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Elusive "penguin" mechanism for K decay

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We observe that the uncertainty in estimating the "penguin" diagram is so large that its contribution to $A(K\rightarrow 2\pi)$ can range from 30% to negligible, and thus no meaningful lower bound can be set for ϵ'/ϵ. We then map out the relations between the experimental measurements of ϵ'/ϵ and m_t and the hadronic dynamical parameters for use in the search for the "penguin."

Theorists have long engaged in a struggle to understand and explain nonleptonic decays. Despite many heroic efforts, we still know very little about the mechanism of these decays. The main obstacle to our understanding is the difficulty in mastering the hadronic dynamics. In 1976, the so-called "penguin" diagram, i.e., the W-loop diagram in the one-gluon-exchange approximation, was proposed by Shifman, Vainshtein, and Zukharov[1] to explain the $\Delta I=\frac{1}{2}$ dominance observed in 1957 in K decays. Gilman and Wise[2,3] subsequently noted that such a diagram in the Kobayashi-Maskawa (KM) scheme can give a nonvanishing ϵ'. Its profound implications for the understanding of CP violation have prompted much theoretical and experimental interest. Recently, Gilman and Hagelin[4] even claimed to have established a lower bound on ϵ'/ϵ, based upon the belief that the hadronic-matrix part of the penguin diagram was calculable. Recent measurements[5] set a new upper bound on ϵ'/ϵ which is close to being inconsistent[6] with the lower bound set by Gilman and Hagelin.[4] This raises the question as to whether the experiments have ruled out the KM scheme for CP violation, or eliminated the penguin mechanism. A clear-cut answer "yes" would open a new era of searching for some other mechanism to account for CP violation. Unfortunately, the uncertainties associated with the calculations at present are too large to justify such a confrontation with the experimental results. Certainly the KM model has provided a framework in which the ϵ' can be nonzero, but the magnitude and the sign of ϵ'/ϵ depend upon other dynamical information.

In the present state of our very crude understanding of nonleptonic decays involving hadronic amplitudes, the claim made in Ref. 4 seemed too strong. We are sure that we are not alone in our skepticism of such a strong claim. However, we want to point out that within the technical framework adopted, the positive lower bound on ϵ'/ϵ given by Ref. 4 is not a conservative estimate. In addition to the calculable part of the penguin diagram, there is the hadronic-matrix element which characterizes how the K^0 and 2π states communicate with the quarks' operator Q_6, which we call

$$B_K'\equiv\sqrt{2}<(\pi\pi)_{I=0}|Q_6|K^0>(1.0\ \mathrm{GeV}^3)^{-1}\ .$$

Using the bag-model calculation of Ref. 7, Gilman and Hagelin obtained $B_K'=1.4$, which they used as a basis for the positive lower bound on ϵ'/ϵ.

Because of the imminence of the publication of the experimental data[5] we redid the earlier calculations, and to our surprise we found that B_K' could be much lower. Reference 4 did not include the anomalous commutator term of Ref. 7, which gives a destructive contribution and reduces B_K' to 1.0–1.15 depending upon the different parameters provided in Ref. 7. Furthermore B_K' can even be negative, if values of quark masses smaller than those in Ref. 7 are used. Even more alarming, using the formulation of the vacuum-insertion calculation by Donoghue,[8] we found[9] $B_K'=0.33$, a factor of 4 below the value of Gilman and Hagelin. With this value of B_K', the penguin contribution to $A(K\rightarrow 2\pi)$ becomes negligible. It is fair to say that not only is there no lower bound on ϵ'/ϵ, but even the mechanism of the penguin diagram for the $\Delta I=\frac{1}{2}$ rule $K\rightarrow 2\pi$ is now also very questionable.

Such wide variations in the estimates remind us of another case of estimating the hadronic-matrix element, i.e., the box graph for $K^0\rightleftarrows\bar{K}^0$. In addition to the calculable quark-W box diagram, there is the hadronic-matrix element

$$B_K\equiv\langle K^0|[\bar{d}\gamma_\mu(1-\gamma_5)s]^2|\bar{K}^0\rangle(-\tfrac{4}{3}f_K{}^2M_K)^{-1}\ .$$

The vacuum-insertion calculation gave $B_K=1$; the bag-model calculation[10] gave results varying from a wrong-signed $B_K=+0.4$ to $-0.42\leq B_K\leq 0.34$. On the basis of PCAC (partial conservation of axial-vector current) and SU(3), B_K was estimated[11] to be 0.33. Just recently this estimate was shown to have a 100%-calculate correction term.[12] This is the state of affairs with the penguin and the box graphs.

Twenty-seven years after the observation of the $\Delta I=\frac{1}{2}$ rule, we still have no real understanding of its origin. The situation calls for more theoretical work to improve our ability to calculate hadronic reactions. As the experimental results get better, we can take an approach[13,14] which is complementary to that of doing calculations in specific models, i.e., to observe the experiments as objectively as possible. In the following we shall map out as best as we can the relation between these dynamical quantities and the experimental measurements of ϵ'/ϵ and m_t, for the search of the elusive penguin, or more generally, the W-loop diagram.

The parameter ϵ' originates primarily from the phase difference[14] between the amplitudes a_0 and a_2, the $I_{\pi\pi}=0,2$ part of the $K \to 2\pi$ amplitude,

$$\epsilon' = \frac{i}{\sqrt{2}} \frac{\mathrm{Re}a_2}{\mathrm{Re}a_0} e^{i(\delta_2-\delta_0)} \left(\frac{\mathrm{Im}a_2}{\mathrm{Re}a_2} - \frac{\mathrm{Im}a_0}{\mathrm{Re}a_0} \right) . \tag{1}$$

First, in the KM scheme of *CP* violation we can show that, in the first order of weak interactions, $\mathrm{Im}a_0$ and $\mathrm{Im}a_2$ can only result from *W*-loop diagrams, based upon the general quark-diagram approach,[13,14]

$$a_2 = (\tfrac{1}{2})^{1/2} V_{ud} V_{us}^*(a+b) e^{-i\delta_2} , \tag{2a}$$

$$a_0 = (\tfrac{1}{6})^{1/2} [V_{ud} V_{us}^*(2a - b + 3c + 3e + 6f) e^{-i\delta_0} + 3 V_{cd} V_{cs}^*(3e+6f) e^{-i\delta_0}] , \tag{2b}$$

where the V's are the KM matrix elements, and the amplitudes *a, b, c, d, e, f* are six distinct, independent quark diagrams which are first order in the weak interactions.[13,14] This classification is independent of strong-interaction schemes, and can incorporate any specific strong-interaction model calculation. With the one-gluon-exchange approximation, amplitudes *e* and *f* become the penguin diagrams. Note that the quantities in the square brackets are real. The phases of a_2 and a_0 are solely from the quark-mixing matrix. In the KM phase convention, $V_{ud}V_{us}^*$ is real, so a_2 is real, $\mathrm{Im}a_2 = 0$. The imaginary part of a_0 is from the *W*-loop diagrams *e* and *f*,

$$\mathrm{Im}a_0 = 3\,\mathrm{Im}(V_{cd}V_{cs}^*)[(e+2f)e^{-i\delta_0}] = -3 s_1 s_2 s_3 s_\delta [(e+2f)e^{-\delta_0}] .$$

However, the real part of a_0 has contributions from many different amplitudes[15] *a, b, c* besides the *W*-loop diagrams *e* and *f*.

Using $\delta_2 - \delta_0 = -41.4^\circ \pm 8.1^\circ \approx \pi/4$ rad, and $\mathrm{Re}a_2/\mathrm{Re}a_0 \approx \frac{1}{20}$ from experiments, and the KM phase-convention result $\mathrm{Im}a_2 = 0$, Eq. (1) becomes

$$\epsilon' \approx [-e^{i\pi/4}/(20\sqrt{2})](\mathrm{Im}a_0/\mathrm{Re}a_0) . \tag{3}$$

A phase-convention-independent expression for ϵ is

$$\epsilon = (e^{i\pi/4}/\sqrt{2})(\mathrm{Im}M_{12}/2\,\mathrm{Re}M_{12} + \mathrm{Im}a_0/\mathrm{Re}a_0) . \tag{4}$$

Note that ϵ' has approximately the same phase $e^{i\pi/4}$ as does ϵ, so $\mathrm{Re}(\epsilon'/\epsilon) \approx \epsilon'/\epsilon$.

Until now our discussion has been general, independent of any dynamical model. Using the quark-mixing matrix, as determined[16,17] by the *b* lifetime and semileptonic decays, we can now take a more direct approach to the implications of the values of ϵ and ϵ'/ϵ on dynamical schemes.

Because $\mathrm{Re}a_0$ can have contributions from so many other amplitudes, we shall adopt the scheme[15] in which $|\mathrm{Re}a_0| \approx |a_0|$ is given by the experimental value of $K \to 2\pi$, and $\mathrm{Im}a_0$ is calculated according to the penguin diagram. The result is

$$\epsilon'/\epsilon = 6.0[\mathrm{sign}(\mathrm{Re}a_0)]\{B_K' s_2 s_3 s_\delta(-\mathrm{Im}\tilde{c}_6/0.1)[\ln(m_t^2/m_c^2) - D(x)]\}/[\sqrt{2}\ln(30\text{ GeV}/m_c)^2] \tag{5a}$$

$$= [\mathrm{sign}\ (\mathrm{Re}a_0)] B_K' s_2 s_3 s_\delta [\ln(m_t^2/m_c^2) - D(x)]/\sqrt{2} , \tag{5b}$$

with the experimental value for ϵ. The quantity in the curly brackets is from the penguin-diagram calculation for $\mathrm{Im}a_0$. We pick $(-\mathrm{Im}\tilde{c}_6/0.1)=1$. *Here we use the exact expression for the m_t dependence in the short-distance part of the penguin diagram,*

$$D(x) = \tfrac{1}{8} x(1-x)^{-3} \times [18 - 11x - x^2 + 2x(1-x)^{-1}(15 - 16x + 4x^2)\ln x] ,$$

where $x \equiv m_t^2/M_W^2$. Such corrections become important as m_t becomes comparable to M_W. (In all previous analyses such m_t dependence was ignored. The corrections can be as large as 20%.)

For the K^0-$\bar{K}^0$ matrix M_{12}, we use the experimental value for $2\,\mathrm{Re}M_{12} = \Delta M = M_S - M_L$. The quantity $\mathrm{Im}M_{12}$ has two parts: $\mathrm{Im}M_{12} = \mathrm{Im}M_{12}^{\mathrm{box}} + \mathrm{Im}M_{12}^{\mathrm{LD}}$ from the box diagram and the long-distance contribution. We use

$$(\mathrm{Im}M_{12}/\mathrm{Re}M_{12})^{\mathrm{LD}} = -2\,\mathrm{Im}a_0/\mathrm{Re}a_0 , \tag{6}$$

based upon the fact that the $\bar{K}^0, K^0 \to \pi\pi$ amplitudes dominate[18] the $K^0 \leftrightarrow \bar{K}^0$ matrix via the completeness relation.[19] Using these relations we can rewrite Eq. (4) as

$$\epsilon = [(e^{i\pi/4}/\sqrt{2})\mathrm{Im}M_{12}^{\mathrm{box}} - 2\,\mathrm{Re}M_{12}^{\mathrm{box}}\,20\epsilon']/\Delta M . \tag{7}$$

Here we use the same calculation for M_{12}^{box} as in Refs. 17 and 20.

Previously, separate analyses were done for ϵ (Refs. 16 and 17) and for ϵ'/ϵ (Ref. 2). Here we do a simultaneous analysis,[21] and we also include the precise m_t dependence in ϵ'/ϵ; both can have a correction up to 30%.

The experimental value of ϵ gives $s_\delta B_K > 0$. The sign of ϵ'/ϵ is determined by the sign of $\mathrm{Im}a_0/\mathrm{Re}a_0$, i.e., the sign of $s_\delta B_K'/\mathrm{Re}a_0$ (the signs of s_1, s_2, s_3 are positive by convention). If the penguin is the dominant amplitude, then the sign of $\mathrm{Im}a_0/\mathrm{Re}a_0$ is determined. However in the situation that the penguin may not be dominating, the sign of $\mathrm{Im}a_0/\mathrm{Re}a_0$ should be treated as a free parameter. We present results for both possibilities $\epsilon'/\epsilon > 0$ as well as $\epsilon'/\epsilon < 0$.

For given values of B_K and B_K' in Eqs. (5) and (7), there are four parameters s_2, s_3, s_δ, and m_t. These parameters are constrained by two experimental measurements: ϵ and $|V_{cb}|$. [The ratio ϵ'/ϵ expressed in terms of these four parameters by Eqs. (5) is used in Eq. (7).] So, fixing m_t (or ϵ'/ϵ), we can solve for s_2 in terms of s_3; i.e., we obtain contours in the s_2-s_3 plane (Fig. 1). More interestingly, we can also plot the allowed range of m_t and ϵ'/ϵ for different values of B_K and B_K' (see Fig. 2). From this figure we can read off the following physical implications.

(1) Given ϵ'/ϵ, there are *both lower and upper bounds on* m_t. The lower bound comes about mainly because the *b*-lifetime restriction renders $|V_{td}^* V_{ts}|^2$ extremely small, so that to fit ϵ, m_t must be large enough. The lower bounds on m_t decreases from 50 to 20 GeV as $|B_K|$ increases from 0.33 to about 1. Now we look at the upper bound on m_t. The dynamic factor inside the second set of square brackets in Eq. (5b) goes to an asymptotic value of $[\ln(M_W^2/m_c^2) - \frac{1}{8}]$ for large m_t. Then for given ϵ'/ϵ, $s_2 s_3 s_\delta$ is bounded below.

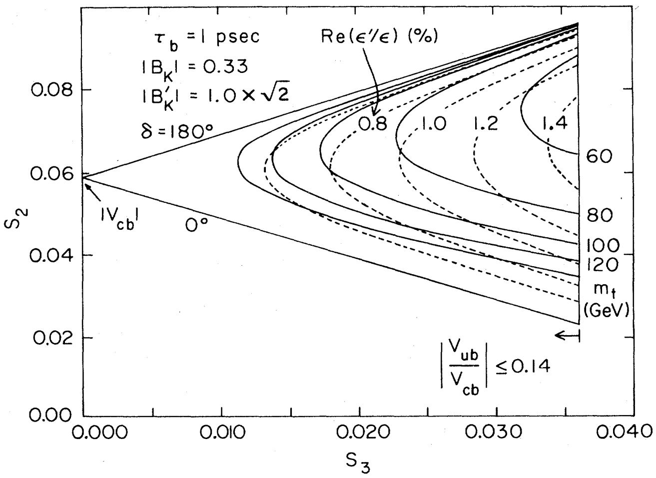

FIG. 1. Contours of constant m_t (in solid curves) and of constant $\epsilon'/\epsilon > 0$ (in dashed curves) are illustrated in the map of KM angles s_2 and s_3. The triangle region is bounded by $s_2 < s_3 + |V_{cb}|$, $s_2 > -s_3 + |V_{cb}|$, and $s_3 < |V_{cb}/s_1|\,|V_{ub}/V_{cb}|_{\max}$ (see Ref. 17).

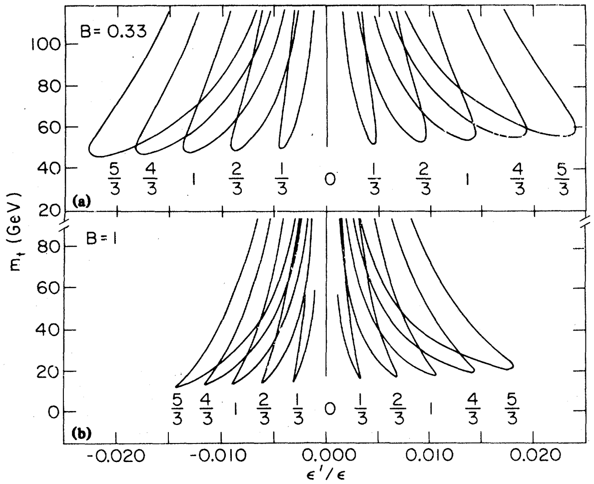

FIG. 2. The allowed regions (inside the curves) of m_t and ϵ'/ϵ. See the text for the conditions on the signs of ϵ'/ϵ. The numbers below the curves indicate the values of $B'/\sqrt{2}$.

To avoid oversaturating ϵ, there is an upper bound on m_t. The upper bound increases as $|\epsilon'/\epsilon|$ decreases.

(2) Given m_t within the allowed range, there are also *upper* and *lower* bounds on $|\epsilon'/\epsilon|$, depending crucially on the values of B_K and B'_K, as shown in Fig. 2.

(3) In our model the result $\epsilon'/\epsilon = 0$ can be obtained by simply setting $B'_K = 0$; i.e., the penguin contribution is zero. (This coincides with previous analysis in Ref. 17.) For $\epsilon'/\epsilon = 0$, there are only lower bounds on m_t, $m_t > 16$–51 GeV, as $|B_K|$ varies from 1 to 0.33.

(4) Taking $|B'_K| = \sqrt{2}$ and $|B_K| = 0.33$, Fig. 2(a) gives lower and upper bounds on $|\epsilon'/\epsilon|$: $0.002 < |\epsilon'/\epsilon| < 0.015$.

(5) If we take the currently measured value of ϵ'/ϵ with, say, a range of one standard deviation, $-0.0123 < \epsilon'/\epsilon < 0.0031$, and a range of $40 < m_t < 50$ GeV, we find that $0.33 \lesssim |B_K|$ independent of the values of B'_K, and there is no upper bound on $|B_K|$; $|B'_K| < 1$–4 as $|B_K|$ varies from 0.33 to 1.0, respectively, with the upper bound increasing as $|B_K|$ increases.

(6) s_8 obtained from the fit is not very small. s_2, s_3, and s_8 are equally responsible for the smallness of $\epsilon = 2.3 \times 10^{-3}$. If $m_t \lesssim 60$ GeV, then s_2, s_3 are actually responsible for the smallness of *CP* violation (see Fig. 1).

In conclusion, we want to emphasize that, with the values of the quark-mixing matrix being well determined from the B decays, the value of ϵ together with the precise measurement of ϵ'/ϵ can provide us with direct information on the dynamics for $K^0 \leftrightarrow \bar{K}^0$, and $\mathrm{Im}\,a_0$, and the allowed range of the t-quark mass within the Kobayashi-Maskawa scheme. Conversely, in the case of the exciting prospect of observing the t quark, our understanding of the nonleptonic dynamics in weak interactions and the quark-mixing angles will be greatly improved.

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