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Atoms in Superstrong Magnetic Fields*

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It is pointed out that because of possibly large radiative corrections, the mass and magnetic moment of an electron in the presence of magnetic fields above 10^{13} G, which may be present in pulsars, are unknown. Calculations of ionization energies of atoms in such fields may therefore be meaningless unless the quantum-electrodynamic corrections are included.

It has been pointed out by Cohen, Lodenquai, and Ruderman[1] that in magnetic fields of the order of 10^{12} G or greater, which are characteristic of neutron-star models of pulsars, the ionization energy of atoms is greatly increased over that in field-free space. This effect would influence the ionization probability of various elements, and hence the distribution of nuclei pulled off the stellar surface by electric fields.

We wish to add the comment that at such superstrong magnetic fields, quantum-electrodynamic corrections are beginning to change such fundamental properties of electrons as their mass and magnetic moment by non-negligible amounts. Radiative corrections of order $\alpha(B/B_c)^2$ and $\alpha(B/B_c)^2\ln(B/B_c)$, where

$$B_c=m_e^2c^3/\hbar e\simeq 4.41\times 10^{13}\text{ G}$$

and α is the fine-structure constant, were calculated some time ago.[2–6]

These calculations may cause one to worry that since for $B\gtrsim B_c$ the size of the radiative effects is unknown, the mass and magnetic moment of the electron are unknown. Hence calculations of the binding energy of atoms in such strong fields without the calculation of radiative effects may be meaningless.

Actually, Ref. 3 contains a calculation of radiative effects (to first order in α) in constant fields without expansion in the field strength, and Jancovici[7] recently extracted from this the limiting value for very strong fields, $L\equiv B/B_c\gg 1$. His result was that the first-order radiative correction to the energy of an electron asymptotically goes as $(\alpha/4\pi)mc^2(\ln L)^2$ and he concluded from this that even when $L\simeq 1/\alpha$, the radiative correction remains small. Now this interesting conclusion does not follow from the asymptotic forms of the correction for $L\gg 1$ and $L\ll 1$, even though it is made quite plausible by it. We can, however, derive a bound for Demeur's formula in a simple manner.

Jancovici's transformation of Demeur's expression for the rest energy of an electron in a constant uniform magnetic field can be written

$$E=mc^2+(\alpha/4\pi)mc^2I,$$

$$I=2\int_0^\infty dz\, e^{-z/L}\int_0^1 dv\, R(t,v),\quad t=z/v,$$

$$R=\frac{(2t+v+1)f(t)-4}{2-vtf(t)},$$

$$f(t)=(e^{-2t}-1+2t)t^{-2}.$$

It is easy to see that $|R|<2$ for $0\leq v\leq 1$ and $0\leq z<\infty$. Furthermore, for $0\leq v\leq 1$ and $1\leq z<\infty$, one finds that

$$|R|<\frac{2(2-c)}{2(z-c)(1-v)+c},$$

where $c=1-e^{-2}=0.87$. If we use the first inequality in the part of the z integral from 0 to 1, we get

$$|I_1|<4L(1-e^{-1/L})<4.$$

Inserting the second inequality in the z integral from 1 to ∞, changing variables to $x=2z/c-1$, and integrating by parts, we obtain

$$\begin{aligned}|I_2|&<\frac{(2-c)^2}{2(1-c)}e^{-1/L'}\int_0^\infty dx\, e^{-x/L'}(\ln x)^2\\&=\frac{(2-c)^2}{2(1-c)}e^{-1/L'}\{\tfrac{1}{6}\pi^2+[\ln(\gamma L')]^2\}\\&<5[\ln(\gamma L')]^2+8,\end{aligned}$$

where $L'=2L/c$, $\gamma=e^{-\epsilon}$, $\epsilon=0.577\cdots$ is Euler's constant, so that $\gamma L'=(2\gamma/c)L\simeq 1.3L$. The result is that for all L

$$|I|<12+5[\ln(1.3L)]^2,$$

which shows that the first radiative correction is less than 10% of the rest mass so long as $L\lesssim 100$, i.e., $B\lesssim 4\times 10^{15}$ G, and less than half the rest mass for $B\lesssim 5\times 10^{18}$ G.

However, it should be pointed out that the results of

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Refs. 5 and 6 are in disagreement with those of Ref. 3 (as well as with one another). If one of these recent calculations is correct, then the mass correction increases more rapidly with the magnetic field strength than Demeur's, and at fields of the order of 10^{13} G or larger it can no longer be assumed to be small.

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Comments on Isospin-Factored Current Algebra at Infinite Momentum

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A solution of the isospin-factored current algebra at infinite momentum was given by Chang, Dashen, and O'Raifeartaigh, who assumed that one could project from the isospin-factored current algebra those solutions which contain a spacelike part which is definitely coupled to a timelike part by the current. This assumption was used to conclude that the operator M_3 vanishes. However, since the resulting solutions still contain a spacelike part which is coupled to the timelike part, it was suspected that this assumption may have excluded an important class of solutions. This, however, is shown not to be the case.

I. INTRODUCTION

Recently a solution of the $SU(2)\otimes SU(2)$ isospin-factored current algebra at infinite momentum was presented by Chang, Dashen, and O'Raifeartaigh[1] in which it was discovered by using an angular condition that the resulting solutions were either physically trivial or that they contained a spacelike part, i.e., $M^2<0$, for the mass operator. The physically trivial solutions are those which correspond to the charge-density current-density algebra and to the free quark model which has the special property that the spacelike and timelike solutions are unconnected by the current. The existence of a spacelike part in other cases of solutions is undesirable, for it makes it very difficult to see how one could carry out a program of saturating the current-algebra equations at infinite momentum.

The solutions to the isospin-factored current algebra were obtained subject to the assumption that the current involved does not connect the spacelike and timelike states, expressed technically as the vanishing of the operator M_3 which is defined later. Since, in fact, it is found that the solutions do indeed contain a spacelike part, there existed the possibility that an important class of solutions had been excluded as a result of this assumption. It is shown, however, that the assumption regarding the connecting of spacelike and timelike states is actually unnecessary so that no solutions have been excluded.

In order to find solutions to the current algebra at infinite momentum, it is necessary to introduce an angular condition[2,3] so that one can generate the Lorentz group, for as a result of transforming to an infinite-momentum frame, the Lorentz group degenerates to an $E(2)\otimes D$ group,[4] i.e., the semidirect product of the group of Euclidean motions in a plane and a boost in a fixed direction. For the isospin-factored algebra, this angular condition can be reduced to a simple set of equations from which one can find various expressions for the mass operator. Central to the problem of finding solutions to this set of equations is the demonstration of the existence of an $E(2)\otimes D$ subgroup of the $SL(2C)$ groups which are generated by the basic operators. It is also shown in this work that the existence of the Lie algebra corresponding to this $E(2)\otimes D$ group is sufficient to make unnecessary the assumption that the current does not connect spacelike and timelike states.

II. ANGULAR CONDITION

The isospin algebra at infinite momentum has the form[5]

$$[F^a(\mathbf{k}),F^b(\mathbf{k}')]=i\epsilon_{abc}F^c(\mathbf{k}+\mathbf{k}'), \qquad (2.1)$$

where $\mathbf{k}$ is the transverse momentum and where ϵ_{abc} is the Levi-Civita symbol. If one makes the assumption that the current is factored into the product of an isotopic spin generator of the Lie algebra of $SU(2)$ and a reduced matrix element such that

$$F^a(\mathbf{k})=I^aF(\mathbf{k})\,, \qquad (2.2)$$

then the reduced matrix element must satisfy the equation

$$F(\mathbf{k})F(\mathbf{k}')=F(\mathbf{k}+\mathbf{k}')\,. \qquad (2.3)$$

This equation forms a unitary representation of the

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