is surely not that usually associated with poles destined to produce resonances at higher energies.

An analogous calculation was performed for scalar Compton scattering. Here the signatured partial-wave amplitudes of interest are $\langle J-1,\ 1|A_{\pm}{}^J|J-1,\ 1\rangle$. Again, the threshold poles near $J=\frac{1}{2}$ are given by an equation similar to (7) with a new $\nu_{\pm}{}^{(\mathrm{scalar})}$:

$$\nu_{\pm}{}^{(\mathrm{scalar})}\approx m_v(2\mu-m_v)\times\exp\left\{\mp\left(\frac{1}{\alpha}\right)\frac{m_v+\mu}{[m_v(2\mu-m_v)]^{1/2}}\right\}, \quad (10)$$

where μ is the mass of the scalar particle.

Another interesting question is to ask if an infinite number of poles also converge at the pseudothresholds. [A psuedothreshold is at $s=s_2=(m-m_v)^2$, in contrast to the physical threshold, which is at $s=s_1=(m+m_v)^2$.] Here we just make a few brief remarks and hope to return to this question in detail elsewhere. For spinless scattering, it is known[9] that if the amplitude possesses an su double-spectral function, then

$$T_l(s)\xrightarrow[s\to s_2]{} c_l(s-s_2)^l+d_l(s-s_2)^{1/2}, \quad (11)$$

where c_l and d_l are independent of s. Thus, for $l<\frac{1}{2}$ (our region of interest) the behavior at s_2 appears analogous to the one at the physical threshold, s_1. Hence we expect an infinite number of poles to arrive at $J=1$ for spinor-vector scattering and at $J=\frac{1}{2}$ for scalar-vector scattering for $s\to s_2$ as well. The answer to this question may also be found by observing the asymptotic behavior of the scattering amplitudes near the pseudothresholds and noting whether or not promotion occurs.

Finally, since both the leading even- and odd-signature threshold poles remain almost stationary near $J=1$ for small energies, it would seem to be the case that neither set is able to generate physical resonances. So, unless the situation is dramatically changed by higher orders, we find no support for the Cheng-Wu conjecture that such generation may occur.

[9] G. Frye and R. Warnock, Phys. Rev. 130, 478 (1963); E. Abers and V. Teplitz, Nuovo Cimento 39, 739 (1965).

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Broken Chiral and Conformal Symmetry and the Kuo Transformation

A. M. Harun-Ar Rashid
International Centre for Theoretical Physics, Trieste, Italy
and
Institute of Physics, University of Islamabad, Rawalpindi, Pakistan

AND

M. A. Rashid*†
International Centre for Theoretical Physics, Trieste, Italy
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The breaking of conformal and chiral symmetry within the framework of effective Lagrangians is studied, and it is shown that the (8,8) type of symmetry breaking leads to unacceptable values of the pion-pion scattering lengths. A combination of $(3,\bar{3})+(\bar{3},3)$ and (8,8) is then proposed, the particular form being uniquely fixed by the requirement of Kuo transformation. This is then shown to lead to improved scattering lengths for meason-meson scattering processes.

I. INTRODUCTION

SINCE conformal symmetry is a physically interesting generalization of the Poincaré symmetry, it is an extremely attractive idea to investigate the spontaneous breakdown of conformal symmetry to the symmetry of the Poincaré group.[1] Although it is fairly straightforward to formulate effective Lagrange functions which are simultaneously invariant under the conformal group of transformations as well as under a chiral group of higher symmetry like $SU(3)\otimes SU(3)$, there is no unique way of introducing explicit symmetry-breaking terms into the Lagrangian. The suggestion has therefore been made by Isham *et al.*[2] that one should choose a symmetry-breaking term which belongs to an irreducible representation of the combined conformal and chiral groups so that one can take advantage of the existence of a fundamental scalar field $\chi(x)$ of conformal weight -1 and of its nonzero vacuum expectation value. For the case of chiral $SU(3)\otimes SU(3)$, one then has the free-

* Present Address: Institute of Mathematics, University of Islamabad, Rawalpindi, Pakistan.
† On leave of absence from the Pakistan Atomic Energy Commission.
[1] C. J. Isham, A. Salam, and J. Strathdee, Phys. Letters 31B, 300 (1970).
[2] C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D 2, 685 (1970).

dom of assigning the symmetry breaking either to $(3,\bar{3})+(\bar{3},3)$ or to (8,8). In the first case, the symmetry-breaking Lagrangain is written as[3]

$$L_1=\epsilon_0 u_0+\epsilon_8 u_8\,, \tag{1}$$

where u_0 and u_8 belong to the linear representation $(3,\bar{3})+(\bar{3},3)$ containing a scalar nonet u_i and a pseudoscalar nonet v_i of the diagonal $SU(3)$. These 18 linearly transforming components can be represented as

$$\begin{aligned} u_i(M,\chi)&=4\chi\,\mathrm{Tr}(\lambda_i\cos(\lambda\cdot M/\chi))\,,\\ v_i(M,\chi)&=-4\chi\,\mathrm{Tr}(\lambda_i\sin(\lambda\cdot M/\chi))\,, \end{aligned} \tag{2}$$

assuming that the fields have the same conformal weight $l=-1$. Here $\lambda\cdot M=\sum_i\lambda_i M_i$ and λ_i's are the Gell-Mann matrices.

However, a difficulty with this type of symmetry breaking has recently been pointed out by Okubo[4] in connection with Kuo transformation.[5] If the Hamiltonian is to remain invariant under the discrete unitary transformation

$$W=\exp(i\tfrac{3}{2}\pi Y_5)\,, \tag{3}$$

where Y_5 is the chiral counterpart of the hypercharge operator, then it is easy to see that the parameters ϵ_0 and ϵ_8 undergo the following transformation:

$$\begin{aligned} \epsilon_0&\to\bar{\epsilon}_0=-\tfrac{1}{3}\epsilon_0-\tfrac{2}{3}\sqrt{2}\epsilon_8\,,\\ \epsilon_8&\to\bar{\epsilon}_8=-\tfrac{2}{3}\sqrt{2}\epsilon_0+\tfrac{1}{3}\epsilon_8\,. \end{aligned} \tag{4}$$

Now the masses of the mesons calculated from the first-order expansion of the Lagrange function are easily found to be

$$\begin{aligned} m_\pi^2&=\frac{8\epsilon_0}{\langle\chi\rangle}(\sqrt{\tfrac{2}{3}})(1+a)\,,\\ m_K^2&=\frac{8\epsilon_0}{\langle\chi\rangle}(\sqrt{\tfrac{2}{3}})(1-\tfrac{1}{2}a)\,,\\ m_\eta^2&=\frac{8\epsilon_0}{\langle\chi\rangle}(\sqrt{\tfrac{2}{3}})(1-a)\,, \end{aligned} \tag{5}$$

where $\sqrt{2}a=\epsilon_8/\epsilon_0$, and these are not invariant under the Kuo transformation

$$\begin{aligned} \epsilon_0&\to\bar{\epsilon}_0=-\tfrac{1}{3}(1+4a)\epsilon_0\,,\\ a&\to\bar{a}=(2-a)/(1+4a)\,. \end{aligned} \tag{6}$$

This is rather surprising since, as already pointed out by Okubo, the Kuo transformation is a canonical transformation, and one knows from the theorem of Kamefuchi *et al.*[6] that such a transformation should have no observable effect. One possible explanation is, of course, that the operator Y_5 is not a well-defined mathematical operator and therefore the theorem of Kamefuchi *et al.* does not, strictly speaking, apply in the present case. On the other hand, one should also recognize that the various terms in the interaction Hamiltonian H' are defined only through their transformation properties and therefore the fact that the parameters ϵ_0 and ϵ_8 in H' get changed under the transformation W can only mean that the assumed transformation properties of H' do not uniquely define its various components. This is the point of view suggested by Kuo, who has thus been led to enunciate the suggestion that H' must be explicitly invariant under the discrete transformation W. The immediate consequence of this suggestion is that the interaction Lagrangian within $(3,\bar{3})+(\bar{3},3)$ can be written uniquely in the form

$$L_1=u_0-\sqrt{2}u_8\,. \tag{7}$$

Unfortunately, with this form of L_1, the mass of the pion turns out to be zero, and the question naturally arises as to whether one can add a second term L_2 which will have the effect of giving a nonvanishing pion mass. One possible suggestion is that it belongs to (8,8) of $SU(3)\otimes SU(3)$. In fact Barnes and Isham,[7] in a different context, have shown that if one wants to construct field theories with currents, the more natural choice of symmetry breaking is of the (8,8) type. The 64 linearly transforming components ω_{ij} of (8,8) can be represented as

$$\omega_{ij}(M,\chi)=\chi[e^{2(O\cdot M)/\chi}]_{ij}\,, \tag{8}$$

where $(O\cdot M)_{ij}=\sum f_{ijk}M_k$. The symmetry-breaking Lagrangian is then written in the form

$$L_2(M,\chi)=(\sqrt{\tfrac{1}{8}})\epsilon_1'\sum_{i=1}^{8}\omega_{ii}+(\sqrt{\tfrac{3}{5}})\epsilon_8'\sum_{i,j=1}^{8}d_{8ij}\omega_{ij}\,. \tag{9}$$

If the symmetry breaking is entirely of this type, then the masses of the mesons can again be calculated by a first-order expansion of the interaction Lagrangian, and one gets

$$\begin{aligned} m_\pi^2&=\frac{6\epsilon_1'}{\langle\chi\rangle}(\sqrt{\tfrac{2}{5}})(\tfrac{1}{2}\sqrt{5}+a')\,,\\ m_K^2&=\frac{6\epsilon_1'}{\langle\chi\rangle}(\sqrt{\tfrac{2}{5}})(\tfrac{1}{2}\sqrt{5}-\tfrac{1}{2}a')\,,\\ m_\eta^2&=\frac{6\epsilon_1'}{\langle\chi\rangle}(\sqrt{\tfrac{2}{5}})(\tfrac{1}{2}\sqrt{5}-a')\,, \end{aligned} \tag{10}$$

where $\sqrt{2}a'=\epsilon_8'/\epsilon_1'$. Under the Kuo transformation, the parameters now change in a complicated manner since one has to consider also the **27** and $\mathbf{8}_F$ components in the decomposition of $\mathbf{8}\otimes\mathbf{8}$. This we shall show explicitly in Sec. III, but we only note here that if we demand the absence of **27** and $\mathbf{8}_F$ in both the original and transformed Hamiltonian, we can obtain a value of this parameter a' under which the masses will be invariant.

[3] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[4] S. Okubo, Phys. Rev. D **3**, 409 (1971).
[5] T. K. Kuo, Phys. Rev. D **2**, 342 (1970).
[6] S. Kamefuchi, S. L. O'Raifeartaigh, and A. Salam, Nucl. Phys. **28**, 529 (1961).
[7] K. Barnes and C. J. Isham, Nucl. Phys. **B17**, 267 (1970).

We shall then use this value of a' to construct our interaction Lagrangian L_2.

One can, of course, start by assuming that the entire interaction Lagrangian L' is of the (8,8) type, but we shall first show in Sec. II that such a choice leads to significant changes in the meson-meson scattering lengths from their accepted Weinberg values. Although there is nothing sacrosanct about Weinberg pion-pion scattering lengths, one is naturally less inclined to accept a situation in which the contributions from the symmetry-breaking term to the scattering lengths are of the same order as those from the symmetric terms, which seems to be happening with only (8,8)-type symmetry breaking.

We therefore prefer to introduce the (8,8)-type symmetry breaking as a small correction to our main symmetry-breaking term which is of the type $(3,\bar{3})+(\bar{3},3)$, but both terms are explicitly invariant under the Kuo transformation. There are then two parameters in the theory which are fixed with the help of pion and η-meson masses, the dilatation mass being obtained in the manner suggested by Isham, Salam, and Strathdee.[2] The Gell-Mann–Okubo mass formula, of course, follows naturally.

In Sec. III we discuss the Kuo transformation and fix the relative parameters in the Lagrangians L_1 and L_2, and in Sec. IV we derive the mass relations as well as the PCAC (partial conservation of axial-vector current) condition which leads to the result $F_\pi=F_K=F_\eta$ for the decay constants. In Sec. V we apply our interaction Lagrangian for calculating the meson-meson scattering lengths which are found to differ, as expected, from Cronin's results[8] by only terms of the order of pion mass.

II. PION-PION SCATTERING LENGTHS WITH (8,8)-TYPE SYMMETRY BREAKING

Consider the symmetry-breaking interaction Lagrangian L_2 given by Eq. (9). The quartic terms in the expansion of meson fields give

$$L_2=\frac{2}{3\langle\chi\rangle^3}\Big[\frac{\epsilon_1'}{\sqrt{8}}\frac{9}{4}(M^2)^2+(\sqrt{\tfrac{3}{5}})\epsilon_8'\Big(\frac{15}{8}M^2d_{8ij}M_iM_j+\cdots\Big)\Big]$$

$$=\frac{3}{2\langle\chi\rangle^3}\epsilon_1'(\boldsymbol{\pi}^2+2\bar{K}K+\eta^2)\Big[\Big(\frac{1}{\sqrt{8}}+\frac{1}{2\sqrt{5}}\Big(\frac{5\sqrt{2}}{3}\Big)a'\Big)\boldsymbol{\pi}^2+\Big(\frac{1}{\sqrt{8}}-\frac{1}{4\sqrt{5}}\Big(\frac{5\sqrt{2}}{3}\Big)a'\Big)2\bar{K}K+\Big(\frac{1}{\sqrt{8}}-\frac{1}{2\sqrt{5}}\Big(\frac{5\sqrt{2}}{3}\Big)a'\Big)\eta^2\Big]. \quad (11)$$

[8] J. A. Cronin, Phys. Rev. 161, 1483 (1967).

Now from the masses given in (10), we get

$$a'=(-\sqrt{5})(m_K^2-m_\pi^2)/(2m_K^2+m_\pi^2)=-0.988. \quad (12)$$

The quantity $\langle\chi\rangle$ can be related to the pion decay constant F_π by the use of the PCAC condition. We first calculate the divergence of the axial-vector current $\partial_\mu A_\mu{}^i$ from the relation

$$\partial_\mu A_\mu{}^i=-i[Q_5{}^i,L_2], \quad (13)$$

which, together with the divergence of the dilatation current, are known to be the consequences of the assumption that the symmetry breaking belongs to a definite irreducible representation of the direct product of the chiral and conformal groups. Using the commutation relation

$$[Q_5{}^i,\omega^{jk}]=f^{ijl}\omega^{lk}-f^{ikl}\omega^{jl}, \quad (14)$$

it is then straightforward to show that (to lowest order)

$$\partial_\mu A_\mu{}^i=m_\pi^2\langle\chi\rangle M^i, \quad i=1,2,3 \quad (15)$$

so that we can make the identification

$$F_\pi=\langle\chi\rangle. \quad (16)$$

Thus, for example, substituting the expression for the pion mass from (10), we obtain the π-π interaction Lagrangian in the form

$$L_2{}^{(\pi\pi)}=\frac{5}{12}f^2\Big(\frac{a'+3/2\sqrt{5}}{a'+\frac{1}{2}\sqrt{5}}\Big)m_\pi{}^2(\boldsymbol{\pi}^2)^2, \quad (17)$$

where we have put

$$F_\pi=1/f\sqrt{2}, \quad (18)$$

following the notation of Cronin,[8] who has also given the $SU(3)\otimes SU(3)$-invariant pion-pion effective Lagrangian. The complete expression for the Lagrangian in the pion-pion sector is thus

$$L_{\pi\pi}=\tfrac{1}{3}f^2[(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2-(\partial_\mu\boldsymbol{\pi})^2\boldsymbol{\pi}^2+\lambda'^2m_\pi{}^2(\boldsymbol{\pi}^2)^2],$$

where

$$\lambda'^2=\frac{5}{4}\frac{a'+3/2\sqrt{5}}{a'+\frac{1}{2}\sqrt{5}}=-3.05, \quad (19)$$

which is a rather large number.

The scattering lengths in the isospin states $T=0$, 2 are easily calculated and one gets

$$\begin{aligned}m_\pi a_0&=[(m_\pi f)^2/12\pi](4+5\lambda'^2),\\ m_\pi a_2&=[(m_\pi f)^2/6\pi](\lambda'^2-1).\end{aligned} \quad (20)$$

Using $(m_\pi f)^2/4\pi=0.056$ as given by Isham and Patani,[9] these scattering lengths turn out to be $a_0=-0.2m_\pi{}^{-1}$ and $a_2=-0.14m_\pi{}^{-1}$, which are rather large (and even of opposite sign in the case of a_0) compared to those of Weinberg.

[9] C. J. Isham and A. A. Patani, Nuovo Cimento 60A, 255 (1969).

The difficulty stems from the fact that the symmetry breaking is contributing too much, contrary to one's expectation. This is in turn due to the fact that λ'^2 is a large number owing to the small number $a'+\frac{1}{2}\sqrt{5}=0.13$ in the denominator. With a symmetry breaking of the type given in Eq. (1), λ'^2 comes out to be $\frac{1}{4}$, and we do not run into this difficulty. It therefore seems to us that the symmetry breaking cannot be entirely of the type (8,8) but rather a suitable mixture of $(\bar{3},3)+(3,\bar{3})$ and (8,8).

III. KUO TRANSFORMATION

Let us now consider the Kuo transformation in detail to fix the form of the interaction Lagrangian.

A general symmetry-breaking Hamiltonian density H' with specified transformation properties under $SU(3)_L\otimes SU(3)_R$ must, of course, commute with the isospin operator $\mathbf{I}$, hypercharge Y, and parity P. Labeling the states with the isospin and hypercharge from $SU(3)_L$ and $SU(3)_R$ subject to the condition that $Y_L=-Y_R=Y$ and $I_L=I_R=I$, we see that a term in H' can be characterized by the eigenvalues Y and I and we can denote such a term as $H'(Y,I)$. Thus, e.g., if $H'\in(3,\bar{3})+(\bar{3},3)$, we easily find from

$$(\mathbf{1})_{Y=0,I=0}=\tfrac{1}{3}\sqrt{3}(\bar{\mathcal{P}}\mathcal{P}+\bar{\mathfrak{N}}\mathfrak{N}+\bar{\lambda}\lambda)\,,$$
$$(\mathbf{8})_{Y=0,I=0}=(1/\sqrt{6})(\bar{\mathcal{P}}\mathcal{P}+\bar{\mathfrak{N}}\mathfrak{N}-2\bar{\lambda}\lambda)\,,$$

where $\mathcal{P}$, $\mathfrak{N}$, λ are the quarks, that the term $H'(Y=-\frac{2}{3}, I=0)$ is given by the following combination:

$$H'(Y=-\tfrac{2}{3},\,I=0)=u_0-\sqrt{2}u_8\,.$$

Similarly,

$$H'(Y=\tfrac{1}{3},\,I=\tfrac{1}{2})=u_0+\tfrac{1}{2}\sqrt{2}u_8\,.$$

Now the action of the operator W,

$$W=\exp[i\tfrac{3}{2}\pi(Y_+-Y_-)]\,,\qquad(21)$$

is simply to transform $H'(Y,I)$ such that

$$WH'(Y,I)W^{-1}=\pm H'(Y,I)\,,\qquad(22)$$

the $\pm$ signs corresponding to $Y=\frac{2}{3}n$ and $Y=\frac{1}{3}(2n+1)$, respectively. Thus from this completely general transformation we deduce that

$$\begin{aligned}WH'(-\tfrac{2}{3},0)W^{-1}&=+(u_0-\sqrt{2}u_8)\,,\\ WH'(\tfrac{1}{3},\tfrac{1}{2})W^{-1}&=-(u_0+\tfrac{1}{2}\sqrt{2}u_8)\,.\end{aligned}\qquad(23)$$

Using these relations, it is a simple matter to show that for a general

$$H'=\epsilon_0u_0+\epsilon_8u_8\,,$$

the Kuo transformation

$$WH'W^{-1}=\bar{\epsilon}_0u_0+\bar{\epsilon}_8u_8$$

results in the new parameters $\bar{\epsilon}_0$ and $\bar{\epsilon}_8$ given in Eq. (4). If we now demand that the interaction Hamiltonian must remain invariant under the Kuo transformation, we are led to keep only the part $H'(-\frac{2}{3},0)$, so that

$$L_1=u_0-\sqrt{2}u_8\,.$$

Let us now consider the case $H'\in(8,8)$. Following the procedure outlined above, we can write

$$\begin{aligned}H'&=C_1H'(1,\tfrac{1}{2})+C_2H'(0,1)+C_3H'(0,0)+C_4H'(-1,\tfrac{1}{2})\\
&=\left(\frac{\sqrt{15}}{10}C_1-\frac{\sqrt{10}}{20}C_2+\frac{3\sqrt{30}}{20}C_3-\frac{\sqrt{15}}{10}C_4\right)(\mathbf{27})_{Y=0,I=0}\\
&\quad+\left(\frac{\sqrt{10}}{10}C_1-\frac{\sqrt{15}}{5}C_2-\frac{\sqrt{5}}{5}C_3-\frac{\sqrt{10}}{10}C_4\right)(\mathbf{8}_D)_{Y=0,I=0}\\
&\quad+\left(\tfrac{1}{2}C_1+\frac{\sqrt{6}}{4}C_2-\frac{\sqrt{2}}{4}C_3-\tfrac{1}{2}C_4\right)(\mathbf{1})_{Y=0,I=0}\\
&\quad+\frac{\sqrt{2}}{2}(C_1+C_4)(\mathbf{8}_F)_{Y=0,I=0}\,.\end{aligned}$$

Requiring the absence of $(\mathbf{27})$ and $(\mathbf{8}_F)$ in H', we then get two equations:

$$\frac{\sqrt{15}}{10}C_1-\frac{\sqrt{10}}{20}C_2+\frac{3\sqrt{30}}{20}C_3-\frac{\sqrt{15}}{10}C_4=0\,,\qquad(24)$$

$$C_1+C_4=0\,.\qquad(25)$$

Under the Kuo transformation

$$H'\to\bar{H}'=WH'W^{-1}\,,\qquad(26)$$

we note that

$$C_1\to -C_1\,,\quad C_2\to C_2\,,\quad C_3\to C_3\,,\quad C_4\to -C_4\,.$$

Again requiring the absence of $(\mathbf{27})$ and $(\mathbf{8}_F)$ in $\bar{H}'$, we get two further equations:

$$\begin{aligned}-\frac{\sqrt{15}}{10}C_1-\frac{\sqrt{10}}{20}C_2+\frac{3\sqrt{30}}{20}C_3+\frac{\sqrt{15}}{10}C_4&=0\,,\\ -C_1-C_4&=0\,.\end{aligned}\qquad(27)$$

Solving the resulting equations (24), (25), and (27), we get

$$C_1=C_4=0\,,\quad C_2/C_3=3\sqrt{3}\,.\qquad(28)$$

Thus we finally get

$$H'=C_3[(\sqrt{8})(\mathbf{1})-2(\sqrt{5})(\mathbf{8}_D)]\,.\qquad(29)$$

This is to be compared with the form (9) given by Barnes and Isham[7] with a different normalization of the individual terms. Taking note of this different normalization in (9), we get

$$\epsilon_1'=(\sqrt{8})C_3\,,\quad \epsilon_8'=-2(\sqrt{5})C_3\,,$$

and thus the Kuo transformation results in the following ratio of the parameters:

$$a'=-\sqrt{(5/4)}\,.\qquad(30)$$

The Lagrangian (29) is manifestly invariant under Kuo transformations [being a combination of $H'(0,1)$ and $H'(0,0)$]. This invariance, nevertheless, does not fix it uniquely when $H'\in(8,8)$. It becomes unique on restriction to a combination of **1** and $\mathbf{8}_s$ of the diagonal $SU(3)$.

IV. MASS FORMULA AND PCAC

We have thus arrived at the interaction Lagrangian

$$L'=A(u_0-\sqrt{2}u_8)+B\left(\frac{1}{\sqrt{8}}\sum_i \omega_{ii}+(\sqrt{\tfrac{3}{2}})\sum_{i,j} d_{8ij}\omega_{ij}\right), \quad (31)$$

where ω_{ij} have been defined in Eq. (8).

Making an expansion to the first order M, we easily get the following expressions for the masses:

$$m_\pi^2=12B/\langle\chi\rangle, \quad m_\eta^2=16(\sqrt{\tfrac{2}{3}})A/\langle\chi\rangle, \quad (32)$$

and

$$m_K^2=\frac{8A}{\langle\chi\rangle}\sqrt{\tfrac{3}{2}}+\frac{12B}{4\sqrt{2}\langle\chi\rangle}=\tfrac{1}{4}(3m_\eta^2+m_\pi^2), \quad (33)$$

which is the expected Gell-Mann–Okubo mass relation.

For the dilatation field, we follow the technique of Isham, Salam, and Strathdee[2] and derive the following relation for the parameter κ defined there:

$$4\kappa\langle\chi\rangle^2=-3(\sqrt{\tfrac{2}{3}})\frac{A}{\langle\chi\rangle}-\frac{(\sqrt{8})B}{\langle\chi\rangle}. \quad (34)$$

In terms of this parameter κ, the dilatation mass is

$$m_\chi^2=-12\kappa\langle\chi\rangle^2=\tfrac{9}{16}m_\eta^2+m_\pi^2. \quad (35)$$

Thus

$$16m_\chi^2=12m_K^2+13m_\pi^2, \quad (36)$$

i.e., the dilatation mass now comes out to be 450 MeV.

For the derivation of the PCAC condition, we make use of Eq. (13) to obtain

$$\partial_\mu A_\mu{}^i=6\sqrt{2}BM^i, \quad i=1,2,3 \quad (37)$$
$$=(8(\sqrt{\tfrac{3}{2}})A+\tfrac{3}{2}\sqrt{2}B)M^i, \quad i=4,5,6,7 \quad (38)$$
$$=16(\sqrt{\tfrac{2}{3}})AM^i, \quad i=8. \quad (39)$$

Using now the expressions for the masses given above, we get

$$\begin{aligned}\partial_\mu A_\mu{}^i&=\langle\chi\rangle m_\pi^2M^i, && i=1,2,3\\ &=\langle\chi\rangle m_\eta^2M^i, && i=8\\ &=\langle\chi\rangle(\tfrac{3}{4}m_\eta^2+\tfrac{1}{4}m_\pi^2)M^i, && i=4,5,6,7.\end{aligned} \quad (40)$$

Making the identification

$$\langle\chi\rangle=F_\pi,$$

we get from the above

$$F_\pi=F_\eta=F_K$$

on using Eq. (33).

In the Appendix, we add a note on the width of the dilatation particle for the decay into two pions.

V. MESON-MESON SCATTERING LENGTHS

The $SU(3)\otimes SU(3)$-invariant chiral Lagrangian for the meson-meson system has been written down before by many people, and all we have to do is to evaluate our interaction Lagrangian L' given in (31) up to fourth order in meson fields. We thus get the following interaction Lagrangians for $\pi\pi$, πK, and KK scattering: For $\pi\pi$ scattering,

$$L_{\pi\pi}=\tfrac{1}{3}f^2[(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2-(\partial_\mu\boldsymbol{\pi})^2(\boldsymbol{\pi})^2+m_\pi^2(\boldsymbol{\pi})^2]; \quad (41)$$

for πK scattering,

$$\begin{aligned}L_{\pi K}=\tfrac{1}{6}f^2\{&(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})[(\partial_\mu\bar{K})K+(\partial_\mu K)\bar{K}]\\ &-\boldsymbol{\pi}^2(\partial_\mu\bar{K})(\partial_\mu K)-(\partial_\mu\boldsymbol{\pi})^2\bar{K}K\\ &+3i(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\cdot(\partial_\mu\bar{K}\boldsymbol{\tau}K-\bar{K}\boldsymbol{\tau}\partial_\mu K)\\ &+(m_K^2+4m_\pi^2)\boldsymbol{\pi}^2\bar{K}K\};\end{aligned} \quad (42)$$

and for KK scattering,

$$\begin{aligned}L_{KK}=\tfrac{1}{3}f^2\{&[(\partial_\mu\bar{K})K]^2+[(\partial_\mu K)\bar{K}]^2-\bar{K}(\partial_\mu K)K(\partial_\mu\bar{K})\\ &+\bar{K}K(\partial_\mu\bar{K})(\partial_\mu K)+m_K^2(\bar{K}K)^2\}.\end{aligned} \quad (43)$$

The scattering amplitudes are then given by

(I) $\pi_a(q_1)+\pi_b(q_2)\to\pi_c(q_3)+\pi_d(q_4)$,

$$\begin{aligned}T_{cd,ab}{}^{\pi\pi}=&2f^2[(s-m_\pi^2)\delta_{ab}\delta_{cd}+(t-m_\pi^2)\delta_{ac}\delta_{bd}\\ &+(u-m_\pi^2)\delta_{ad}\delta_{bc}]-\tfrac{1}{3}f^2(2\textstyle\sum_i q_i^2-14m_\pi^2)\\ &\times(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}),\end{aligned} \quad (44)$$

(II) $\pi_a(q_1)+K_\alpha(k)\to\pi_b(q')+K_\beta(k')$,

$$\begin{aligned}T_{b\beta,a\alpha}{}^{\pi K}=\tfrac{1}{6}f^2\{&[3t-(q^2+k^2+q'^2+k'^2)+2(m_K^2+4m_\pi^2)]\\ &\times\delta_{ab}\delta_{\alpha\beta}+3i(s-u)\epsilon_{bac}(\tau_c)_{\beta\alpha}\},\end{aligned}$$

(III) $K_\alpha(q_1)+K_\beta(q_2)\to K_\gamma(q_3)+K_\delta(q_4)$,

$$T_{\gamma\delta,\alpha\beta}{}^{KK}=\tfrac{1}{3}f^2[(-3s+\textstyle\sum_i q_i^2+2m_K^2)(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})].$$

The scattering lengths are given by the following expressions: For $\pi\pi$ scattering,

$$a_0=\left(\frac{7}{16\pi}+\frac{5}{16\pi}\right)(fm_\pi)^2m_\pi^{-1}=\frac{3}{4\pi}(fm_\pi)^2m_\pi^{-1},$$

$$a_1=0,$$

$$a_2=\left(-\frac{1}{8\pi}+\frac{1}{8\pi}\right)(fm_\pi)^2m_\pi^{-1}=0;$$

for πK scattering,

$$a_{3/2}=-\frac{1}{4\pi}(fm_\pi)^2\left(1+\frac{m_\pi}{m_K}\right)^{-1}\left(1-\frac{m_\pi}{2m_K}\right)m_\pi^{-1},$$

$$a_{1/2}=\frac{1}{2\pi}(fm_\pi)^2\left(1+\frac{m_\pi}{m_K}\right)^{-1}\left(1+\frac{m_\pi}{4m_K}\right)m_\pi^{-1};$$

and lastly for KK scattering,

$$a_0=0\,,$$
$$a_1=-\frac{1}{8\pi}(fm_\pi)^2\frac{m_K}{m_\pi}\frac{1}{m_\pi}.$$

Notice that in all cases the correction to Cronin's values[8] of the scattering lengths is of the order of m_π as expected.

VI. DISCUSSION

We have shown here that the symmetry breaking entirely of the (8,8) type leads to large changes of the pion-pion scattering lengths and may not therefore be considered justifiable. It is well known that all chiral models result in the following general form of the pion-pion scattering amplitude:

$$T_{cd,ab}{}^{\pi\pi}=2f^2[(s-\lambda^2m_\pi{}^2)\delta_{ab}\delta_{cd}+(t-\lambda^2m_\pi{}^2)\delta_{ac}\delta_{bd}+(u-\lambda^2m_\pi{}^2)\delta_{ad}\delta_{bc}],$$

where the values of $\lambda=1,\frac{4}{3},\frac{3}{2}$ corerspond to the σ model of Gell-Mann and Lévy, the exponential model of Gürsey, Weinberg, Cronin, and others, and the rational model of Schwinger. In the (8,8)-type symmetry breaking, however, we get the corresponding factor from Eq. (19) to be

$$\lambda^2=\tfrac{4}{3}(1-\lambda'^2)\,,$$

i.e., $\lambda^2=16/3$.

We are therefore led to believe that the symmetry breaking cannot be entirely of the (8,8) type. Since the $(\bar{3},3)+(3,\bar{3})$ type of symmetry breaking advocated by Gell-Mann has been studied extensively in the literature, we have tried here the hypothesis that a mixture of these two types of symmetry breaking may be a suitable alternative. An interaction Lagrangian consisting of both these pieces will, of course, contain four parameters and would not have much to recommend itself, and we are therefore led to consider seriously the point of view proposed by Kuo. Following him, we demand the invariance of the Lagrangian under the transformation (3) which limits the number of parameters to two. These two parameters can then be fixed by a reference to the meson masses. The resulting pion-pion scattering amplitude has now a value of $\lambda^2=0$.

We should like to point out that we have used the same dilatation field χ for the two pieces of the interaction Lagrangian L_1 and L_2 leading to the result for the decay constants $F_\pi=F_\eta=F_K$. One may, of course, assume the existence of two dilatation fields χ_1 and χ_2, but even apart from the unaesthetic nature of this assumption, the masses of the two corresponding particles turn out to be $m_{\chi_1}=\frac{3}{4}m_\eta$ and $m_{\chi_2}=m_\kappa$ in such a scheme. Also, one gets the following equation for the decay constants:

$$4m_K{}^2F_K=3m_\eta{}^2F_\eta+m_\pi{}^2F_\pi\,,$$

assuming the existence of two dilatation fields. However, as we have mentioned, there seems to us no reason to assume two dilatation fields.

Note added in proof. It has been pointed out to us that a model similar to that in Sec. III has been proposed by T. K. Kuo, Phys. Rev. D (to be published). J. Ellis (private communication) has pointed out that our result, Eq. (16), depends upon the special choice of coordinates in Eq. (2).

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APPENDIX: WIDTH OF χ-FIELD QUANTA

We have found the mass of the dilatation quanta to be given by

$$m_\chi{}^2=\tfrac{9}{16}m_\eta{}^2+m_\pi{}^2.$$

The coupling of the χ field to the pion field can be easily calculated, and we get

$$L_{\chi\pi\pi}=(m_\pi{}^2/2F_\pi)\bar{\chi}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\,,$$

and hence we have the effective $\chi\pi\pi$ coupling constant

$$g_{\chi\pi\pi}=m_\pi{}^2/2F_\pi=7\text{ MeV}.$$

This leads to an effective half-width Γ_χ given by

$$\tfrac{1}{2}m_\chi\Gamma_x=g_{\chi\pi\pi}{}^2\,,$$

i.e.,

$$\Gamma_\chi=2\times125=250\text{ MeV}.$$

It is to be noted that only the (8,8)-type symmetry-breaking term L_2 contributes to $L_{\chi\pi\pi}$ since the pion mass term in L_1 does not exist because of our choice $u_0-\sqrt{2}u_8$. We may mention here that Ellis[10] has obtained the width $\Gamma_\chi=380$ MeV from the (8,8)-type symmetry-breaking Lagrangian, but his method is different.

[10] J. Ellis, University of Cambridge report, 1970 (unpublished).