{\mu\nu}$ is equivalent to $\tau^{(\mu\nu)}$ modulo perfect divergences at post-Newtonian order). Through the $(S^0)^2$ terms appearing in $\Theta^{\mu0}$, the number density of fermion particles is seen to contribute to the energy and momentum of a system in NGT at post-Newtonian order.

At post-Newtonian order, the matter response equation (2.28) splits up in the following manner: the time equation is the energy conservation equation,

$$\rho^*\frac{d\Pi}{dt}+pv^j{}_{,j}=O(\epsilon^5|\nabla\rho_0|)\,, \tag{4.24}$$

while the spatial equations are the Newton-Euler equations of motion,

$$\rho^*\frac{dv^i}{dt}-\rho^*U_{,i}+\tilde{p}_{,i}+O(\epsilon^4\rho_{0,i})\,, \tag{4.25}$$

where the comoving derivative $d/dt\equiv\partial/\partial t+v^k\partial/\partial x^k$.

The previous post-Newtonian results are now applied to the assumed binary system. From Eq. (4.19a), the mass of body A is given by

$$m_A=\int_{\text{body }A}\Theta^{00}d^3x$$
$$=\int_{\text{body }A}\left[\rho^*(1+\Pi-\tfrac{1}{2}\bar{U})-\frac{2\pi}{3}S^0S^0\right]d^3x\,, \tag{4.26}$$

where

$$\bar{U} \equiv \int_{\text{body } A} \rho^*(\mathbf{x}') \, |\mathbf{x}-\mathbf{x}'|^{-1} d^3x \;, \tag{4.27}$$

while its center of mass is given by

$$\mathbf{x}_A = \frac{1}{m_A} \int_{\text{body } A} \Theta^{00} \mathbf{x} \, d^3x$$
$$= \int_{\text{body } A} \left[\rho^*(1+\Pi-\tfrac{1}{2}\bar{U}) - \frac{2\pi}{3} S^0 S^0 \right] \mathbf{x} \, d^3x \;. \tag{4.28}$$

The full Newtonian potential U is given by

$$U(x) = \begin{cases} \bar{U}_A + \sum_{B \neq A} m_B \, |\mathbf{x}-\mathbf{x}_B|^{-1} & (x \text{ inside body } A) \;, \\ \sum_B m_B \, |\mathbf{x}-\mathbf{x}_B|^{-1} & (x \text{ outside body } A) \;. \end{cases} \tag{4.29}$$

Then the total energy of the system P^0 is, from Eq. (4.23), given by

$$P^0 = \sum_A \left[m_A + \tfrac{1}{2} m_A v_A{}^2 - \tfrac{1}{2} \sum_{B \neq A} m_A m_B / r_{AB} \right] , \tag{4.30}$$

where $r_{AB} \equiv |\mathbf{x}_A - \mathbf{x}_B|$ and higher-order corrections arising from expanding $|\mathbf{x}-\mathbf{x}_B|^{-1}$ about $\mathbf{x}=\mathbf{x}_A$ have been dropped. The orbital terms in Eq. (4.30) may be evaluated to the required order using the equations for a Keplerian orbit

$$r = r_{AB} = a(1-e^2)(1+e\cos\psi)^{-1} \;, \tag{4.31}$$

$$\dot{\psi} = d\psi/dt = [ma(1-e^2)]^{1/2}/r^2 \;, \tag{4.32}$$

where ψ is the angle between the periastron direction and the direction of $\mathbf{r}$. The result is

$$P^0 = m + E \;, \quad E = -\tfrac{1}{2} m_1 m_2 / a \;, \tag{4.33}$$

where $m = m_1 + m_2$. The rate at which the system loses energy due to gravitational waves generated by the orbital motions is then given by

$$dP^0/dt = (dE/dt)_{\text{gravitational waves}} \tag{4.34}$$

(where m_1 and m_2 are constant as long as tidal interactions are ignored).

From Eq. (3.6), it can be seen that in the radiation zone the fields are determined by moments of their source densities:

$$\psi(\mathbf{x},t) = -\frac{4}{R} \left[\int \tau \, d^3x + \hat{\mathbf{n}}^i \frac{d}{dt} \int \tau x^i d^3x + \tfrac{1}{2} \hat{\mathbf{n}}^i \hat{\mathbf{n}}^j \frac{d^2}{dt^2} \int \tau x^i x^j d^3x + \cdots \right] , \tag{4.35}$$

where the first integral is the monopole moment of τ, the second is the dipole moment, and the third is the quadrupole moment. Using the post-Newtonian source densities given by Eqs. (4.16)—(4.18) and the Newtonian equations of motion (4.25), we have evaluated these integrals to appropriate order for the case of the assumed binary system.

Since the center of mass of the binary system, defined by

$$\mathbf{X} = (m_1 \mathbf{x}_1 + m_2 \mathbf{x}_2)/(m_1 + m_2) \;, \tag{4.36}$$

is unaccelerated, the relative orbital motion may be separated from the center-of-mass motion. To analyze the relative orbital motion, a coordinate frame can be chosen whose origin coincides with the center of mass. The relative orbital variables are

$$\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2 \;, \quad \mathbf{r} = \frac{d\mathbf{r}}{dt} \;, \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} \;,$$
$$r \equiv |\mathbf{r}| \;, \quad \dot{r} \equiv \frac{dr}{dt} = \frac{\mathbf{v}\cdot\mathbf{r}}{r} \;. \tag{4.37}$$

From Eq. (4.25) follows the equations of motion

$$\mathbf{a}_1 = -m_2 \mathbf{r}/r^3, \quad \mathbf{a}_2 = m_1 \mathbf{r}/r^3 \;. \tag{4.38}$$

With the assumption of static structure, the moment integrals in Eq. (4.35) when evaluated for the appropriate source densities given by Eqs. (4.16)—(4.18) lead to the following results for the fields generated by the binary system (fields evaluated in the radiation zone to lowest post-Newtonian order):

$$h_{\mathrm{TT}}^{(ij)} = -(4\mu/R)(v^i v^j - m r^i r^j / r^3)_{\mathrm{TT}} \;, \tag{4.39a}$$

$$h_{\mathrm{TT}}^{[ij]} = -(2\mu m d/R)(r^{[i} \hat{\mathbf{n}}^{j]}/r^3)_{\mathrm{TT}} \;, \tag{4.39b}$$

$$h^{[0i]} = -(\mu m d / R r^3)(7 r^i + \hat{\mathbf{n}}^i \hat{\mathbf{n}} \cdot \mathbf{r}) \;, \tag{4.39c}$$

$$A_i = -(12 \mu m d / R r^3)(v^i - 3 r^i \dot{r}/r) \;, \tag{4.39d}$$

where d is the dipole parameter defined as

$$d \equiv l_1{}^2/m_1 - l_2{}^2/m_2 \;, \tag{4.40}$$

with

$$l_A{}^2 \equiv \int_{\text{body } A} S_A^0 \, d^3x \;. \tag{4.41}$$

The result that the fields given by Eqs. (4.39b)—(4.39d) are generated by dipole moments of S^0 arises from evaluating terms involving $\int S^0 \mathbf{x} \, d^3x$ that occur in the moment expansions of the fields. With the assumption of static structure, it can be shown that

$$\frac{d^2}{dt^2} \int S^0 x^i d^3x = -(\mu m \, d r^i / r^3)[1 + O(\epsilon^2)] \;, \tag{4.42}$$

where use has been made of the fact that (at lowest post-Newtonian order)

$$\frac{d}{dt} \int S^0(\mathbf{x},t) \mathbf{x} \, d^3x = \int S^0(\mathbf{x},t) \frac{d\mathbf{x}}{dt} d^3x \;, \tag{4.43}$$

where $d/dt = \partial/\partial t + v^k \partial/\partial x^k$ [from Eq. (A41)].

Inserting the gravitational waveforms as given by Eq. (4.39) into Eq. (3.17) and integrating over solid angle using the identity

$$\int \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j \, d\Omega = \frac{4\pi}{3} \delta_{ij} \;, \tag{4.44}$$

one finds that the energy loss-rate dE/dt for the binary system is given by

$$(dE/dt)_{\rm NGT}=(dE/dt)_{\rm GR}+(dE/dt)_{\rm dipole}+(\text{post-Newtonian quadrupole and higher-order corrections})\ , \qquad (4.45)$$

where $(dE/dt)_{\rm GR}$ and $(dE/dT)_{\rm dipole}$ are, respectively, given by Eqs. (1.3) and (1.4). With the orbital period given by Kepler's third law

$$(P/2\pi)^2=a^3/m\ , \qquad (4.46)$$

and after evaluating the orbital terms in Eqs. (1.3) and (1.4) using Eqs. (4.31) and (4.32), it follows that $(dE/dt)_{\rm GR}$ is the Peters and Mathews[49] result that

$$(dE/dt)_{\rm GR}=-\tfrac{32}{5}(\mu^2m^3/a^5)f_{\rm GR}(e)\ , \qquad (4.47a)$$

$$f_{\rm GR}(e)=(1-e^2)^{-7/2}(1+\tfrac{73}{24}e^2+\tfrac{37}{96}e^4)\ , \qquad (4.47b)$$

while $(dE/dt)_{\rm dipole}$ is given by

$$(dE/dt)_{\rm dipole}=-\tfrac{88}{3}(\mu^2m^3d^2/a^7)f_{\rm dipole}(e)\ , \qquad (4.48a)$$

$$f_{\rm dipole}(e)=(1-e^2)^{-11/2}(1+\tfrac{19}{2}e^2+\tfrac{69}{8}e^4+\tfrac{9}{16}e^6)\ . \qquad (4.48b)$$

From Eqs. (4.33) and (4.46), it follows that the loss of energy from the binary system results in a secular change in the orbital period given by

$$(\dot P/P)=-\tfrac{3}{2}(\dot E/E)\ . \qquad (4.49)$$

Equations (4.47)–(4.48) then give the result of NGT that

$$(\dot P/P)_{\rm NGT}=(\dot P/P)_{\rm GR}+(\dot P/P)_{\rm dipole}\ , \qquad (4.50a)$$

$$(P/P)_{\rm GR}=-\tfrac{96}{5}\mu m^{2/3}(2\pi/P)^{8/3}f_{\rm GR}(e)\ , \qquad (4.50b)$$

$$(\dot P/P)_{\rm dipole}=-88\mu d^2(2\pi/P)^4f_{\rm dipole}(e)\ . \qquad (4.50c)$$

The relative size of $(\dot P/P)_{\rm dipole}$ to $(\dot P/P)_{\rm GR}$ is of order

$$(\dot P/P)_{\rm dipole}(\dot P/P)_{\rm GR}{}^{-1}\sim(d/a)^2\ . \qquad (4.51)$$

Hence, NGT makes corrections of order $(d/a)^2$ to the predictions of general relativity for gravitational radiation from binary systems. For a system consisting of two solar mass stars orbiting each other at a minimum distance of one solar radius, the correction is of order $(l_1{}^2-l_2{}^2)^2\times10^{-12}$, where l_A is in kilometers.

V. THE BINARY PULSAR PSR 1913 + 16 AS A TEST OF NGT

For a system such as the binary pulsar in which there is a neutron star with a highly relativistic interior, one can no longer apply the assumptions of the post-Newtonian limit everywhere, except possibly in the interbody region between the relativistic bodies. Even so, it can be assumed that the previous results carry over unchanged. It is expected that the worst that can happen is for there to arise complications due to Einstein-Infeld-Hoffman effects of the Will-Eardley type, i.e., self-gravitational corrections in the sources of the fields (see Ref. 43, Sec. 11.3, for a complete discussion of these effects).

In addition to the observed secular decrease in the orbital period of the binary pulsar, two additional orbital parameters of the system that are relevant to testing NGT are the mean rate of the precession of the periastron and the red-shift–Doppler parameter. The mean rate of periastron precession for a binary system in NGT has been reported by Moffat and Woolgar[20] to be given by

$$\langle\dot\omega\rangle_{\rm NGT}=\langle\dot\omega\rangle_{\rm GR}[1-l^4(1+e^2/4)/m^2a^2(1-e^2)^2]\ , \qquad (5.1)$$

where $l^2=l_1{}^2+l_2{}^2$ and $\langle\dot\omega\rangle_{\rm GR}$ is the general relativistic result

$$\langle\dot\omega\rangle_{\rm GR}=6\pi Gm/a(1-e^2)P\ . \qquad (5.2)$$

The red-shift–Doppler parameter γ is derived by comparing proper time at the pulsar's point of emission to coordinate time.[50] The ratio of the NGT to GR contribution to γ is of order $l_{\rm pulsar}{}^4/ma^3$. For PSR 1913 + 16, one expects that $m\lesssim3M_\odot$ and $a\sim R_\odot$, giving

$$l_{\rm pulsar}{}^4/ma^3\lesssim10^{-18}l_{\rm pulsar}{}^4\ {\rm km}^{-4}\ . \qquad (5.3)$$

It has been reported that the interior solution for a neutron star in NGT yields an upper bound for the l parameter of ~10 km (Ref. 40). With this value for $l_{\rm pulsar}$, NGT makes a negligible correction to the general relativistic result for γ:

$$\gamma=m_2(m_1+2m_2)m^{-4/3}(P/2\pi)^{1/3}e\ . \qquad (5.4)$$

Using the measured values of P and e (see Table I) in Eqs. (4.50), (5.1), and (5.4), one finds that the masses and the l parameters of the pulsar and its companion are constrained by the relations

$$|\dot P|=1.7\times10^{-12}\mu m^{2/3}(1+7.7\times10^{-12}d^2m^{-2/3})\ , \qquad (5.5a)$$

$$\langle\dot\omega\rangle=2.113m^{2/3}(1-6.9\times10^{-13}l^4m^{-8/3})\ , \qquad (5.5b)$$

$$\gamma=2.9\times10^{-3}m_cm^{-4/3}(m_p+2m_c)\ , \qquad (5.5c)$$

where m_p and m_c are the respective masses of the pulsar and its companions measured in solar mass units, and where their corresponding l parameters are measured in kilometers.

The observed value of $\langle\dot\omega\rangle$ constrains the total mass of the system to be $m\gtrsim2.8m_\odot$, depending upon the value of $l^2=l_p{}^2+l_c{}^2$ appearing in Eq. (5.5b), where l_p and l_c are the l parameters of the pulsar and its companion, respectively. If one starts by assuming that $m\sim2.8m_\odot$, i.e., that l makes a small correction to $\langle\dot\omega\rangle$, then the constraint imposed by the observed value of γ is that $m_p\cong m_c$. If it is then assumed that, phenomenologically, l_c varies linearly with mass over the mass region of interest according to $l_c\cong m_c\cong\frac{1}{2}\alpha m$, and neglecting l_p, since it is expected that $l_p\lesssim10$ km, it follows that

$$\langle\dot\omega\rangle=2.113m^{2/3}(1-4.3\times10^{-6}\tilde\alpha^4m^{4/3}) \qquad (5.6)$$

and

$$|\dot P|=4.25\times10^{-13}m^{5/3}(1+2\times10^{-4}\tilde\alpha^4m^{4/3})\ , \qquad (5.7)$$

where $\tilde\alpha\equiv\alpha/100$.

For $\tilde\alpha>0$, NGT is seen to increase the mass inferred

TABLE I. The measured orbital parameters of PSR 1913 + 16 that are relevant to testing NGT.[a]

Orbital period	$P = 27\,906.981\,63 \pm 0.000\,02$ s
Orbital period derivative	$\dot{P} = (-2.40 \pm 0.09) \times 10^{-12}$ ss^{-1}
Eccentricity	$e = 0.617\,127 \pm 0.000\,003$
Mean rate of periastron advance	$\langle \dot{\omega} \rangle = 4.2263 \pm 0.0003$ deg yr^{-1}
Redshift-Doppler parameter	$\gamma = 0.004\,38 \pm 0.000\,12$ s

[a]From Weisberg and Taylor, Ref. 45.

from $\langle \dot{\omega} \rangle$ and to decrease the mass inferred from $\dot{P}$. The measured values of $\langle \dot{\omega} \rangle$ and $\dot{P}$ imply that the largest value of $\tilde{\alpha}$ that Eqs. (5.6) and (5.7) can accommodate is $\tilde{\alpha} \sim 2.5$. This then constrains $l_c \lesssim 350$ km. With such small l parameters, the results for the masses do not differ significantly from those obtained within general relativity:[45] $m_p = (1.42 \pm 0.03) m_\odot$ and $m_c = (1.40 \pm 0.03) m_\odot$.

VI. CONCLUSIONS

The binary pulsar provides a stringent test of alternative gravitational theories that predict dipole gravitational radiation[48] (see also Ref. 43, Sec. 12.3). For the case of NGT, we found that binary pulsar observations impose tight constraints upon the l parameters of the pulsar and its companion: $l_p \lesssim 10$ km, $l_c \lesssim 350$ km, respectively. If the companion is a neutron star, for which l_c is expected to be $\lesssim 10$ km, the effects of dipole radiation in NGT are negligible, however, and the theory is not constrained by binary pulsar observations. Whether this conclusion will hold in other close binary systems containing, say, white dwarfs, is a question currently under study.

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APPENDIX A: THE NGT FIELD EQUATIONS

The fundamental field variables of NGT are the nonsymmetric metric $g_{\mu\nu}$ and the nonsymmetric connection $W^\rho_{\mu\nu}$. Covariant and contravariant components of $g_{\mu\nu}$ are related by

$$g^{\mu\nu} g_{\sigma\nu} = g^{\nu\mu} g_{\nu\sigma} = \delta^\mu_\sigma \,, \tag{A1}$$

where the order of the indices is important. Field equations are obtained from an action principle with the Lagrangian

$$\mathscr{L}_{\rm NGT} = \mathscr{L}_G + \mathscr{L}_{\rm NG} \,, \tag{A2}$$

where $\mathscr{L}_G$ is the gravitational part given by

$$\mathscr{L}_G = -\sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(W) \,, \tag{A3}$$

where

$$R_{\mu\nu}(W) \equiv W^\beta_{\mu\nu,\beta} - \tfrac{1}{2}(W^\beta_{\mu\beta,\nu} + W^\beta_{\nu\beta,\mu}) - W^\beta_{\alpha\nu} W^\alpha_{\mu\beta} + W^\beta_{\alpha\beta} W^\alpha_{\mu\nu} \,, \tag{A4}$$

$$g \equiv \det \| g_{\mu\nu} \| \,, \tag{A5}$$

and $\mathscr{L}_{\rm NG}$ is the nongravitational or matter Lagrangian. Using a form of $\mathscr{L}_{\rm NG}$ appropriate for perfect fluids, Vincent[21] has derived a perfect-fluid stress-energy tensor within NGT that has the *same structure* as for general relativity:

$$T^{\mu\nu} = [\rho_0(1+\Pi) + p] u^\mu u^\nu - p g^{\mu\nu} \,, \tag{A6}$$

where ρ_0 is the fluid rest-mass density, Π is the rest specific internal energy density, and p is the pressure. Note that $T^{\mu\nu}$ is nonsymmetric because of the presence of $g^{\mu\nu}$. In addition to T, there occurs a new source for the fields in the form of a number-density current of fermion particles, S, having components

$$S^\mu = f^2 n u^\mu \,. \tag{A7}$$

With this perfect-fluid model of matter, $\mathscr{L}_{\rm NG}$ has the phenomenological form

$$\mathscr{L}_{\rm NG} = 8\pi \sqrt{-g}\, g_{\mu\nu} T^{\mu\nu} + \frac{8\pi}{3} \sqrt{-g}\, W_\mu S^\mu \,, \tag{A8}$$

where $T^{\mu\nu}$ and $S^{\mu\nu}$ are, respectively, given by Eqs. (A6) and (A7), and $W_\mu \equiv W^\lambda_{[\mu\lambda]}$.

The primary field equation of NGT is obtained by varying $\mathscr{L}_{\rm NGT}$ with respect to $g_{\mu\nu}$, giving

$$G^{\mu\nu}(W) = 8\pi T^{\mu\nu} \,, \tag{A9}$$

where

$$G^{\mu\nu}(W) \equiv R^{\mu\nu}(W) - \tfrac{1}{2} g^{\mu\nu} R(W) \,, \tag{A10a}$$

$$R^{\mu\nu}(W) \equiv g^{\mu\beta} g^{\alpha\nu} R_{\alpha\beta}(W) \,, \tag{A10b}$$

$$R(W) \equiv g^{\alpha\beta} R_{\alpha\beta}(W) \,. \tag{A10c}$$

The metric compatibility equation and secondary field equation for $g^{[\mu\nu]}$ is obtained by varying $\mathscr{L}_{\rm NGT}$ with respect to $W^\lambda_{\mu\nu}$, giving

$$\hat{g}^{\mu\nu}{}_{,\lambda} - \delta^\nu_\lambda \hat{g}^{(\mu\alpha)}{}_{,\alpha} + \hat{g}^{\alpha\nu} W^\mu_{\alpha\lambda} + \hat{g}^{\mu\alpha} W^\nu_{\lambda\alpha} - \delta^\nu_\lambda \hat{g}^{\alpha\beta} W^\mu_{\alpha\beta} - \hat{g}^{\mu\nu} W^\alpha_{\lambda\alpha} = \frac{4\pi}{3} \sqrt{-g}\,(S^\mu \delta^\nu_\lambda - S^\nu \delta^\mu_\lambda) \,, \tag{A11}$$

where for any variable X, $\hat{X} \equiv \sqrt{-g}\, X$. Contracting on μ and λ in Eq. (A11) yields the secondary field equation for $g^{[\mu\nu]}$,

$$\hat{g}^{[\mu\nu]}{}_{,\nu} = 4\pi \sqrt{-g}\, S^\mu \,, \tag{A12}$$

while contracting on ν and λ in Eq. (A11) yields

$$\hat{g}^{(\mu\nu)}{}_{,\nu}+\hat{g}^{\alpha\beta}W^{\mu}_{\alpha\beta}+\tfrac{2}{3}\hat{g}^{\mu\alpha}W_{\alpha}=0\ . \tag{A13}$$

Substituting Eq. (A13) back into Eq. (A11) puts the metric compatibility equation into the form

$$\hat{g}^{\mu\nu}{}_{,\lambda}+\hat{g}^{\alpha\nu}W^{\mu}_{\alpha\lambda}+\hat{g}^{\mu\alpha}W^{\nu}_{\lambda\alpha}-\hat{g}^{\mu\nu}W^{\alpha}_{\lambda\alpha}+\tfrac{2}{3}\delta^{\nu}_{\lambda}\hat{g}^{\mu\alpha}W_{\alpha} =\frac{4\pi}{3}\sqrt{-g}\,(S^{\mu}\delta^{\nu}_{\lambda}-S^{\nu}\delta^{\mu}_{\lambda})\ . \tag{A14}$$

The metric compatibility equation (A14), can be written in a more conventional form by introducing three additional connection fields: $\Gamma^{\rho}_{\mu\nu}$, $D^{\rho}_{\mu\nu}$, and $\Lambda^{\rho}_{\mu\nu}$ defined, respectively, from $W^{\rho}_{\mu\nu}$ and S^{ρ} by the relations

$$\Gamma^{\lambda}_{\mu\nu}\equiv W^{\lambda}_{\mu\nu}+\tfrac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}\ , \tag{A15a}$$

$$\Gamma_{\mu}\equiv\Gamma^{\lambda}_{[\mu\lambda]}=0\ , \tag{A15b}$$

$$g_{\rho\nu}D^{\rho}_{\mu\sigma}+g_{\mu\rho}D^{\rho}_{\sigma\nu}=-\frac{4\pi}{3}S^{\rho}(g_{\mu\sigma}g_{\rho\nu}-g_{\mu\rho}g_{\sigma\nu}+g_{\mu\nu}g_{[\sigma\rho]})\ , \tag{A16}$$

and

$$\Lambda^{\rho}_{\mu\nu}\equiv\Gamma^{\rho}_{\mu\nu}+D^{\rho}_{\mu\nu}\ . \tag{A17}$$

Using these definitions in Eq. (A14) gives the results

$$g_{\mu\nu,\lambda}-g_{\alpha\nu}\Lambda^{\alpha}_{\mu\lambda}-g_{\mu\alpha}\Lambda^{\alpha}_{\lambda\nu}=0\ , \tag{A18}$$

$$\sqrt{-g}_{,\lambda}-\sqrt{-g}\,\Gamma^{\mu}_{(\lambda\mu)}=\frac{4\pi}{3}\sqrt{-g}\,S^{\mu}g_{[\mu\lambda]}\ . \tag{A19}$$

If the definition of $\Gamma^{\rho}_{\mu\nu}$, Eq. (A15), is put into the definition of $R_{\mu\nu}(W)$, Eq. (A4), one obtains

$$R_{\mu\nu}(W)=R_{\mu\nu}(\Gamma)+\tfrac{2}{3}W_{[\mu,\nu]}\ , \tag{A20}$$

with

$$R_{\mu\nu}(\Gamma)=\Gamma^{\beta}_{\mu\nu,\beta}-\tfrac{1}{2}(\Gamma^{\beta}_{(\mu\beta),\nu}+\Gamma^{\beta}_{(\nu\beta),\mu})+\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{(\alpha\beta)}-\Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}\ . \tag{A21}$$

In terms of $\Lambda^{\rho}_{\mu\nu}$ and $D^{\rho}_{\mu\nu}$, $R_{\mu\nu}(\Gamma)$ can then be written in terms of its symmetric and antisymmetric parts, respectively, given by

$$R_{(\mu\nu)}(\Gamma)=R_{(\mu\nu)}(\Lambda)-R_{(\mu\nu)}(D)-R_{(\mu\nu)}(\Lambda D)\ , \tag{A22}$$

where

$$R_{(\mu\nu)}(\Lambda)=\Lambda^{\beta}_{(\mu\nu),\beta}-\tfrac{1}{2}(\Lambda^{\beta}_{(\mu\beta),\nu}+\Lambda^{\beta}_{(\nu\beta),\mu}) +\Lambda^{\alpha}_{(\mu\nu)}\Lambda^{\beta}_{(\alpha\beta)}-\Lambda^{\beta}_{(\mu\alpha)}\Lambda^{\alpha}_{(\beta\nu)}-\Lambda^{\beta}_{[\mu\alpha]}\Lambda^{\alpha}_{[\beta\nu]}\ , \tag{A23a}$$

$$R_{(\mu\nu)}(D)=D^{\beta}_{(\mu\nu),\beta}-\tfrac{1}{2}(D^{\beta}_{(\mu\beta),\nu}+D^{\beta}_{(\nu\beta),\mu}) -D^{\alpha}_{(\mu\nu)}D^{\beta}_{(\alpha\beta)}+D^{\beta}_{(\mu\alpha)}D^{\alpha}_{(\beta\nu)}+D^{\beta}_{[\mu\alpha]}D^{\alpha}_{[\beta\nu]}\ , \tag{A23b}$$

$$R_{(\mu\nu)}(\Lambda D)=\Lambda^{\alpha}_{(\mu\nu)}D^{\beta}_{(\alpha\beta)}+\Lambda^{\beta}_{(\alpha\beta)}D^{\alpha}_{(\mu\nu)} -\Lambda^{\beta}_{(\mu\alpha)}D^{\alpha}_{(\beta\nu)}-\Lambda^{\beta}_{(\nu\alpha)}D^{\alpha}_{(\beta\mu)} -\Lambda^{\beta}_{[\mu\alpha]}D^{\alpha}_{[\beta\nu]}-\Lambda^{\alpha}_{[\beta\nu]}D^{\beta}_{[\mu\alpha]}\ , \tag{A23c}$$

while

$$R_{[\mu\nu]}(\Gamma)=R_{[\mu\nu]}(\Lambda)-R_{[\mu\nu]}(D)-R_{[\mu\nu]}(\Lambda D)\ , \tag{A24}$$

where

$$R_{[\mu\nu]}(\Lambda)=\Lambda^{\beta}_{[\mu\nu],\beta}+\Lambda^{\alpha}_{[\mu\nu]}\Lambda^{\beta}_{(\alpha\beta)}-\Lambda^{\beta}_{(\mu\alpha)}\Lambda^{\alpha}_{[\beta\nu]}-\Lambda^{\beta}_{[\mu\alpha]}\Lambda^{\alpha}_{(\beta\nu)}\ , \tag{A25a}$$

$$R_{[\mu\nu]}(D)=D^{\beta}_{[\mu\nu],\beta}-D^{\alpha}_{[\mu\nu]}D^{\beta}_{(\alpha\beta)}+D^{\beta}_{(\mu\alpha)}D^{\alpha}_{[\beta\nu]} +D^{\beta}_{[\mu\alpha]}D^{\alpha}_{(\beta\nu)}\ , \tag{A25b}$$

$$R_{[\mu\nu]}(\Lambda D)=\Lambda^{\alpha}_{[\mu\nu]}D^{\beta}_{(\alpha\beta)}+\Lambda^{\beta}_{(\alpha\beta)}D^{\alpha}_{[\mu\nu]}-\Lambda^{\beta}_{(\mu\alpha)}D^{\alpha}_{[\beta\nu]} -\Lambda^{\beta}_{[\mu\alpha]}D^{\alpha}_{(\beta\nu)}+\Lambda^{\beta}_{(\nu\alpha)}D^{\alpha}_{[\beta\mu]}+\Lambda^{\beta}_{[\nu\alpha]}D^{\alpha}_{(\beta\mu)}\ . \tag{A25c}$$

$G^{\mu\nu}(W)$ can be written in terms of $\Gamma^{\rho}_{\mu\nu}$ by using Eq. (A20) in Eq. (A10):

$$G^{\mu\nu}(W)=G^{\mu\nu}(\Gamma)+\tfrac{2}{3}(g^{\mu\beta}g^{\alpha\nu}-\tfrac{1}{2}g^{\mu\nu}g^{[\alpha\beta]})W_{[\alpha,\beta]}\ , \tag{A26}$$

where

$$G^{\mu\nu}(\Gamma)=R^{\mu\nu}(\Gamma)-\tfrac{1}{2}g^{\mu\nu}R(\Gamma)\ , \tag{A27a}$$

$$R^{\mu\nu}(\Gamma)=g^{\mu\beta}g^{\alpha\nu}R_{\alpha\beta}(\Gamma)\ , \tag{A27b}$$

$$R(\Gamma)=g^{\alpha\beta}R_{\alpha\beta}(\Gamma)\ . \tag{A27c}$$

Symmetrizing and antisymmetrizing,

$$G^{(\mu\nu)}(W)=G^{(\mu\nu)}(\Gamma)+\tfrac{1}{3}(2s^{\mu\beta\alpha\nu}-g^{(\mu\nu)}g^{[\alpha\beta]})W_{[\alpha,\beta]}\ , \tag{A28}$$

where

$$G^{(\mu\nu)}(\Gamma)=R^{(\mu\nu)}(\Gamma)-\tfrac{1}{2}g^{(\mu\nu)}R(\Gamma)\ , \tag{A29}$$

$$R^{(\mu\nu)}(\Gamma)=s^{\mu\beta\alpha\nu}R_{(\alpha\beta)}(\Gamma)\ , \tag{A30}$$

and

$$s^{\mu\beta\alpha\nu}=s^{\nu\beta\alpha\mu}\equiv\tfrac{1}{2}(g^{\mu\beta}g^{\alpha\nu}+g^{\nu\beta}g^{\alpha\mu})\ , \tag{A31}$$

while

$$G^{[\mu\nu]}(W)=G^{[\mu\nu]}(\Gamma)+\tfrac{1}{3}(2a^{\mu\beta\alpha\nu}-g^{[\mu\nu]}g^{[\alpha\beta]})W_{[\alpha,\beta]}\ , \tag{A32}$$

where

$$G^{[\mu\nu]}(\Gamma)=R^{[\mu\nu]}(\Gamma)-\tfrac{1}{2}g^{[\mu\nu]}R(\Gamma)\ , \tag{A33}$$

$$R^{[\mu\nu]}(\Gamma)=a^{\mu\beta\alpha\nu}R_{[\alpha\beta]}(\Gamma)\ , \tag{A34}$$

and

$$a^{\mu\beta\alpha\nu}=-a^{\nu\beta\alpha\mu}\equiv\tfrac{1}{2}(g^{\mu\beta}g^{\alpha\nu}-g^{\nu\beta}g^{\alpha\mu})\ . \tag{A35}$$

With the previous results, the field equations of NGT can now be summarized. They are

$$G^{(\mu\nu)}(\Gamma)=8\pi T^{(\mu\nu)}-\tfrac{1}{3}(2s^{\mu\beta\alpha\nu}-g^{(\mu\nu)}g^{[\alpha\beta]})W_{[\alpha,\beta]}\ , \tag{A36a}$$

$$G^{[\mu\nu]}(\Gamma)=8\pi T^{[\mu\nu]}-\tfrac{1}{3}(2a^{\mu\beta\alpha\nu}-g^{[\mu\nu]}g^{[\alpha\beta]})W_{[\alpha,\beta]}\ , \tag{A36b}$$

$$\hat{g}^{[\mu\nu]}{}_{,\nu}=4\pi\sqrt{-g}S^{\mu}\ , \tag{A36c}$$

and the metric compatibility equation

$$g_{\mu\nu,\lambda}-g_{\alpha\nu}\Lambda^{\alpha}_{\mu\lambda}-g_{\mu\alpha}\Lambda^{\alpha}_{\lambda\nu}=0\ . \tag{A37}$$

The matter response equations in a gravitational theory determine how matter responds to the gravitational fields that are present. The matter response equations for T result from general coordinate invariance of the nongravitational action under the coordinate transformation $x^{\alpha'}(P)=x^{\alpha}(P)+\xi^{\alpha}(P)$ characterized by the four infinitesimal generators ξ^{α} (Ref. 51). In NGT, these equations are

$$g_{(\rho\mu)}\hat{T}^{(\mu\nu)}{}_{,\nu}+g_{[\mu\rho]}\hat{T}^{[\mu\nu]}{}_{,\nu}+g_{(\sigma\rho)}\hat{T}^{(\mu\nu)}\Lambda^{\sigma}_{(\mu\nu)}$$
$$+2g_{[\mu\sigma]}\hat{T}^{(\mu\nu)}\Lambda^{\sigma}_{[\nu\rho]}+g_{(\sigma\rho)}\hat{T}^{[\mu\nu]}\Lambda^{\sigma}_{[\mu\nu]}$$
$$+2g_{(\mu\sigma)}\hat{T}^{[\mu\nu]}\Lambda^{\sigma}_{[\nu\rho]}+\tfrac{1}{3}W_{[\rho,\nu]}\hat{S}^{\nu}=0\ . \tag{A38}$$

The gravitational action for NGT is invariant under the gauge transformation of W_{μ} given by

$$\tilde{W}_{\mu}=W_{\mu}+\phi_{,\mu}\ , \tag{A39}$$

where ϕ is some scalar field. This results in the conservation law for S:

$$\hat{S}^{\mu}{}_{,\mu}=0\ . \tag{A40}$$

With S^{μ} given by Eq. (A7), Eq. (A40) can be written in a $(t,\mathbf{x})$ coordinate system in the form of a continuity equation for the proper fermion-particle number density n^*:

$$\partial n^*/\partial t+\nabla\cdot n^*\mathbf{v}=0\ , \tag{A41}$$

where

$$n^*\equiv nu^0\sqrt{-g}\ , \tag{A42}$$

$$v^i=u^i/u^0\ . \tag{A43}$$

Conservation of rest mass, expressed by

$$(\sqrt{-g}\,\rho_0u^{\mu})_{,\mu}=0\ , \tag{A44}$$

is built into $T^{\mu\nu}$ given by Eq. (A6). Similarly, Eq. (A44) can be written in the form of a continuity equation for the proper rest-mass density ρ^*:

$$\partial\rho^*/\partial t+\nabla\cdot\rho^*\mathbf{v}=0\ , \tag{A45}$$

where

$$\rho^*\equiv\rho_0u^0\sqrt{-g}\ . \tag{A46}$$

[1]A. Einstein, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl., Part II, 844 (1915).
[2]A. Pais, *Subtle is the Lord. . . The Science and the Life of Albert Einstein* (Clarendon, Oxford, 1982), Chap. 17.
[3]A. Einstein, *The Meaning of Relativity* (Princeton, New Jersey, 1956), Appendix II; A. Coley, Gen. Relativ. Gravit. **16**, 459 (1984).
[4]J. Callaway, Phys. Rev. **92**, 1567 (1953).
[5]S. Weinberg, Rev. Mod. Phys. **52**, 515 (1980); A. Salam, *ibid.* **52**, 525 (1980); S. L. Glashow, *ibid.* **52**, 539 (1980).
[6]J. W. Moffat, Phys. Rev. D **19**, 3554 (1979).
[7]G. Kunstatter, J. W. Moffat, and P. Savaria, Phys. Rev. D **19**, 3559 (1979).
[8]J. Moffat, Phys. Rev. D **19**, 3562 (1979).
[9]J. W. Moffat, J. Math. Phys. **21**, 1798 (1980).
[10]R. B. Mann, J. W. Moffat, and J. G. Taylor, Phys. Lett. **97B**, 73 (1980).
[11]J. W. Moffat, Phys. Rev. D **23**, 2870 (1981).
[12]R. B. Mann and J. W. Moffat, Can. J. Phys. **59**, 1723 (1981).
[13]R. B. Mann and J. W. Moffat, Can. J. Phys. **59**, 656(E) (1983).
[14]R. B. Mann and J. W. Moffat, Can. J. Phys. **59**, 1730 (1981).
[15]J. W. Moffat, in *Proceedings of the VII International School of Gravitation and Cosmology,* Erice, Sicily, edited by V. de Sabbata (World Scientific, Singapore, 1982), pp. 127–180.
[16]R. B. Mann and J. W. Moffat, Phys. Rev. D **26**, 1858 (1982).
[17]J. C. McDow and J. W. Moffat, Can. J. Phys. **60**, 1545 (1982).
[18]J. C. McDow and J. W. Moffat, Can. J. Phys. **60**, 1556 (1982).
[19]L. Campbell and J. W. Moffat, Astrophys. J. Lett. **275**, L77 (1983).
[20]J. W. Moffat and E. Woolgar, Can. J. Phys. **63**, 428 (1985).
[21]D. E. Vincent, Class. Quantum Gravit. (to be published).
[22]J. W. Moffat, Can. J. Phys. **59**, 283 (1981).
[23]J. W. Moffat, Can. J. Phys. **59**, 1289(E) (1981).
[24]R. B. Mann and J. W. Moffat, Can. J. Phys. **59**, 1592 (1981).
[25]J. W. Moffat, Phys. Rev. Lett. **50**, 709 (1983).
[26]L. Campbell, J. C. McDow, J. W. Moffat, and D. Vincent, Nature **305**, 508 (1983).
[27]J. W. Moffat, Lett. Nuovo Cimento **32**, 281 (1981).
[28]R. B. Mann and J. W. Moffat, Can. J. Phys. **58**, 1595 (1980).
[29]J. W. Moffat and J. G. Taylor, Phys. Lett. **99B**, 396 (1981).
[30]L. Campbell and J. W. Moffat, University of Toronto report, 1982 (unpublished).
[31]G. Kunstatter, J. W. Moffat, and P. Savaria, Can. J. Phys. **58**, 729 (1980).
[32]J. W. Moffat and D. Vincent, Can. J. Phys. **60**, 659 (1982).
[33]R. B. Mann and J. W. Moffat, J. Phys. A **14**, 2367 (1981).
[34]R. B. Mann and J. W. Moffat, J. Phys. A **15**, 1055(E) (1982).
[35]G. Kunstatter and R. Yates, J. Phys. A **14**, 847 (1981).
[36]G. Kunstatter, J. W. Moffat, and J. Malzan, J. Math. Phys. **24**, 886 (1983).
[37]R. B. Mann, Nucl. Phys. **B231**, 418 (1984).
[38]J. W. Moffat, J. Math. Phys. **25**, 347 (1984).
[39]R. B. Mann, Class. Quantum Gravit. **1**, 561 (1984).
[40]J. W. Moffat, Astrophys. J. Lett. **287**, L77 (1984).
[41]R. A. Hulse and J. H. Taylor, Astrophys. J. Lett. **195**, L51 (1975).
[42]L. L. Smarr and R. Blandford, Astrophys. J. **207**, 574 (1976).
[43]C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981).
[44]R. V. Wagoner, Astrophys. J. Lett. **196**, L63 (1975).
[45]J. M. Weisberg and J. H. Taylor, Phys. Rev. Lett. **52**, 1348 (1984).
[46]R. Epstein and R. V. Wagoner, Astrophys. J. **197**, 717 (1975).
[47]R. V. Wagoner and C. M. Will, Astrophys. J. **210**, 764 (1976).
[48]C. M. Will, Astrophys. J. **214**, 826 (1977).
[49]P. C. Peters and J. Mathews, Phys. Rev. **131**, 435 (1963).
[50]R. Blandford and S. A. Teukolsky, Astrophys. J. **205**, 580 (1976).
[51]D. L. Lee, A. P. Lightman, and W.-T. Ni, Phys. Rev. D **10**, 1685 (1974).