An effective Lagrangian for quantum chromodynamics: The structure of hadrons

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We develop an effective Lagrangian for the study of color-singlet hadrons which has some relation to quantum chromodynamics. We assume the QCD vacuum contains a condensate of gluon pairs, and introduce an order parameter $\chi(x)$, which describes the modification of this condensate in the presence of quarks. In addition, we introduce another order parameter which is related to the degree of local color neutrality of the system. We assume that there are kinetic terms in the effective Lagrangian for only the first of these order parameters, $\chi(x)$. Therefore the functional derivative of the effective Lagrangian with respect to the second-order parameter yields a constraint equation determining the value of the second-order parameter in terms of the (colored) quark current and the order parameter $\chi(x)$. This analysis leads to a model in which the field that the quark sees has an attractive part dependent on $\chi(x)$, and a confining field that depends on $\chi^{-2}(x)$. While the equations of the model are nonlinear, they can be solved by iteration and yield a covariant description of a hadron. In this model the colored quark currents in a hadron are locally cancelled by screening currents. This leads to the vanishing of the coherent parts of $\mathbf{E}^a(x)$ and $\mathbf{B}^a(x)$, the color electric and magnetic fields. Thus we can remark that the QCD system can be thought of as both a perfect diamagnetic material ($\mu=0$) and a perfect "dia-electric" material ($\epsilon=\infty$) when considering low-momentum-transfer phenomena. If we neglect the term describing confinement in (color-singlet) hadrons, the model described here reduces to our previously developed model of "covariant soliton dynamics," a model which was able to give a good account of the structure of the nucleon and various mesons as (covariantly described) nontopological solitons. The new feature that emerges in the work described here is a specific model for confinement related to the screening aspect of the theory noted above. While we have not *derived* this model from QCD we show in this work that our effective Lagrangian is not inconsistent with the highly nonlinear equations of QCD and may contain the key to understanding confinement in hadron physics. This situation is typical of that found in condensed-matter physics, where one does not *solve* the highly nonlinear equations of the theory but searches for the key elements governing the dynamics so that a model which explains the salient physical phenomena can be constructed.

I. INTRODUCTION

Over the past few years we have seen intense activity in the construction of models of hadron structure which attempt to use as much guidance as possible from our limited understanding of QCD dynamics. We assume the reader is familiar with the various models proposed such as the MIT bag model, the chiral bag model, the cloudy bag model, topological models based upon the ideas of Skyrme, and other nontopological models such as that of Friedberg and Lee. We do not attempt to review this rather large literature here. We will, however, note that we have recently developed a quite promising model of hadron structure which we called "covariant soliton dynamics."[1–3] We have discussed the structure of the nucleon[1] and various mesons[3] (ρ, ω, charmonium, and the Υ system) in earlier works. That analysis was carried forth using an extremely simple Lagrangian,

$$\mathscr{L}(x)=\tfrac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x)-\frac{m_\chi{}^2}{2}\chi^2(x)+\bar{q}(x)[i\gamma^\mu\partial_\mu-m_q-g_\chi\chi(x)]q(x)\ , \tag{1.1}$$

which described quarks coupled to a scalar field $\chi(x)$. We made a covariant analysis of the field equations

$$(\partial^\mu\partial_\mu+m_\chi{}^2)\chi(x)=-g_\chi\bar{q}(x)q(x)\ , \tag{1.2}$$

$$(i\gamma^\mu\partial_\mu-m_q)q(x)=g_\chi q(x)\chi(x)\ , \tag{1.3}$$

and found that one could provide a unified approach to the structure of the nucleon and various mesons with our very simple effective Lagrangian. In our analysis we identified $\chi(x)$ as an order parameter of the QCD vacuum but did not explore the relation of our model to QCD. In this work we attempt to relate our model to QCD and, at the same time, extend our analysis to include a description of confinement.

The development of this model leads to a picture of "color screening." That is, we find that, under certain assumptions, a model emerges in which the colored matter (quark) currents are shielded by screening currents such that (on the average) the current sources of color electric and magnetic fields vanish. This mechanism appears to lead to a theory which has *confined* soliton solutions, which can be described in a covariant fashion using the techniques we have developed.[1–3]

TABLE I. Some analogies between superconductivity, condensed Bose-Einstein systems, and our model of QCD.

	Superconductivity, Bose-Einstein systems	QCD
Special expectation values	$\langle \psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\rangle$ $\langle a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger\rangle$ electron pairing	$\langle \mathscr{A}_\mu^a(x)\mathscr{A}_a^\mu(x)\rangle$ gluon pairs
Order parameter	$\Psi(x)$: Landau-Ginzburg BCS theory $\Psi(x)\sim\Delta(x)=\langle\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\rangle$	$\chi^2(x)\sim\delta\langle\mathscr{A}_\mu^a(x)\mathscr{A}_a^\mu(x)\rangle$
Gauge invariance of model	no	no
Supercurrent or screening current	$\mathbf{j}_{\rm sc}(x)=\frac{-2e^2}{m}\mid\Psi(x)\mid^2\mathbf{A}(x)$	$j_{\mu,{\rm sc}}^a(x)=\frac{-\chi^2(x)}{\sigma_v}\overline{A}_\mu^a(x)$
	$\mathbf{B}=0$ Meissner effect (perfect diamagnetism)	$\langle\mathbf{B}^a\rangle=0$; $\langle\mathbf{E}^a\rangle=0$ or $\langle F_a^{\mu\nu}(x)\rangle=0$
	Abelian theory: $\epsilon\mu=1$ in vacuum	Non-Abelian: $\epsilon=\infty$, $\mu=0$
Semiclassical aspect	Bose system $\phi(x)=\xi_0+\sum_{k\neq0}a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}$ $\xi_0=n_0{}^{1/2}$ condensate parameter $\langle\hat\xi_0^\dagger\hat\xi_0\rangle=n_0=N_0/V$ condensate density	$A_\mu^a=\overline{A}_\mu^a(x)+\mathscr{A}_\mu^a(x)$ $\overline{A}_\mu^a(x)$ coherent gluon field $\mathscr{A}_\mu^a(x)$ fluctuating field
Effective-Lagrangian theory	Free energy density in Landau-Ginzburg theory: $F_S=F_{n0}+a\mid\Psi\mid^2+b\mid\Psi\mid^4/2+\frac{\mathbf{B}^2}{8\pi}+\frac{1}{2m^*}\left\|\left[-i\nabla+\frac{e^*\mathbf{A}}{c}\right]\Psi\right\|^2$	Effective Lagrangian $\mathscr{L}(x)=\frac{1}{2}\partial^\mu\chi\,\partial_\mu\chi-\frac{m_\chi^2}{2}\chi^2-\frac{\chi^2}{2\sigma_v}\overline{A}_a^\mu\overline{A}_\mu^a+\overline{q}\left[i\gamma^\mu\partial_\mu-m_q-g_\chi\chi+g\gamma^\mu\frac{\lambda^a}{2}A_\mu^a\right]q$
Currents: supercurrents, screening currents, normal currents	Supercurrent: (electromagnetic) $\mathbf{j}_S=\frac{-e^*}{2m^*i}(\Psi^*\nabla\Psi-\Psi\nabla\Psi^*)-\frac{(e^*)^2}{m^*}\mid\Psi\mid^2\mathbf{A}$ $\mathbf{j}(x)=\mathbf{j}_n(x)+\mathbf{j}_S(x)$	Colored current: (matter current plus screening currents) $J_\mu^a(x)=g\overline{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)-\frac{\chi^2(x)}{\sigma_v}\overline{A}_\mu^a(x)$ Note: χ is colorless
Solutions of nonlinear field equations	Vortex filaments, surface energy and penetration depth, etc.	Nontopological soliton models of hadrons: Covariant soliton dynamics (Refs. 1–3)

In order to understand the basis for our analysis it is useful to draw some parallels between our description of QCD dynamics and the description of nonrelativistic systems that exhibit a condensate structure.[4] We have in mind both the theory of superconductivity and the theory of a Bose system such as liquid ^{4}He.

Various analogies are presented in a schematic fashion in Table I. For example, in the (microscopic) theory of superconductivity the expectation value $\langle\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\rangle$ or $\langle\psi_\uparrow(x)\psi_\downarrow(x)\rangle$ provides a useful order parameter which is related to the presence in the ground state of zero-momentum electron pairs with opposed spin. The introduction of such order parameters requires that matrix elements be calculated for an ensemble that does not have a fixed number of electrons. We also remark that the theory of superconductivity rests on the assumption that electron pairing is the physical mechanism responsible for the observed effects. The extraordinary success of this model, of course, leaves no doubt that this is the correct identification of the essentially dynamical feature governing superconductivity.

At this point we can be a bit more precise concerning the averaging procedure we will use in this work. We envision a hadron of typical length scale of about 1 fm propagating in space-time. We can then consider the regions of space-time to be divided into segments of characteristic length of, say, 10^{-2} fm. In these small volumes we can define averaged values of the fundamental fields. Thus, the notation

$$\langle A_\mu^a(x)A_a^\mu(x)\rangle,\ \langle F_a^{\mu\nu}(x)\rangle,\ \bar{A}_a^\mu(x)\equiv\langle A_a^\mu(x)\rangle$$

is used to denote average values of these operators, averaged in the space-time region about the point x. This notation is then analogous to that used in condensed-matter theories where the spatial variation of the order parameter may be important, the theory of superconductivity being just one example. We here propose that hadrons are color neutral on the small length scale introduced here, say, 10^{-2} fm. Therefore the order parameter $\bar{A}_a^\mu(x)$, which, as we will see, measures the local deviation from color neutrality, is considered to be a small quantity and various higher-order terms in $\bar{A}_a^\mu(x)$ are neglected in the analysis which follows. While QCD is a gauge theory, the effective theory we consider here is not. We ascribe the absence of gauge invariance in the effective theory to the space-time averaging introduced in the construction of our model of low-energy dynamics.

In Appendix A we summarize some of the equations of quantum chromodynamics. As we turn to a consideration of QCD, we might expect that gluons with opposite momentum and opposed spins would be present in a condensate, presumably taking advantage of some attractive short-range interaction which we will not attempt to describe. This suggests that $\langle A_\mu^a(x)A_a^\mu(x)\rangle$ might be a useful *non-gauge-invariant* order parameter. (We realize that there are useful gauge-invariant order parameters such as

$$\left\langle 0\left|\frac{g_s^{\,2}}{4\pi^2}F_a^{\mu\nu}F_{\mu\nu}^a\right|0\right\rangle ,$$

which have received much attention in work making use of QCD sum rules.[5]) Thus, the concepts we develop are analogous to the ideas used in the BCS theory of superconductivity: Since the ground state has a large number of ordered pair states, the addition of another pair leads to a state that is coherent with the ground state and therefore $\langle a_{\mathbf{k}\uparrow}^\dagger a_{-\mathbf{k}\downarrow}^\dagger\rangle$ is a large matrix element. Similar remarks can be made concerning gluon pairs; however, it is not profitable to consider an explicit form of the ground-state wave function in the *relativistic* theory, although such a wave function can, of course, be constructed in a nonrelativistic theory.

We recall that the Ginzburg-Landau theory is quite useful in the study of superconductivity.[6] These authors wrote the free energy of a superconductor in terms of an order parameter $\Psi(x)$, which was found, in the microscopic analysis, to be proportional to the gap parameter

$$\Delta(x)=\langle\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\rangle\ .$$

As we will see, a corresponding order parameter

$$\chi^2(x)\sim\langle A_\mu^a(x)A_a^\mu(x)\rangle-\langle A_\mu^a(x)A_a^\mu(x)\rangle_{\rm vac}$$

will play a central role in our considerations. The notation here anticipates a specific identification of the order parameter in Eqs. (1.1)–(1.3) with some specific average of the gluon field operators—see Sec. III.

One of the most remarkable features of superconductivity is the Meissner effect associated with the presence of a screening current which makes a perfect superconductor a perfect diamagnetic material. Some authors have attempted to consider the QCD vacuum to be a perfect "dia-electric," that is, a color superconductor. They have defined such a material as having $\epsilon=0$, while we prefer to consider a perfect dia-electric material to have $\epsilon=\infty$. (The analogy to the Meissner effect is more meaningful if one has $\mathbf{E}_a=0$ rather than $\mathbf{D}_a=0$ for color fields.)

The Meissner effect is related to the presence of a screening current whose magnitude depends on the order parameter $|\Psi(x)|^2$; see Table I. Of course, in this case one is screening an *external* field. In QCD there are no external *color* fields and therefore "screening" is of a somewhat different nature in our model. We do suggest the presence of a screening current in QCD, however. This current serves to cancel the color (quark) matter current. Thus, *on the average,* there are no sources for the color electric and magnetic fields, $\mathbf{E}^a$ and $\mathbf{B}^a$. That, therefore, is our non-Abelian version of the Meissner effect, a cancellation of both $\mathbf{E}^a$ *and* $\mathbf{B}^a$ on a length scale significantly smaller than that defined by typical hadron radii.

For the moment, let us turn to the nonrelativistic theory of condensed Bose-Einstein systems. In the *nonrelativistic* theory one can single out a single state for macroscopic occupation, usually the state with $\mathbf{k}=0$. The occupation factor for this state introduces a semiclassical element in the theory and one may write the (boson) field operator as[4]

$$\phi(x)=\xi_0+\sum_{k\neq0}a_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}\ . \tag{1.4}$$

Here $a_{\mathbf{k}}$ destroys a boson in a state $\mathbf{k}=0$ and ξ_0 is a parameter defined such that $\xi_0^{\,2}=n_0=N_0/V$ where N_0 is the number of condensed particles and V is the volume of the system. Further, N_0/N would be the fraction of condensed particles, if N is used to denote the total number of particles. In a relativistic theory one cannot single out a single state for macroscopic occupation; however, one expects that an operator that creates or destroys gluon states may have a large matrix element. Therefore, in analogy to the nonrelativistic case, we separate $A_\mu^a(x)$ into a semiclassical part and a fluctuation field $\mathscr{A}_\mu^a(x)$:

$$A_\mu^a(x)=\bar{A}_\mu^a(x)+\mathscr{A}_\mu^a(x)\ . \tag{1.5}$$

Now let us again consider the Ginzburg-Landau theory. The free energy of the superconductor is written in Table I in terms of the order parameter $\Psi(x)$. (Here we use the notation of Fetter and Walecka.[4] Note that $m^*=2m$ and $e^*=2e$ where e and m are the charge and mass of the electron.) In Table I we also present the effective Lagrangian that results from our study of QCD; see Sec. III.

One difference which appears upon comparing the Ginzburg-Landau result with our QCD-based result should be noted. In the theory of superconductivity the (complex) order parameter $\Psi(x)$ represents an electrically charged condensate. However $\mathcal{X}(x)$ is a *color singlet.* Therefore the nonrelativistic supercurrent has a convection term

$$[-e^*/(2m^*i)](\Psi^*\nabla\Psi-\Psi\nabla\Psi^*)\ ,$$

which has no correspondence in the colored screening current (see Table I),

$$j^a_{\mu,\mathrm{sc}}(x)=-\frac{\mathcal{X}^2(x)}{\sigma_v}\bar{A}^{\,a}_{\mu}(x)\ . \quad (1.6)$$

Here σ_v is a constant, $\sigma_v>0$.

In order to see the origin of this screening current we can study the "field current" in QCD and apply similar techniques to those which will be used in Sec. III to expand the Lagrangian in terms of order-parameter fields.

We recall that in QCD one has (see Appendix A),

$$\begin{aligned}-\partial_\mu F^{\mu\nu}_a(x)&=J^\nu_a(x)\\ &=g\bar{q}(x)\gamma^\nu\frac{\lambda^a}{2}q(x)+gf^{abc}A^{\,b}_{\mu}(x)F^{\mu\nu}_c(x)\ ,\end{aligned} \quad (1.7)$$

with $\partial_\nu J^\nu_a(x)=0$. Thus, we have an octet of conserved colored currents in QCD. Writing Eq. (1.7) in a more explicit fashion, we have

$$\begin{aligned}J^\nu_a(x)=\ &g\bar{q}(x)\gamma^\nu\frac{\lambda^a}{2}q(x)+gf^{abc}A^{\,b}_{\mu}(x)[\partial^\mu A^\nu_c(x)-\partial^\nu A^\mu_c(x)]\\ &+g^2\cdot f^{abc}f^{cde}A^{\,b}_{\mu}(x)A^\mu_d(x)A^\nu_e(x)\ ,\end{aligned} \quad (1.8)$$

where

$$F^a_{\mu\nu}(x)=\partial_\mu A^{\,a}_{\nu}(x)-\partial_\nu A^{\,a}_{\mu}(x)+gf^{abc}A^{\,b}_{\mu}(x)A^{\,c}_{\nu}(x)\ . \quad (1.9)$$

In developing Eq. (1.8) we will use the assumption that $\delta\langle\mathscr{A}^a_\mu(x)\mathscr{A}^\mu_a(x)\rangle$ is a large quantity while $\bar{A}^{\,\mu}_a(x)=\langle A^\mu_a(x)\rangle$ is small. Thus we will neglect all terms of order $(\bar{A})^3$ and $(\bar{A})^4$ when they appear. Also the second term in Eq. (1.8) can be considered to be of order $(\bar{A})^2/R$, where R is the radius of the hadron, and we will also drop that term. Therefore we concentrate on the last term in Eq. (1.8) and obtain the coherent part by putting $b=d$. We will further consider $\delta\langle\mathscr{A}^b_\mu(x)\mathscr{A}^\mu_b(x)\rangle$ to be largely independent of b except for quite small fluctuations. Therefore, if we note that

$$\sum_{bc}f^{abc}f^{cbe}=-\sum_{bc}f^{abc}f^{ebc}=-3\delta_{e,a}$$

we may write the last term in Eq. (1.8) as $\mathcal{X}^2(x)\bar{A}^{\,\nu}_a(x)/\sigma_v$, where $\sigma_v>0$ is a constant. Therefore we have, for the coherent part of $J^\nu_a(x)$,

$$\langle J^\nu_a(x)\rangle=\left\langle g\bar{q}(x)\gamma^\nu\frac{\lambda^a}{2}q(x)\right\rangle-\frac{\mathcal{X}^2(x)}{\sigma_v}\bar{A}^{\,\nu}_a(x)\ . \quad (1.10)$$

In Sec. III we will show that, given our approximations,

$$\langle J^\nu_a(x)\rangle=0\ , \quad (1.11)$$

providing the basis for our remarks concerning the role of the screening current, $-\mathcal{X}^2(x)\bar{A}^{\,\nu}_a(x)/\sigma_v$. In that section we will extract the coherent part of the Lagrangian and obtain an *effective* Lagrangian for the study of hadron physics. In Appendix B we show that the screening current and the matter current in this model are *separately conserved.*

The total current of Eq. (1.10) may be compared to the total current in a superconductor

$$\mathbf{j}(x)=\mathbf{j}_n(x)+\mathbf{j}_S(x)\ , \quad (1.12)$$

which is the sum of the normal electron current $\mathbf{j}_n(x)$ and a supercurrent

$$\begin{aligned}\mathbf{j}_S(x)=&-\frac{e^*}{2m^*i}[\Psi^*(x)\nabla\Psi(x)-\Psi(x)\nabla\Psi^*(x)]\\ &-\frac{(e^*)^2}{m^*}\,|\Psi(x)|^2\mathbf{A}(x)\ .\end{aligned} \quad (1.13)$$

We further note that it is $|\Psi(x)|^2$ that is proportional to the number of superconducting electrons. [One specific normalization is $|\Psi|^2=n_s^*$ (Ref. 7).] Therefore the correspondence is to be made between $|\Psi(x)|^2$ and $\mathcal{X}^2(x)$, as may be noted from Table I or from comparing Eqs. (1.13) and (1.10).

We note that the nonlinear Ginzburg-Landau equations are particularly useful for those cases in which the order parameter has a marked spatial dependence. Again, quite analogously, our effective Lagrangian for QCD is quite useful for the study of hadron structure. The physical mechanism for building a hadron is quite clear in our model of covariant soliton dynamics:[1–3] the quarks are assumed to be coupled to the order parameter, their presence disturbs the vacuum structure and in a sense, the quarks "dig a hole" in the vacuum. The hadron is then a *covariantly described* vacuum bubble containing quarks.

II. ANALYSIS OF THE QCD LAGRANGIAN

We begin by writing the Lagrangian of QCD in an explicit fashion,

$$\begin{aligned}\mathscr{L}(x)=&-\tfrac{1}{4}[\partial_\mu A^{\,a}_{\nu}(x)-\partial_\nu A^{\,a}_{\mu}(x)+gf^{abc}A^{\,b}_{\mu}(x)A^{\,c}_{\nu}(x)]\\ &\times[\partial^\mu A^\nu_a(x)-\partial^\nu A^\mu_a(x)+gf^{ade}A^\mu_d(x)A^\nu_e(x)]\\ &+\bar{q}(x)\left[i\gamma^\mu\partial_\mu+g\gamma^\mu\frac{\lambda^a}{2}A^{\,a}_{\mu}(x)-m_q\right]q(x)\ .\end{aligned} \quad (2.1)$$

To analyze this Lagrangian we make two important assumptions. As in the last section, we assume that the vacuum contains a gluon condensate[8,9] and therefore techniques of many-body theory used for the analysis of similar problems are appropriate. Of course, such techniques have to be generalized for the consideration of relativistic systems. We adopt the physical picture that the ground state has gluon pairs of opposite momenta $\mathbf{k}$ and $-\mathbf{k}$ and opposed spin projections, that is, the same helicity. This leads us to consider that the expectation value of $A^\mu_a(x)A^{\,a}_{\mu}(x)$ will be anomalous. More precisely we make a correspondence between $\delta\langle\mathscr{A}^\mu_a(x)\mathscr{A}^b_\nu(x)\rangle$ and $\delta_{\mu\nu}\delta_{b,a}\mathcal{X}^2(x)$ where $\mathcal{X}^2(x)$ measures the deviation of the value of this (averaged) quantity from the value it would have in the absence of quark sources. We will use brackets to denote the coherent or semiclassical part of the operator in question as in the last section.

We will further assume that the other anomalous expectation value in the model is $\langle A_a^\mu(x)\rangle = \bar{A}_a^\mu(x)$. This disturbance carries color. However we can form a color-singlet order parameter $\bar{A}_a^\mu(x)\bar{A}_\mu^a(x)$. We also assume that this parameter is not excited without an associated excitation of the χ field, which in turn requires the presence of quark sources. [As in the last section, we may find it useful to expand $A_\mu^a(x)$ in terms of a semiclassical field and a fluctuating field—see Eq. (1.5).]

With these assumptions stated, we can concentrate on the quantity

$$\mathscr{H}_4(x) = -\mathscr{L}_4(x)$$

$$\equiv \frac{g^2}{4} f^{abc} f^{ade} A_\mu^b(x) A_\nu^c(x) A_d^\mu(x) A_e^\nu(x) , \qquad (2.2)$$

and consider the coherent part of $\mathscr{H}_4(x)$ which we will write as $\langle \mathscr{H}_4(x)\rangle$. This coherent part can be considered as a functional of *color-singlet* order parameters $\chi(x)$ and $\bar{A}_a^\mu(x)\bar{A}_\mu^a(x)$. We are interested in $\delta\langle \mathscr{H}_4(x)\rangle$ which is the variation of this quantity from the value it would have in vacuum, that is in the absence of quarks. We expand $\delta\langle \mathscr{H}_4(x)\rangle$ as

$$\delta\langle \mathscr{H}_4(x)\rangle = \frac{m_\chi{}^2}{2}\chi^2(x) + \frac{\chi^2(x)}{2\sigma_v}\bar{A}_a^\mu(x)\bar{A}_\mu^a(x)$$

$$+\lambda[\bar{A}_a^\mu(x)\bar{A}_\mu^a(x)][\bar{A}_b^\nu(x)\bar{A}_\nu^b(x)] \qquad (2.3)$$

$$\approx \frac{m_\chi{}^2}{2}\chi^2(x) + \frac{\chi^2(x)}{2\sigma_v}[\bar{A}_a^\mu(x)\bar{A}_\mu^a(x)] , \qquad (2.4)$$

where we have dropped the last term in Eq. (2.3). Some comments concerning the neglect of such terms were given in the last section.

One way to motivate the expansion of Eq. (2.4) is to consider the coherent part of $\mathscr{L}_4(x) = -\mathscr{H}_4(x)$. We again assume the extraction of the coherent part requires the identification of pairs of color indices. That is, as a first step we put $b=d$ and $c=e$ in Eq. (2.2) and consider the expansion given in Eq. (1.5).

We have

$$\begin{aligned}
\delta\langle \mathscr{L}_4(x)\rangle &= \langle \mathscr{L}_4(x)\rangle - \langle \mathscr{L}_4(x)\rangle_{\rm vac}\\
&= -\frac{g^2}{4} f^{abc} f^{ade} [\langle A_\mu^b(x) A_\nu^c(x) A_d^\mu(x) A_e^\nu(x)\rangle - \langle A_\mu^b(x) A_\nu^c(x) A_d^\mu(x) A_e^\nu(x)\rangle_{\rm vac}]\\
&\simeq -\frac{g^2}{4} f^{abc} f^{abc} [\langle A_\mu^b(x) A_b^\mu(x)\rangle\langle A_c^\nu(x) A_\nu^c(x)\rangle - \langle A_\mu^b(x) A_b^\mu(x)\rangle_{\rm vac}\langle A_c^\nu(x) A_\nu^c(x)\rangle_{\rm vac}]\\
&\simeq -\frac{g^2}{4} f^{abc} f^{abc} \{[\bar{A}_\mu^b(x)\bar{A}_b^\mu(x) + \langle \mathscr{A}_\mu^b(x)\mathscr{A}_b^\mu(x)\rangle][\bar{A}_c^\nu(x)\bar{A}_\nu^c(x) + \langle \mathscr{A}_\nu^c(x)\mathscr{A}_c^\nu(x)\rangle]\\
&\qquad - \langle \mathscr{A}_\mu^b(x)\mathscr{A}_b^\mu(x)\rangle_{\rm vac}\langle \mathscr{A}_c^\nu(x)\mathscr{A}_\nu^c(x)\rangle_{\rm vac}\}\\
&\simeq -\frac{m_\chi{}^2}{2}\chi^2(x) - \frac{\chi^2(x)}{2\sigma_v}\bar{A}_a^\mu(x)\bar{A}_\mu^a(x) ,
\end{aligned} \qquad (2.5)$$

where

$$\chi^2(x) \equiv \langle \mathscr{A}_\nu^c(x)\mathscr{A}_c^\nu(x)\rangle - \langle \mathscr{A}_\nu^c(x)\mathscr{A}_c^\nu(x)\rangle_{\rm vac} . \qquad (2.6)$$

In Eq. (2.5) we have neglected a term that would give a mass to the $\bar{A}_a^\mu(x)$ field. That approximation may be justified if (χ^2/σ_v) is large compared to $m_\chi{}^2$.

We have assumed that a quantity such as $\langle \mathscr{A}_\mu^b(x)\mathscr{A}_b^\mu(x)\rangle$, *for a fixed b*, is independent of b in a color-singlet hadron. That is, color dependence of this quantity would be related to local fluctuations away from color neutrality, which are here considered to be quite small compared to the average value of $\delta\langle \mathscr{A}_\mu^b(x)\mathscr{A}_b^\mu(x)\rangle$. We can then use the fact that

$$\sum_{a,b} (f^{abc})^2 = 3 ,$$

leading to the result given in Eq. (2.5).

The presence of a semiclassical value of $A_a^\mu(x)$ in this expression implies the possibility of a local disturbance of the condensate which carries color. However, as we will see, this disturbance, that is the screening current, is specified in terms of the color matter current, and serves to keep the total color current equal to zero. With the assumption that $\bar{A}_\mu^a(x)$ is a small quantity we found the coherent part of $\mathscr{L}_4(x)$ to be

$$\delta\langle \mathscr{L}_4(x)\rangle = -\frac{m_\chi{}^2\chi^2(x)}{2} - \frac{\chi^2(x)}{2\sigma_v}\bar{A}_\nu^c(x)\bar{A}_c^\nu(x) . \qquad (2.7)$$

On the other hand, we see that the quantity

$$\mathscr{L}_3(x) \equiv \{ g f^{abc} A_\mu^b(x) A_\nu^c(x)[\partial^\mu A_a^\nu(x) - \partial^\nu A_a^\mu(x)]$$

$$+ g[\partial^\mu A_\nu^a - \partial_\nu A_\mu^a(x)] f^{ade} A_d^\mu(x) A_e^\nu(x)\}(-\tfrac{1}{4}) \qquad (2.8)$$

is small relative to the terms in Eq. (2.7), given our assumptions about the relative size of $\chi(x)$ and $\bar{A}_\mu^a(x)$.

At this point a further comment concerning gauge invariance may be in order. Again let us recall the factorization made in the theory of superconductivity. One usually considers the number-conserving interaction

$$\tfrac{1}{2} v(\mathbf{x}_1 - \mathbf{x}_2)\psi^\dagger(\mathbf{x}_1)\psi^\dagger(\mathbf{x}_2)\psi(\mathbf{x}_2)\psi(\mathbf{x}_1) ,$$

often in the limit of zero-range potentials. One then makes a factorization of the form

$$\begin{aligned}
\langle \psi_\downarrow^\dagger(x_1)\psi_\uparrow^\dagger(x_2)\psi_\uparrow(x_2)\psi_\downarrow(x_1)\rangle &= \psi_\uparrow^\dagger(x_2)\psi_\uparrow(x_2)\langle \psi_\downarrow^\dagger(x_1)\psi_\downarrow(x_1)\rangle - \psi_\downarrow^\dagger(x_1)\psi_\uparrow(x_2)\langle \psi_\uparrow^\dagger(x_1)\psi_\uparrow(x_2)\rangle\\
&\quad + \langle \psi_\downarrow^\dagger(x_1)\psi_\uparrow^\dagger(x_2)\rangle\psi_\uparrow(x_2)\psi_\downarrow(x_1) + \psi_\downarrow^\dagger(x_1)\psi_\uparrow^\dagger(x_2)\langle \psi_\uparrow(x_2)\psi_\downarrow(x_1)\rangle ,
\end{aligned} \qquad (2.9)$$

for example. It is clear that while the original theory conserves charge, a violation of gauge invariance is introduced by replacing matrix elements which would be of the form $\langle N+2 | \psi_\downarrow^\dagger(x_1)\psi_\uparrow^\dagger(x_2) | N \rangle$ by ground-state expectation values $\langle \psi_\uparrow^\dagger(x_1)\psi_\downarrow(x_2)\rangle$ which refer to an ensemble where only the *expectation value* of the number operator is specified.

In this section we have made a somewhat analogous approximation. We have essentially taken *gauge-invariant* quantity, $\frac{1}{4}F_a^{\mu\nu}(x)F_{\mu\nu}^a(x)$, and made the replacement

$$\delta\langle \tfrac{1}{4}F_a^{\mu\nu}(x)F_{\mu\nu}^a(x)\rangle \to +\frac{\chi^2(x)}{2\sigma_v}\bar{A}_\mu^a(x)\bar{A}_a^\mu(x) + \frac{m_\chi^2}{2}\chi^2(x) \,. \tag{2.10}$$

This then becomes our prescription for obtaining the coherent part of what was a manifestly gauge-invariant quantity. [Note that works that deal with QCD sum rules deal directly with the gauge-invariant quantity $\langle(\alpha_s^2/\pi)F_{\mu\nu}^a F_a^{\mu\nu}\rangle$, which is found to have a value of 0.012 GeV^4, when fits are made to experimental data on hadron masses.[5,9]]

We now require two additional features. As in the case of the Ginzburg-Landau theory of superconductivity, it is necessary to add kinetic terms associated with the variation of the order parameter. We will add $\frac{1}{2}\partial^\mu\chi(x)\partial_\mu\chi(x)$ to the (coherent) Lagrangian, and as a final step we can assume that the quark field has an effective coupling to the order-parameter field. (This feature is analogous to the coupling of electrons to the Higgs fields in electroweak theories.)

Finally we have for our *effective Lagrangian*,

$$\begin{aligned}\langle \mathscr{L}(x)\rangle = &-\frac{\chi^2(x)}{2\sigma_v}\bar{A}_\mu^a(x)\bar{A}_a^\mu(x) \\ &+\tfrac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x) - \frac{m_\chi^2}{2}\chi^2(x) \\ &+\bar{q}(x)[\,i\gamma^\mu\partial_\mu - m_q - g_\chi\chi(x) \\ &+g\gamma^\mu\frac{\lambda^a}{2}\bar{A}_\mu^a(x)]q(x) \,,\end{aligned} \tag{2.11}$$

where m_q is a flavor-dependent mass matrix. *Note that the absence of a kinetic term involving the order parameter $A_\mu^a x$ is a specific assumption of the model and leads to the color-screening mechanism described earlier.*

Further, we see that this Lagrangian is constructed so as to provide a model for a hadron. (The mass of the hadron calculated using this Lagrangian is the energy necessary to create this hadron out of the physical vacuum and may thus be identified with the experimental value of the hadron mass.)

We now have

$$\frac{\delta\langle \mathscr{L}(x)\rangle}{\delta\bar{A}_\mu^a(x)} = -\frac{\chi^2(x)}{\sigma_v}\bar{A}_a^\mu(x) + g\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x) = 0 \,. \tag{2.12}$$

We have already identified

$$j_{a,\mathrm{sc}}^\mu(x) = -\chi^2(x)\bar{A}_a^\mu(x)/\sigma_v \tag{2.13}$$

as a screening current and may use the notation

$$j_a^\mu(x) \equiv g\bar{q}(x)\gamma^\mu\lambda^a q(x)/2$$

for the matter current. Equation (2.12) is then

$$\langle J_a^\mu(x)\rangle = j_{a,\mathrm{sc}}^\mu(x) + j_a^\mu(x) = 0 \tag{2.14}$$

which in turn implies

$$\partial_\mu\langle F_a^{\mu\nu}(x)\rangle = 0 \,. \tag{2.15}$$

We further see that

$$(\partial^\mu\partial_\mu + m_\chi^2)\chi(x) = \frac{\chi(x)}{\sigma_v}\bar{A}_\mu^a(x)\bar{A}_a^\mu(x) - g_\chi\bar{q}(x)q(x) \tag{2.16}$$

and

$$(i\gamma^\mu\partial_\mu - m_q)q(x) = g_\chi\chi(x)q(x) - g\bar{A}_\mu^a(x)\gamma^\mu\frac{\lambda^a}{2}q(x) \,, \tag{2.17}$$

or

$$\begin{aligned}&(i\gamma^\mu\partial_\mu - m_q)q(x) \\ &\quad = g_\chi\chi(x)q(x) - g^2\sigma_v\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)\gamma_\mu\frac{\lambda^a}{2}q(x)\chi^{-2}(x) \,.\end{aligned} \tag{2.18}$$

For the moment let us drop the first term on the right-hand side of Eq. (2.16) and write

$$(\gamma^\mu\partial_\mu + m_\chi^2)\chi(x) = -g_\chi\bar{q}(x)q(x) \,, \tag{2.19}$$

$$\begin{aligned}(i\gamma^\mu\partial_\mu - m_q)q(x) = {}& g_\chi\chi(x)q(x) \\ &-g^2\sigma_v\left[\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)\right]\gamma_\mu\frac{\lambda^a}{2}q(x) \\ &\times\chi^{-2}(x) \,.\end{aligned} \tag{2.20}$$

In this approximation we believe that the theory will exhibit confinement due to the term involving $\chi^{-2}(x)$ in Eq. (2.20). That is, we anticipate that when we solve Eqs. (2.19) and (2.20) to obtain the structure of a color-singlet hadron we will find that our equations have only bound-state solutions.

Thus far we have introduced an averaging procedure to pass from the Lagrangian of QCD to an effective Lagrangian expressed in terms of semiclassical order parameters. Rather than solve Eqs. (2.19) and (2.20) as classical (c-number) equations we can usefully consider these equations to be operator field equations. This procedure, which was fully developed in Refs. 1—3, has the advantage of allowing us to provide a fully covariant analysis. If we were to solve a classical version of these equations, we would find the restoration of translation invariance to the theory an exceedingly cumbersome task.

III. DISCUSSION

Analysis of Eqs. (2.19) and (2.20) has been carried forward using the techniques discussed extensively in previous publications.[1-3] It is clear from Eq. (2.19) that the quark scalar density provides the source for the field $\chi(x)$. The first term on the right-hand side of Eq. (2.20) in turn provides a mean field which binds the quarks, either into a nucleon[1,2] or a meson.[3] Covariant self-consistent solutions of Eqs. (1.2) and (1.3) have been obtained, however, as noted earlier, these equations do not lead to confinement of the quarks.

The last term in Eq. (2.20) leads to a rather unusual model of confinement. Without this term, we can consider the energy of the soliton as a function of its volume. For small volume the energy would be largely due to the increased quark kinetic energy. For large volume, the energy would go over to $2m_q$, in the case of a meson such as the ρ, or $3m_q$, in the case of the nucleon. The interaction of the quarks with the field $\chi(x)$ produces a minimum in the energy versus volume curve, at some finite volume, when one solves Eqs. (1.2) and (1.3). However, the inclusion of the second term on the right-hand side of Eq. (2.20) will change the character of the energy versus volume curve. Instead of going over to solutions containing free quarks, we anticipate an increased energy for large volume, essentially proportional to the volume of the soliton. That is, of course, what is *assumed* when constructing the energy expression for the MIT bag model, a term in the energy proportional to the volume of the hadrons. It may be seen from the analysis of Eq. (2.20) that this effect arises from what is a relatively short-range interaction between the quarks. The confinement term in Eq. (2.20), when included in the equation which determines the quark motion, gives rise to a short-range repulsive interaction *which becomes increasingly more repulsive with increasing volume of the soliton.* The short-range character of the effect may be understood by noting that the last term in Eq. (2.20) could be obtained by including in the Hamiltonian density a term such as

$$-\frac{\sigma_v}{2}\frac{j_\mu^a(x)j_a^\mu(x)}{\chi^2(x)}$$

which can be interpreted as a local interaction of the quark color currents whose strength is governed by the value $[\chi(x)]^{-2}$. Estimates of how such a term contributes to the energy of the soliton leads to the physical model described above. A detailed (covariant) analysis of our field equations and various numerical results will appear in a future work.

IV. APPLICATIONS

Extensive applications[1-3] have been made of Eqs. (2.19) and (2.20), excluding the last term in Eq. (2.20). In the absence of this term we do not have a model of confinement and therefore our studies of hadron states were limited to relatively low-lying excitations. For example, in our study of the Υ system[3] we found the physical size (the rms radius) of the $1S$, $2S$, and $3S$ states of the Υ to be 0.25 fm, 0.48 fm, and 0.81 fm, respectively. The $4S$ state was already in the continuum of our model. We also found that the size of the ρ and ω mesons was about 0.84 fm, while the physical size of the nucleon was about 0.63 fm. [Note that the electromagnetic radius of the nucleon was given correctly in our model as about 0.86 fm (Ref. 1). The electromagnetic radius is defined via the slope of the electromagnetic form factor, in the standard manner.] These results suggest that the confining potential in Eq. (2.20) will become important for hadrons that are about 1 fm or larger in size. This speculation may be supported by further numerical analysis which is presently underway.

The simple model represented by Eqs. (1.2) and (1.3) leads to quite a satisfactory description of the nucleon, the ρ and ω mesons, the ground state of charmonium, the $J/\psi(3100)$ particle, and the three states of the Υ system mentioned above. (In our study of the nucleon[1] we also included some boson fields with the quantum numbers of the π, ρ, and ω mesons in the analysis. However, these fields were seen to play only a minor role.)

Another interesting feature of the model of hadron structure we have developed lies in our ability to calculate the modification of the properties of the nucleon in nuclear matter. This program is carried out in Ref. 2. The basic mechanism giving rise to modified nucleon properties relates to the fact that there are intense scalar fields in nuclear matter. We have argued that these fields signal a modification of vacuum order, that is they can be treated on the same footing as the χ field.[2,10] Indeed, we have coupled these scalar fields to the quarks, obtaining the coupling constant from our knowledge of the coupling of these fields to the nucleon.[2] It was found that the nucleon increased in size in the presence of these fields. Indeed, the size increase was precisely that needed to explain the EMC effect, including the mass number dependence of this effect.[11] In Ref. 2 we also calculated the modification of the electromagnetic form factors of the nucleon which accompanied this size increase. In some recent works we showed that there is significant evidence that our model of medium-modified form factors is correct. In Refs. 12 and 13 we showed that one could make a good account of the longitudinal response for (e,e') inclusive reactions on ^{56}Fe, ^{40}C, and ^{12}C near the nucleon quasielastic peak. For example, in ^{56}Fe and ^{40}Ca the cross section is overestimated by a factor of about 2 in the impulse approximation. This discrepancy is repaired when we introduce our medium-modified form factors. In Refs. 14, 15, and 16 we have also shown that the use of medium-modified form factors leads to very much improved agreement when comparing theoretical charge distributions with those extracted in the study of electron scattering from ^{208}Pb. In particular, the fact that the nucleons are larger in nuclei than in vacuum leads to the quenching of the oscillations found in the theoretical charge distribution obtained using the standard analysis.[14-16]

V. CONCLUDING REMARKS

We have developed a rather specific model to describe hadron structure. We have shown, in other works, that the simplest form of this model, as represented by Eqs. (1.1) and (1.2), is able to give a good account of the structure of various hadrons.[1–3] Indeed, our fit to the properties of the nucleon is quite remarkable.[1] Except for finding a somewhat too large value for the nucleon mass, we found a good fit to the magnetic moments, charge radii, electromagnetic form factors, and the value of g_A. Even more remarkable is that there is definite experimental evidence that the increase in nucleon size and the modification of electromagnetic form factors we predict is indeed correct.[11–16]

We can suggest that the intense scalar fields found in modern theories of nuclear matter[17] are essentially the same kind of fields as those used to construct the hadrons themselves. This further indicates the possibility of a unified approach to nuclear and hadron physics,[10] the unifying element being the importance of scalar order parameters of the QCD vacuum in both problems. Our ability to calculate the modification of nucleon properties in nuclear matter indicates that these ideas may not be as speculative as they might appear at first.

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APPENDIX A

In this appendix we list some equations of quantum chromodynamics for ease of reference. The Lagrangian is

$$\mathscr{L}(x)=-\tfrac{1}{4}F_a^{\mu\nu}(x)F_{\mu\nu}^a(x)+\bar{q}(i\gamma^\mu D_\mu-m_q)q(x)\ ,$$
$$D_\mu q(x)=\left[\partial_\mu-igA_\mu^a(x)\frac{\lambda^a}{2}\right]q(x)\ . \tag{A1}$$

One writes

$$F_{\mu\nu}=\sum_a F_{\mu\nu}^a\frac{\lambda^a}{2},\quad A_\mu(x)=\sum_a\frac{\lambda^a}{2}A_\mu^a(x)\ , \tag{A2}$$

with

$$F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)-ig[A_\mu(x),A_\nu(x)]\ , \tag{A3}$$

or

$$F_{\mu\nu}^a(x)=\partial_\mu A_\nu^a(x)-\partial_\nu A_\mu^a(x)+gf^{abc}A_\mu^b(x)A_\nu^c(x)\ , \tag{A4}$$

which follows from the relation

$$\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right]=if_{abc}\frac{\lambda^c}{2}\ . \tag{A5}$$

One has

$$D^\mu F_{\mu\nu}(x)=\partial^\mu F_{\mu\nu}(x)-ig[A^\mu(x),F_{\mu\nu}(x)]$$
$$=-j_\nu(x) \tag{A6}$$

or

$$-\partial^\mu F_{\mu\nu}^a(x)-gf^{abc}A_b^\mu(x)F_{\mu\nu}^c(x)=j_\nu^a(x)\ . \tag{A7}$$

Further,

$$-\partial^\mu F_{\mu\nu}(x)=J_\nu(x)=j_\nu(x)-ig[A^\mu(x),F_{\mu\nu}(x)]\ , \tag{A8}$$

where

$$J_\nu^a(x)\equiv j_\nu^a(x)+gf^{abc}A_b^\mu(x)F_{\mu\nu}^c(x)\ . \tag{A9}$$

The equation for the quark field is

$$\left[i\gamma^\mu\partial_\mu-m_q+g\frac{\lambda^a}{2}A_\mu^a(x)\gamma^\mu\right]q(x)=0\ , \tag{A10}$$

where m_q is a (flavor-dependent) mass matrix.

APPENDIX B

In this appendix we wish to demonstrate that the matter current is conserved in our model, implying the conservation of the screening current as well. We start with Eq. (2.17),

$$(i\gamma^\mu\partial_\mu-m_q)q(x)=g_\chi\chi(x)q(x)-g\bar{A}_\mu^a(x)\gamma^\mu\frac{\lambda^a}{2}q(x) \tag{B1}$$

and multiply this equation first by $\lambda^b/2$ and they by $q(x)$. One finds

$$i\bar{q}(x)\frac{\lambda^b}{2}\gamma^\mu\partial_\mu q(x)-\bar{q}(x)\frac{\lambda^b}{2}m_q q(x)$$
$$=g_\chi\bar{q}(x)\frac{\lambda^b}{2}\chi(x)q(x)-g\bar{q}(x)\frac{\lambda^b}{2}\gamma^\mu\frac{\lambda^a}{2}q(x)\bar{A}_\mu^a(x)\ . \tag{B2}$$

Proceeding in a similar manner for the adjoint equation, we obtain

$$i\partial_\mu\left[\bar{q}(x)\frac{\lambda^b}{2}\gamma^\mu q(x)\right]$$
$$=-g\bar{q}(x)\left[\frac{\lambda^b}{2}\frac{\lambda^a}{2}-\frac{\lambda^a}{2}\frac{\lambda^b}{2}\right]\gamma^\mu q(x)\bar{A}_\mu^a(x)$$
$$\doteq -g\bar{q}(x)\frac{\lambda^c}{2}\gamma^\mu q(x)f^{abc}\bar{A}_\mu^a(x)$$
$$=-\frac{\chi^2(x)}{\sigma_v}\bar{A}_c^\mu(x)\bar{A}_\mu^a(x)f^{abc}=0\ , \tag{B3}$$

where we have used the fact that f^{abc} is antisymmetric in all its indices and the fact that

$$-\frac{\chi^2(x)}{\sigma_v}\bar{A}_\mu^c(x)+\left\langle g\bar{q}(x)\gamma_\mu\frac{\lambda^c}{2}q(x)\right\rangle=0 \tag{B4}$$

in our model.

It then follows that

$$\partial^\mu j_{\mu,\mathrm{sc}}^a(x)=0\ , \tag{B5}$$

where $j_{\mu,\mathrm{sc}}^a(x)\equiv-\chi^2(x)\bar{A}_\mu^a(x)/\sigma_v$.

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