where

$$
\begin{aligned}
d_3(1) &= -\alpha\beta_1+(K_1-K)^2-K_1^2\,,\\
D_2(1) &= -[m^2-\beta_1(1-\beta_1)(m^2+\alpha-t)]\\
&\quad +(1-\beta_1)K_1^2+\beta_1(K_1+Q-K)^2\,,\\
D_2(3) &= -[m^2-\beta_1(1-\beta_1)(m^2-\alpha)]+\beta_1K_1^2\\
&\quad +(1-\beta_1)(K_1-K)^2\,, \qquad \text{(B2)}\\
D_4(1) &= -[m^2-\beta_1(1-\beta_1)(m^2+\alpha-t)]+(1-\beta_1)K_1^2\\
&\quad +\beta_1(K_1+Q-K)^2\,,\\
D_4(3) &= -[m^2-\beta_1(1-\beta_1)(m^2-t)]\\
&\quad +\beta_1(K_1-K+Q)^2+(1-\beta_1)(K_1-K)^2\,.
\end{aligned}
$$

We immediately see that at the end points of integration—$\beta_1=0$, 1—the terms $D_i(j)$ are strictly negative and cannot vanish. The term $d_3(1)$ can vanish, but its residue is zero [i.e., $d_3(1)$ is a factor of the terms in the brackets]. Hence we conclude that the terms $\beta_1^{\phi_1}$ and $(1-\beta_1)^{\phi_2}$ do not introduce any new singularities.

Finally, we observe that A_1 is certainly a real quantity when the $D_i(j)$'s are negative for all β_1 between 0 and 1. This occurs when the terms in brackets of Eq. (B2) are positive. Since the maximum value of $\beta_1(1-\beta_1)$ is $\frac{1}{4}$, this condition is satisfied for

$$4m^2>m^2+\alpha-t\,,\quad m^2-\alpha\,,\quad m^2-t\,.$$

This is equivalent to the region

$$s_1<4m^2+K^2\,,\quad -3m^2<t<0\,,\quad s_1+t>-2m^2+K^2\,,$$

which overlaps with the region D_2 of Fig. 8.

This method can also be applied to the amplitude A_1 of Fig. 17, but it becomes very tedious. The Feynman parameter method is considerably easier.

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Higher-Order Terms in the Current-Current Theory of Weak Interactions*

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We examine the consequences of the hypothesis that, for some range of energies, the second-order terms in the perturbation expansion of the Fermi theory are small but non-negligible corrections to the first-order terms, and responsible for reactions which violate first-order selection rules. We compute matrix elements through second order in the weak coupling constant G and introduce subtraction constants which are necessary to render the matrix elements finite to this order. From this point of view we consider a number of weak-interaction problems, including the experimental parameters of μ decay, the universality of the μ-decay coupling constant and the Fermi β-decay coupling constant, K_{l3} decays, and $\Delta S=\Delta Q$ and $\Delta S=2$ semileptonic decays.

I. INTRODUCTION

THE phenomenological vector and axial-vector current-current description of weak interactions gives a good account[1] of a large number of experimental data. Nevertheless, it has been realized for some time that the phenomenological current-current matrix elements cannot be exact, because they lead to cross sections for leptonic reactions which violate the unitarity limit at very high energy. This suggests that the phenomenological current-current matrix elements are the first-order terms in a perturbation expansion of the S matrix. If this perturbation expansion is derived from a current-current interaction Lagrangian, it is not renormalizable in the conventional perturbation-theoretic sense. Thus one cannot predict whether higher-order effects are large or small (there is no small dimensionless coupling constant). The approach of this paper is based on the empirical observation that the lowest-order current-current matrix elements do give a good account of a large number of experimental data, so the higher-order effects are small at the energies presently accessible experimentally. This observed weakness of higher-order effects may occur term by term in a perturbation expansion, or it may only come out of a summation of all orders or some other nonperturbative approach. In this paper we explore the consequences of the former possibility. We offer no solution to the fundamental problem of divergences in a nonrenormalizable theory. We compute matrix elements through second order in the weak coupling constant G and introduce subtraction constants which are necessary to render the matrix elements finite to this order. We parametrize divergent integrals by undetermined subtraction constants in subtracted dispersion relations. We are able to show that some of these subtraction constants are "universal" (i.e., appear in the same way in more than one process); some others we guess, on more or less plausible grounds, to be

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[1] See, for example, the detailed description in R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969).

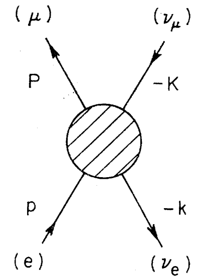

FIG. 1. Kinematics for reaction (2.4).

related; some remain for which we see no relation to the others.[2]

In spite of the predictive weakness of a nonrenormalizable theory, some results do follow from the hypothesis that, for some range of energies, second-order weak terms are small but non-negligible corrections to first-order weak terms, and responsible for reactions which violate first-order selection rules. We predict that the second-order weak corrections to the experimental parameters of μ decay are too small to be observable. The universality of the Fermi coupling constant in β decay and the μ-decay coupling constant is lost; i.e., the value of $\cos\theta_c$ determined from comparison of the O^{14} and μ decays, after electromagnetic radiative corrections have been made, contains an undertermined second-order weak contribution. We show that the second-order weak contribution to $\cos\theta_c$ can be determined (or bounded) if there are sufficiently accurate experimental values of the correlation parameters for the decay of free polarized neutrons and if C_A is taken from the Adler-Weisberger relation. The largest effects for any allowed semileptonic decay should occur in the K_{l3} decays because these decays have the largest q^2 and, for $K_{\mu 3}$, the muon mass appears in those effects which are proportional to the square of the lepton mass. If the existence of $\Delta S=-\Delta Q$ processes should be established with a strength anywhere near the present experimental upper limits (e.g., $\Sigma^+\to n l\bar{\nu}$ or in $K^0\to\pi l\nu$), our approach leads to relations between the branching ratio for $\Sigma^+\to n l\bar{\nu}$, the $\Delta S=-\Delta Q$ parameter x in K^0 decay, and the branching ratio for the $\Delta S=2$ semileptonic decay $\Xi\to N l\nu$.

II. LEPTONIC PROCESSES

We take as interaction Lagrangian

$$\mathcal{L}_I(x)=-(G_0/\sqrt{2}):\mathcal{J}_\lambda{}^\dagger(x)\mathcal{J}^\lambda(x): +\text{mass counterterms}\,, \quad (2.1)$$

where

$$\mathcal{J}_\lambda(x)=j_\lambda(x)+J_\lambda(x)\,, \quad (2.2)$$

$$j_\lambda(x)=\sum_{l=e,\mu}\bar{\Psi}_l(x)(1+\gamma_5)\gamma_\lambda\Psi_{\nu_l}(x)\,, \quad (2.3\text{a})$$

$$J_\lambda(x)=\cos\theta_c\left[V_\lambda{}^{1-i2}(x)+A_\lambda{}^{1-i2}(x)\right] +\sin\theta_c\left[V_\lambda{}^{4-i5}(x)+A_\lambda{}^{4-i5}(x)\right]. \quad (2.3\text{b})$$

The hadronic currents are included in $\mathcal{L}_I$ even when discussing purely leptonic reactions because they contribute to virtual intermediate states when one goes beyond first order in perturbation theory.

Consider, for example, the reaction

$$\bar{\nu}_e+e\to\bar{\nu}_\mu+\mu\,. \quad (2.4)$$

The kinematics are illustrated in Fig. 1. The first order of perturbation theory gives the usual phenomenological matrix element[3]

$$\frac{G_0}{\sqrt{2}}M^{(1)}=\frac{G_0}{\sqrt{2}}[\bar{u}(P)\Gamma_\lambda v(K)][\bar{v}(k)\Gamma^\lambda u(p)]\,, \quad (2.5)$$

$$\Gamma_\lambda=(1+\gamma_5)\gamma_\lambda\,. \quad (2.6)$$

In the second order we obtain

$$\tfrac{1}{2}G_0{}^2M^{(2)}=\tfrac{1}{2}G_0{}^2[\bar{u}(P)\Gamma^\lambda v(K)]J_{\lambda\sigma}(Q)[\bar{v}(k)\Gamma^\sigma u(p)], \quad Q=p+k=P+K\,, \quad (2.7)$$

where

$$J_{\lambda\sigma}(Q)=i\int d^4x\, e^{iQx}\langle 0|T(\mathcal{J}_\lambda{}^\dagger(x)\mathcal{J}_\sigma(0))|0\rangle \quad (2.8)$$

$$=J_{\lambda\sigma}{}^{(e)}(Q)+J_{\lambda\sigma}{}^{(\mu)}(Q)+J_{\lambda\sigma}{}^{(H)}(Q)\,. \quad (2.9)$$

The leptonic contributions are given explicitly in terms of a quadratically divergent integral

$$J_{\lambda\sigma}{}^{(l)}(Q)\sim i\int\frac{d^4r}{(2\pi)^4}\,\mathrm{Tr}\left[\frac{1}{\gamma(r-Q)}\Gamma_\lambda\frac{1}{\gamma r-m_l}\Gamma_\sigma\right]. \quad (2.10)$$

These integrals correspond to the Feynman diagrams of Fig. 2. One approach to the integral (2.10) is to introduce a cutoff and compute the coefficient of the quadratic divergence. Then

$$J_{\lambda\sigma}{}^{(l)}\approx g_{\lambda\sigma}\Lambda^2/4\pi^2\,. \quad (2.11)$$

This estimate has been used in a number of investigations[4,5] of first-order forbidden processes with the conclusion that the effective cutoff Λ is small (compared to the so-called unitarity limit). However, if the effective cutoff is very small, then it is not clear that (2.11) dominates the integral; and the nonquadratic terms cannot be computed unambiguously from the divergent

[2] From this point of view the assumption of a universal cutoff Λ is seen to be a very strong assumption in the absence of any detailed understanding of the origin of the cutoff.

[3] See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1965). We use the metric and γ matrices of this text. We differ in the normalization of spinors and states. We use $\bar{u}(p)u(p)=2m$ and

$$\langle p'|p\rangle=(2\pi)^3 2p_0\delta^3(p-p').$$

[4] B. L. Ioffe and E. P. Shabalin, Yadern. Fiz. **6**, 828 (1967) [Soviet J. Nucl. Phys. **6**, 603 (1968)].

[5] R. N. Mohapatra, J. Subba Rao, and R. E. Marshak, Phys. Rev. **171**, 1502 (1968).

Feynman integral. When we also consider that we have no understanding of the physical basis for a small cutoff Λ, it appears that a somewhat more general parametrization of the second-order weak contributions may be appropriate. Such a parametrization is provided by the subtraction constants in the dispersion relations satisfied by (2.8). The question is: How many subtraction constants? For the leptonic contributions to (2.9) this question can be answered by straightforward calculation. We write

$$J_{\lambda\sigma}^{(l)}(Q)=i\int_{(\text{regulated})}\frac{d^4r}{(2\pi)^4}\operatorname{Tr}\left[\frac{1}{\gamma(r-Q)}\Gamma_\lambda\frac{1}{\gamma r-m_l}\Gamma_\sigma\right]$$
$$=g_{\lambda\sigma}A^{(l)}(Q^2)+Q_\lambda Q_\sigma B^{(l)}(Q^2).\qquad(2.12)$$

The regulated (invariantly cutoff) integral defines two functions $A^{(l)}(Q^2)$ and $B^{(l)}(Q^2)$ which are analytic in the Q^2-plane cut along the positive axis from m_l^2 to ∞, and the discontinuities across the cut may be computed straightforwardly from (2.12), and are independent of the (invariant) cutoff or regulator masses:

$$2i\operatorname{Im}A^{(l)}(Q^2)=-\frac{i}{6\pi}\left(1-\frac{m_l^2}{Q^2}\right)^2\times(2Q^2+m_l^2)\theta(Q^2-m_l^2),\qquad(2.13\text{a})$$

$$2i\operatorname{Im}B^{(l)}(Q^2)=\frac{i}{6\pi}\left(1-\frac{m_l^2}{Q^2}\right)^2\times\left(2+4\frac{m_l^2}{Q^2}\right)\theta(Q^2-m_l^2).\qquad(2.13\text{b})$$

We see that the dispersion relation for $A^{(l)}(Q^2)$ requires two subtractions, and for $B^{(l)}(Q^2)$ one subtraction. It is useful to note that these results are the same as one would obtain by simple dimensional considerations. The function $A^{(l)}(Q^2)$ has the dimensions of mass squared, which suggests a first-order polynomial in Q^2 for $Q^2\to\infty$, while $B^{(l)}(Q^2)$ is dimensionless, which suggests a zeroth-order polynomial in Q^2 for $Q^2\to\infty$.

Since we do not treat the strong interactions perturbatively, we cannot reduce $J_{\lambda\sigma}^{(H)}(Q)$ to any explicit integral, divergent or otherwise. But we can still decompose[6]

$$J_{\lambda\sigma}^{(H)}(Q)=i\int d^4x\,e^{iQx}\langle 0|T^*(J_\lambda^\dagger(x)J_\sigma(0))|0\rangle$$
$$=g_{\lambda\sigma}A^{(H)}(Q^2)+Q_\lambda Q_\sigma B^{(H)}(Q^2).\qquad(2.14)$$

[6] The covariant decomposition (2.12) and (2.14) is required for (2.7) to be a Lorentz-invariant matrix element. Thus the T symbol in (2.8) and (2.14) should be understood as T^*, meaning the time-ordered product plus possible polynomial in Q_0 required to produce a Lorentz-covariant tensor. See K. Johnson, Nucl. Phys. **25**, 431 (1961), and J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

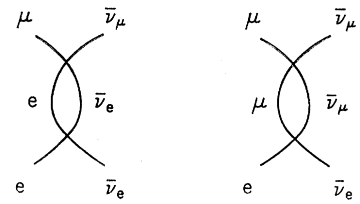

FIG. 2. Leptonic contributions to (2.9).

On the basis of the above dimensional considerations, we assume that $A^{(H)}[B^{(H)}]$ satisfy at most twice (once) subtracted dispersion relations. Then

$$M=\frac{G_0}{\sqrt2}[\bar u(P)\Gamma^\lambda v(K)]\Big\{g_{\lambda\sigma}\Big[1+\frac{G_0}{\sqrt2}[a_0+a_1s+\bar A^{(e)}(s)+\bar A^{(\mu)}(s)+\bar A^{(H)}(s)]\Big]+(p+k)_\lambda(p+k)_\sigma\frac{G_0}{\sqrt2}\times[[b_1+\bar B^{(e)}(s)+\bar B^{(\mu)}(s)+\bar B^{(H)}(s)]]\Big\}\times[\bar v(k)\Gamma^\sigma u(p)],\qquad(2.15)$$

where $s=(p+k)^2$.

Although the functions $A(s)$ and $B(s)$ are independent of the choice of subtraction point s_0, the separation into subtraction terms plus remainder functions does depend on this choice. It is convenient to choose $|s_0|\ll(1\text{ BeV})^2$. Then the remainder functions are bounded by

$$\bar A(s)\lesssim s_{\max}\quad\text{or}\quad|s_0|,\qquad(2.16\text{a})$$
$$\bar B(s)\lesssim 1.\qquad(2.16\text{b})$$

These are order-of-magnitude bounds and neglect $\ln(s/|s_0|)$ factors. For the leptonic functions $\bar A^{(e,\mu)}$ and $\bar B^{(e,\mu)}$ these bounds may be verified[7] explicitly by doing the subtracted dispersion integrals with absorptive parts given by (2.13a) and (2.13b). If the subtraction constants are small, i.e., $a_0\lesssim s_{\max}$, a_1, $b_1\lesssim 1$, then $G_0\approx G_\mu\approx 10^{-5}m_p^{-2}$ and the entire second-order weak contribution is negligible relative to the first order for energies $\lesssim 100$ BeV. In the rest of this paper we will be concerned with the alternative possibility that the subtraction constants are large, i.e.,

$$|\bar A(s),\ \bar B(s)s_{\max}|\ll|a_0,\ a_1s_{\max},\ b_1s_{\max}|\ll(300\text{ BeV})^2.\qquad(2.17)$$

The upper bound is necessary for the applicability of perturbation theory. As the result of the above discussion, under the stated conditions (2.17), we are led to the following parametrization of the matrix element for the process (2.4), including the second-order weak-

[7] The results are given in the dissertation of one of the authors (J.M.T.) (unpublished).

interaction contribution

$$M=\frac{G_0}{\sqrt{2}}(\bar{u}(P)\Gamma^\lambda v(K))\left\{g_{\lambda\sigma}\left[1+\frac{G_0}{\sqrt{2}}(a_0+a_1s)\right]\right.$$
$$\left.+(p+k)_\lambda(p+k)_\sigma\frac{G_0}{\sqrt{2}}b_1\right\}(\bar{v}(k)\Gamma^\sigma u(p)). \quad (2.18)$$

The matrix element for μ^+ decay can be worked out in the same way and is simply related to (2.18) by the substitution rule. A standard calculation of the decay distribution for a polarized μ^+ (in the muon rest frame) gives

$$d\Gamma(x,\theta)=\frac{G_0{}^2m_\mu{}^5}{128\pi^4}\left\{(1-\tfrac{2}{3}x)+\frac{\alpha}{6\pi}f(x)\right.$$
$$+\sqrt{2}G_0\left[(1-\tfrac{2}{3}x)a_0+\tfrac{1}{3}x(1-\tfrac{1}{2}x)a_1m_\mu{}^2\right.$$
$$\left.+\frac{1-x}{x}b_1m_e{}^2\right]-\cos\theta\left[\tfrac{1}{3}(1-2x)+\frac{\alpha}{6\pi}g(x)\right.$$
$$\left.\left.+\sqrt{2}G_0[\tfrac{1}{3}(1-2x)a_0-\tfrac{1}{6}x^2a_1m_\mu{}^2]\right]\right\}$$
$$\times x^2dxd\Omega, \quad (2.19)$$
$$x=2(E_e/m_\mu).$$

We have dropped terms proportional to $m_e{}^2$, except the one multiplying the unknown subtraction constant b_1. We have also included the second-order electromagnetic radiative corrections $(\alpha f(x),\alpha g(x))$ computed by Kinoshita and Sirlin.[8] The lifetime is computed by integrating over x and θ:

$$\tau^{-1}=\frac{G_0{}^2m_\mu{}^5}{192\pi^3}\left[1-\frac{\alpha}{2\pi}\left(\pi^2-\frac{25}{4}\right)\right.$$
$$\left.+\sqrt{2}G_0(a_0+\tfrac{3}{10}a_1m_\mu{}^2+b_1m_e{}^2)\right]$$
$$\equiv\frac{G_\mu{}^2m_\mu{}^5}{192\pi^3}\left[1-\frac{\alpha}{2\pi}\left(\pi^2-\frac{25}{4}\right)\right]\equiv\frac{(G_\mu{}^{\text{obs}})^2m_\mu{}^5}{192\pi^3}. \quad (2.20)$$

From the measured μ lifetime, we have

$$G_\mu m_p{}^2=1.026\times10^{-5}. \quad (2.21)$$

We invert (2.20) and take the square root to obtain

$$G_0=G_\mu[1-\tfrac{1}{2}\sqrt{2}G_\mu(a_0+\tfrac{3}{10}a_1m_\mu{}^2+b_1m_e{}^2)$$
$$+O(G_\mu\alpha,G_\mu{}^2)]. \quad (2.22)$$

[8] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).

Substitute (2.22) back into (2.19) to obtain

$$d\Gamma(x,\theta)=\frac{G_\mu{}^2m_\mu{}^5}{128\pi^4}\left\{(1-\tfrac{2}{3}x)+\frac{\alpha}{6\pi}f(x)+\sqrt{2}G_\mu\right.$$
$$\times[(-\tfrac{3}{10}+8/15x-\tfrac{1}{6}x^2)a_1m_\mu{}^2+(1/x-2+\tfrac{2}{3}x)b_1m_e{}^2]$$
$$-\cos\theta\left[\tfrac{1}{3}(1-2x)+\frac{\alpha}{6\pi}g(x)+\sqrt{2}G_\mu[(-\tfrac{1}{10}+\tfrac{1}{5}x-\tfrac{1}{6}x^2)\right.$$
$$\left.\left.\times a_1m_\mu{}^2+(-\tfrac{1}{3}+\tfrac{2}{3}x)b_1m_e{}^2]\right]\right\}x^2dxd\Omega. \quad (2.23)$$

Note that a_0 has dropped out when $d\Gamma$ is rewritten in terms of the coupling constant G_μ. This could be seen directly from the matrix element (2.18), in which it is evident that a_0 could be eliminated (through order G^2) by a redefinition of the coupling constant.

Equation (2.23) is to be compared with the standard form

$$d\Gamma(x,\theta)_{\mu^+\text{ decay}}=\text{const}\{6(1-x)+\tfrac{4}{3}\rho(4x-3)$$
$$+(\alpha/2\pi)f(x)-\xi\cos\theta[-2(1-x)$$
$$-\tfrac{4}{3}\delta(4x-3)+(\alpha/2\pi)g(x)]\}x^2dxd\Omega. \quad (2.24)$$

The parameters ρ, ξ, and δ are determined by a least-squares fit of (2.24) to the experimentally determined electron momentum spectrum and angular distortion relative to the spin direction of the polarized muon. The values of ρ, ξ, and δ as functions of a_1 and b_1 predicted by (2.23) may be determined by a least-squares fit of (2.24) to (2.23). However, we will find below[9] that $G_\mu a_1m_\mu{}^2$ and $G_\mu b_1m_e{}^2$ are very small, so we do not give these formulas here.[7] The present experimental values[10] of ρ and δ agree with the lowest-order $V+A$ values within the $\frac{1}{2}\%$ experimental accuracy, while the value of ξ differs from 1 by 3% with a quoted experimental error of 1.5%. We do not attribute this 1-standard-deviation effect to higher-order weak interactions.

III. SEMILEPTONIC PROCESSES

Consider the reaction

$$\bar{\nu}_e+e\to\alpha, \quad (3.1)$$

where α is some hadronic final state, e.g., $n+\bar{p}$. The first order of perturbation theory in the weak interactions gives the phenomenological semileptonic matrix element

$$\frac{G_0}{\sqrt{2}}M^{(1)}=\frac{G_0}{\sqrt{2}}\langle\alpha|J_\lambda(0)|0\rangle\bar{v}(k)\Gamma^\lambda u(p). \quad (3.2)$$

In second order in the weak interaction, we have to

[9] We are anticipating results of the discussion of K_{l3} decays in the next section.

[10] S. E. Derenzo, Phys. Rev. **181**, 1854 (1969).

compute

$$\tfrac{1}{2}i^2(\tfrac{1}{2}G_0{}^2)\int d^4z_1d^4z_2\langle\alpha|T(\psi_{\nu_e}(x')\bar{\psi}_e(x):\mathcal{J}_\lambda{}^\dagger(z_1)\mathcal{J}^\lambda(z_1)::\mathcal{J}_\sigma{}^\dagger(z_2)\mathcal{J}^\sigma(z_2):)|0\rangle$$
$$=-\tfrac{1}{2}G_0{}^2\int d^4z_1d^4z_2\{\langle\alpha|T(\psi_{\nu_e}(x')\bar{\psi}_e(x):j_\lambda{}^{(e)\dagger}(z_1)j^{(e)\lambda}(z_1)::j_\sigma{}^{(e)\dagger}(z_2)J^\sigma(z_2):)|0\rangle$$
$$+\langle\alpha|T(\psi_{\nu_e}(x')\bar{\psi}_e(x):j_\lambda{}^{(e)\dagger}(z_1)j^{(\mu)\lambda}(z_1)::j_\sigma{}^{(\mu)\dagger}(z_2)J^\sigma(z_2):)|0\rangle$$
$$+\langle\alpha|T(\psi_{\nu_e}(x')\bar{\psi}_e(x):j_\lambda{}^{(e)\dagger}(z_1)J^\lambda(z_1)::J_\sigma{}^\dagger(z_2)J^\sigma(z_2):)|0\rangle\}_{j_\lambda{}^{(l)}=\bar{\psi}_l\Gamma_\lambda\psi_{\nu_l}}. \quad (3.3)$$

The first term in (3.3) is

$$-\tfrac{1}{2}G_0{}^2\int d^4z_1d^4z_2\langle\alpha|J^\sigma(z_2)|0\rangle S_F(x'-z_1)\Gamma_\lambda S_F(z_1-x)\times\langle 0|T(j^{(e)\lambda}(z_1)j_\sigma{}^{(e)\dagger}(z_2))|0\rangle. \quad (3.4)$$

After amputation of the external legs, this contributes to the second-order invariant matrix element

$$\tfrac{1}{2}G_0{}^2\langle\alpha|J^\sigma(0)|0\rangle J_{\sigma\lambda}{}^{(e)}(Q)\bar{v}(k)\Gamma^\lambda u(p)\,,\quad Q=p+k\,. \quad (3.5)$$

The second term in (3.3) gives a similar result with $J_{\sigma\lambda}{}^{(e)}$ replaced by $J_{\sigma\lambda}{}^{(\mu)}$. The third term in (3.3) is

$$-\tfrac{1}{2}G_0{}^2\int d^4z_1d^4z_2\langle\alpha|T(J^\lambda(z_1):J_\sigma{}^\dagger(z_2)J^\sigma(z_2):)|0\rangle\times S_F(x'-z_1)\Gamma_\lambda S_F(x-z_1). \quad (3.6)$$

We rewrite the matrix element as

$$\langle\alpha|T(J^\lambda(z_1):J_\sigma{}^\dagger(z_2)J^\sigma(z_2):)|0\rangle=\langle\alpha|J^\sigma(z_2)|0\rangle\langle 0|T(J^\lambda(z_1)J_\sigma{}^\dagger(z_2))|0\rangle+\langle\alpha|t^\lambda(z_1,z_2)|0\rangle\,, \quad (3.7)$$

where

$$t^\lambda(z_1,z_2)=T(J^\lambda(z_1):J_\sigma{}^\dagger(z_2)J^\sigma(z_2):)-J^\sigma(z_2)\langle 0|T(J^\lambda(z_1)J_\sigma{}^\dagger(z_2))|0\rangle\,. \quad (3.8)$$

We require that the definition of the double dot product of the interacting hardonic currents (hadronic part of $\mathfrak{L}_I$) satisfies the condition

$$\langle 0|:J_\sigma{}^\dagger(z_2)J^\sigma(z_2):|0\rangle=0\,; \quad (3.9)$$

this ensures that there are no disconnected vacuum diagram contributions to (3.6). [Note that the first term in (3.7) is connected. If we did perturbation theory in the hadronic current this term would correspond to connected closed-loop diagrams of the same structure as the leptonic diagrams of Fig. 2.] When we substitute (3.7) into (3.6), and carry out the external-leg amputation, and combine with (3.5) and the similar muon contribution, we obtain

$$\tfrac{1}{2}G_0{}^2M^{(2)}=\tfrac{1}{2}G_0{}^2[\langle\alpha|J^\sigma(0)|0\rangle J_{\sigma\lambda}(Q)+K_\lambda(\alpha)]\bar{v}(k)\Gamma^\lambda u(p)\,, \quad (3.10)$$

where

$$K^\lambda(\alpha)=i\int d^4z\langle\alpha|[T(J^\lambda(z):J_\sigma{}^\dagger(0)J^\sigma(0):)-J^\sigma(0)\langle 0|T(J^\lambda(z)J_\sigma{}^\dagger(0))|0\rangle]|0\rangle\,, \quad (3.11)$$

and $J_{\sigma\lambda}(Q)$ is the same tensor which appears in (3.7) and is discussed in detail in the preceding section. Because of the factorization, the first term in (3.10) provides a sort of universal correction to all first-order allowed semileptonic processes and is also directly related to the second-order weak correction to purely leptonic processes given in (2.7). The second term in (3.10) destroys this universality and also gives rise to $\Delta S=-\Delta Q$ and $\Delta S=2$ semileptonic reactions.

Let us discuss these first-order forbidden processes first. On general invariance grounds, we have

$$K_\lambda(n\bar{\Sigma}^-)=\sin\theta_c\,\bar{u}_n(P')[\gamma_\lambda\tilde{f}_1(Q^2)+i\sigma_{\lambda\rho}Q^\rho\tilde{f}_2(Q^2)+Q_\lambda\tilde{f}_3(Q^2)+\gamma_5\gamma_\lambda\tilde{h}_1(Q^2)+i\gamma_5\sigma_{\lambda\rho}Q^\rho\tilde{h}_2(Q^2)+\gamma_5Q_\lambda\tilde{h}_3(Q^2)]v_{\Sigma^+}(P)\,. \quad (3.12)$$

The factor $\sin\theta_c$ is included explicitly because the current $J^\dagger$ which appears in (3.11) for $\alpha=n\bar{\Sigma}^-$ must be the strangeness-changing current. On dimensional grounds we would take $\tilde{f}_1$, $\tilde{h}_1$ to be twice subtracted, and $\tilde{f}_2$, $\tilde{f}_3$, $\tilde{h}_2$, $\tilde{h}_3$ once subtracted. Thus

$$\begin{aligned}\tilde{f}_1(Q^2)&\approx\tilde{f}_1(0)+Q^2\tilde{f}_1{}'(0)\,,\\ \tilde{h}_1(Q^2)&\approx\tilde{h}_1(0)+Q^2\tilde{h}_1{}'(0)\,,\\ \tilde{f}_2(Q^2)&\approx\tilde{f}_2(0)\,,\quad\text{etc.}\end{aligned} \quad (3.13)$$

To get an order-of-magnitude estimate, let us neglect all of these constants except $\tilde{f}_1(0)$ and $\tilde{h}_1(0)$. Then

$$\begin{aligned}M(\Sigma^+\to n\bar{l}\nu)&\approx\tfrac{1}{2}G_0{}^2\sin\theta_c\bar{u}_n(P')[\gamma_\lambda\tilde{f}_1(0)+\gamma_5\gamma_\lambda\tilde{h}_1(0)]\times u_{\Sigma^+}(P)(\bar{u}(k)\Gamma^\lambda v(p))\\&\approx\tfrac{1}{2}G_\mu{}^2\sin\theta_c\bar{u}_n(P')[\gamma_\lambda\tilde{f}_1(0)+\gamma_5\gamma_\lambda\tilde{h}_1(0)]\times u_{\Sigma^+}(P)(\bar{u}(k)\Gamma^\lambda v(p))+O(G_\mu{}^3)\,.\end{aligned} \quad (3.14)$$

The matrix element for the allowed decay $\Sigma^-\to nl\bar{\nu}$ is given by the standard Cabibbo theory as

$$M(\Sigma^-\to nl\bar{\nu})=(G_\mu/\sqrt{2})\sin\theta_c\bar{u}_n(P')[\gamma_\lambda+(1-2\alpha)C_A\gamma_5\gamma_\lambda]\times u_\Sigma(P)(\bar{u}(p)\Gamma^\lambda v(k))\,, \quad (3.15)$$
$$\alpha\approx\tfrac{2}{3}\,,\quad C_A\approx 1.25\,.$$

The partial rate is

$$\Gamma=(G_\mu{}^2/2\pi^3)\sin^2\theta_c m_l{}^5[|f_1(0)|^2+3|h_1(0)|^2]f\,, \quad (3.16)$$

where $f_1{}^{(n\Sigma^-)}(0)\approx 1$, $h_1{}^{(n\Sigma^-)}(0)\approx(1-2\alpha)C_A$, and f is the phase-space factor (essentially the same for $\Sigma^\pm$ decays). Thus

$$\Gamma(\Sigma^+\to n\bar{l}\nu)/\Gamma(\Sigma^-\to nl\bar{\nu})\approx G_\mu{}^2\tilde{a}_0{}^2\,, \quad (3.17)$$

where

$$\tilde{a}_0=[\tfrac{1}{3}|\tilde{f}_1(0)|^2+|\tilde{h}_1(0)|^2]^{1/2} \quad (3.18)$$

has the dimensions of mass squared. Now consider the effect of the neglected constants in (3.13). $\tilde{f}_1'(0)$ and $\tilde{h}_1'(0)$ come with factors of Q^2. The constants $\tilde{f}_2, \ldots, \tilde{h}_3$ come with factors of Q^μ in the matrix element. After squaring and summing over spins, these are converted into factors of Q^2 or of M_n^2 or M_Σ^2. For the small momentum transfer involved in decay processes, we expect the terms proportional to Q^2 to be negligible compared to (3.18). If they are in fact comparable to (3.18) for small Q^2, then they will make a very large contribution to the $\Delta S=-\Delta Q$ reaction $\nu_l+n\to l+\Sigma^+$ for momentum transfers of several BeV/c. In the absence of experimental evidence for this reaction, we neglect the Q^2-dependent terms in the decay process. The constant terms may be lumped into the constant $\tilde{a}_0$. Thus Eq. (3.17) is unchanged while the definition of $\tilde{a}_0$ is modified. It is not important to determine the precise combination of subtraction constants (3.13) which replaces (3.18).

K_λ will also give rise to a $\Delta S=-\Delta Q$ amplitude in neutral kaon semileptonic decays and to a $\Delta S=2$ decay $\Xi^-\to nl\bar{\nu}$. In order to relate these different processes, we have to assume that the constant $\tilde{a}_0$ is essentially the same for each of these processes. This is plausible if $\tilde{a}_0$ is much greater than any of the hadron masses (squared) involved in the matrix elements $K_\lambda(\alpha)$. This is still not a very precise assumption, since $\tilde{a}_0$ was defined as some combination of the subtraction constants $\tilde{f}_1(0)$, $\tilde{h}_1(0)$, etc., and one has to make a similar specification of the relation of $\tilde{a}_0$ to the various subtraction constants which appear in the other forbidden decay matrix elements. For an order-of-magnitude estimate we will assume that all form factors of dimension mass squared in the decomposition of $K_\lambda(n\Sigma^+)$, $K_\lambda(\pi^-K^0)$, and $K_\lambda(\bar{n}\Xi^-)$ are equal to $\tilde{a}_0$. Then we obtain[11]

$$\Gamma(\Sigma^+\to nl\bar{\nu})/\Gamma(\Sigma^-\to nl\bar{\nu})\sim G_\mu^2\tilde{a}_0^2\,, \quad (3.19a)$$

$$x\sim G_\mu\tilde{a}_0\,, \quad (3.19b)$$

$$\frac{\Gamma(\Xi^-\to nl\bar{\nu})}{\Gamma(\Xi^-\to \Lambda l\bar{\nu})}\sim G_\mu^2\tilde{a}_0^2\sin^2\theta_c\frac{f(\Xi^-\to nl\bar{\nu})}{f(\Xi^-\to \Lambda l\bar{\nu})}\sim G_\mu^2\tilde{a}_0^2\,. \quad (3.19c)$$

In (3.19b), x is the ratio of the $\Delta S=-\Delta Q$ amplitude to the $\Delta S=+\Delta Q$ amplitude in $K_{e3}{}^0$ decays. The experimental situation is the following. There are a small number of possible $\Sigma^+\to nl\nu$ events reported. The experimental situation is discussed by Filthuth,[12] who quotes an upper limit of about 10% for the branching ratio relative to the $\Delta S=+\Delta Q$ decay $\Sigma^-\to nl\bar{\nu}$. This upper limit, substituted into (3.19a), determines $G_\mu\tilde{a}_0\lesssim 0.32$ or $\tilde{a}_0\lesssim(175m_p)^2$. A branching ratio a factor of 25 below this current upper limit would still correspond to $\tilde{a}_0\lesssim(35m_p)^2\gg m_\Xi{}^2$. This would also be consistent with the current experimental upper limit for x, $x\lesssim 0.06$ (when x is constrained to be real). There are not yet enough Ξ events for the branching ratio (3.19c) to be looked for. We note that the order-of-magnitude relation implied by (3.19a) and (3.19b) would probably also hold in any theory which introduced a $\Delta S=-\Delta Q$ current with a small coefficient to explain a nonzero value of either as a first-order effect. However, such a theory would not predict any $\Xi^-\to nl\bar{\nu}$ decays (although it would predict *nonleptonic* $\Delta S=2$ decays).

Now take up allowed semileptonic decays. We have

$$M(\alpha\to\beta\bar{l}\nu)=\frac{G_0}{\sqrt{2}}\left\{\langle\beta|J^\sigma(0)|\alpha\rangle\left[g_{\sigma\lambda}\left[1+\frac{G_0}{\sqrt{2}}(a_0+a_1q^2)\right]+q_0q_\lambda\frac{G_0}{\sqrt{2}}b_1\right]+\frac{G_0}{\sqrt{2}}K\lambda(\alpha\to\beta)\right\}\bar{u}(k)\Gamma^\lambda v(p)\,, \quad (3.20)$$

$$q=-p-k=P_{(\beta)}-P_{(\alpha)}\,.$$

We make some general observations. First, the constant a_0 can be absorbed in a redefinition of the coupling constant, so it contributes only a universal coupling-constant renormalization, the same as in μ decay. Second, the a_1q^2 term will be most important for those decays with the largest Q values. Third, the contribution from the b_1 term will be proportional to the lepton mass, hence negligible when the lepton is an electron. Fourth, after decomposition into form factors, as in (3.12), $K_\lambda(\alpha\to\beta)$ will contribute one or more subtraction constants, as in (3.13). These will contribute a nonuniversal coupling-constant renormalization.

For the two-body decays $\pi_{l\nu}$ and $K_{l\nu}$, the only effect of the second-order weak contribution is a renormalization of the products $\cos\theta_cF_\pi$ and $\sin\theta_cF_K$:

$$(\cos\theta_cF_\pi)\approx(\cos\theta_cF_\pi)_{\rm obs}\{1-(G_\mu/\sqrt{2})\times[\tilde{a}_0(\pi\to 0)+a_1m_\pi{}^2+b_1m_\pi{}^2]\}\,, \quad (3.21)$$

$$(\sin\theta_cF_K)\approx(\sin\theta_cF_K)_{\rm obs}\{1-(G_\mu/\sqrt{2})\times[\tilde{a}_0(K\to 0)+a_1m_K{}^2+b_1m_K{}^2]\}\,. \quad (3.22)$$

The $\pi_{e\nu}/\pi_{\mu\nu}$ and $K_{e\nu}/K_{\mu\nu}$ branching ratios are unaffected.

If the corrections in (3.21) and (3.22) are to be small enough to be consistent with the perturbation approximation (and the surprisingly good fit of the one-angle Cabibbo theory to all the semileptonic data), then the a_1 and b_1 terms in neutron and nuclear β decay are completely negligible because $q_{\rm max}$ is only a few MeV and the lepton mass is the electron mass. Taking this observation into account, the matrix element for free

[11] Since the kinematical structure of $K_\lambda(\bar{n}\Xi^-)$ is the same as that of $K_\lambda(n\bar{\Sigma}^-)$, Eq. (3.12), the relation implied by (3.19a) and (3.19c) is probably more accurate than an order of magnitude, if $\Sigma^+\to nl\nu$ does indeed go through second-order weak at a rate not too much smaller than the present experimental upper limit.

[12] H. Filthuth, in Topical Conference on Weak Interactions, CERN, 1969 (unpublished).

neutron decay is

$$M(n\to pe\bar{\nu})\approx\frac{G_0}{\sqrt{2}}\left[\langle p|J_\rho{}^\dagger(0)|n\rangle g^{\rho\lambda}\left(1+\frac{G_0}{\sqrt{2}}a_0\right)+\frac{G_0}{\sqrt{2}}K^\lambda(n\to p)\right]\bar{u}(p)\Gamma_\lambda v(k)\,,\quad(3.23)$$

with

$$\langle p|J_\rho{}^\dagger(0)|n\rangle\approx\cos\theta_c\bar{u}_p(P')[\gamma_\rho f_1(0)+\gamma_5\gamma_\rho h_1(0)]u_n(P)\,,\quad(3.24)$$

$$K^\lambda(n\to p)\approx\cos\theta_c\bar{u}_p(P')[\gamma^\lambda\tilde{f}_1(0)+\gamma_5\gamma^\lambda\tilde{h}_1(0)]u_n(P)\,.\quad(3.25)$$

In the nonrelativistic limit appropriate for neutron decay (we have already neglected form factors which come with kinematic factors proportional to q^λ), these expressions reduce to $[f_1(0)=1,\ h_1(0)=C_A]$

$$\begin{aligned}M(n\to pe\bar{\nu})&\approx\frac{G_0}{\sqrt{2}}\cos\theta_c\,2m_p\left[\left\{1+\frac{G_0}{\sqrt{2}}[a_0+\tilde{f}_1(0)]\right\}(\chi_p{}^\dagger\chi_n)\delta_{\lambda0}-\left(C_A+\frac{G_0}{\sqrt{2}}[C_Aa_0+\tilde{h}_1(0)]\right)(\chi_p{}^\dagger\sigma^j\chi_n)\delta_{\lambda j}\right]\bar{u}(p)\Gamma_\lambda v(k)\\&\approx\frac{G_\mu}{\sqrt{2}}\cos\theta_c\,2m_p\left\{\left[1+\frac{G_\mu}{\sqrt{2}}\tilde{f}_1(0)\right](\chi_p{}^\dagger\chi_n)\delta_{\lambda0}-\left[C_A+\frac{G_\mu}{\sqrt{2}}\tilde{h}_1(0)\right](\chi_p{}^\dagger\sigma^j\chi_n)\delta_{\lambda j}\right\}\bar{u}(p)\Gamma_\lambda v(k)\,.\quad(3.26)\end{aligned}$$

We have used the renormalization condition (2.22) and neglected the terms $0.3a_1m^2$ and $b_1m_e{}^2$. Then, by a standard calculation for a polarized neutron, we have

$$d\Gamma=\text{const}\,\xi\left[1+a\frac{\mathbf{p}}{\epsilon}\cdot\hat{k}+\langle\boldsymbol{\sigma}\rangle_n\cdot\left(A\frac{\mathbf{p}}{\epsilon}+B\hat{k}+D\frac{\mathbf{p}}{\epsilon}\times\hat{k}\right)\right].\quad(3.27)$$

Experiment is consistent with $D=0$; hence C_A, $f_1(0)$, and $h_1(0)$ are real. Then

$$\xi/\cos^2\theta_c=\tilde{C}_V{}^2+3\tilde{C}_A{}^2=1+3C_A{}^2+\sqrt{2}G_\mu[\tilde{f}_1(0)+3C_A\tilde{h}_1(0)]\,,\quad(3.28\text{a})$$

$$\xi a/\cos^2\theta_c=\tilde{C}_V{}^2-\tilde{C}_A{}^2=1-C_A{}^2+\sqrt{2}G_\mu[\tilde{f}_1(0)-C_A\tilde{h}_1(0)]\,,\quad(3.28\text{b})$$

$$\xi A/\cos^2\theta_c=2\tilde{C}_V\tilde{C}_A-2\tilde{C}_A{}^2=2C_A(1-C_A)+\sqrt{2}G_\mu[\tilde{f}_1(0)C_A+\tilde{h}_1(0)-2C_A\tilde{h}_1(0)],\quad(3.28\text{c})$$

$$\xi B/\cos^2\theta_c=2\tilde{C}_V\tilde{C}_A+2\tilde{C}_A{}^2=2C_A(1+C_A)+\sqrt{2}G_\mu[\tilde{f}_1(0)C_A+\tilde{h}_1(0)+2C_A\tilde{h}_1(0)].\quad(3.28\text{d})$$

The neutron lifetime is given by

$$\frac{1}{\tau(n)}=\frac{G_\mu{}^2\xi}{2\pi^3}m_e{}^5f\,,\quad(3.29)$$

where f is the (Coulomb-corrected) phase-space factor.

For the $0^+\to0^+$ β-decay $O^{14}\to N^{14*}$, the spin-dependent terms in (3.26) are missing, so after coupling-constant renormalization only one subtraction constant remains, and

$$\frac{1}{f\tau(O^{14})}=\frac{G_\mu{}^2\cos^2\theta_c}{2\pi^3}m_e{}^5[1+O(\alpha)+\sqrt{2}G_\mu\tilde{a}_0(O^{14}\to N^{14*})]\,,\quad(3.30)$$

where $O(\alpha)$ stands for electromagnetic radiative corrections. Here we see explicitly how the second-order weak interaction, as well as electromagnetic radiative corrections, destroy the universality which follows from the conservation of the vector current to all orders in the strong interactions. Since $\tilde{a}_0(O^{14}\to N^{14*})$ cannot be computed (perturbatively), the O^{14} decay does not determine $\cos\theta_c$ if the second-order weak contribution is not negligible. So we return to the decay of the polarized neutron. Of Eqs. (3.28) only three are independent, i.e., we can determine only the combinations $\tilde{C}_V{}^2$, $\tilde{C}_A{}^2$, and $\tilde{C}_V\tilde{C}_A$. The equations are consistent if $B-A=1-a$, which is satisfied within the experimental errors. Thus we cannot determine four unknowns, $\cos\theta_c$, C_A, $\tilde{f}_1(0)$, and $\tilde{h}_1(0)$ [ξ is determined from the lifetime (3.29)]. Of these parameters, C_A is calculated theoretically by Adler[13] and by Weisberger.[14] This calculation is believed to be reliable to within about 10%. To this accuracy one can eliminate $\tilde{f}_1(0)$ and $\tilde{h}_1(0)$ from Eqs. (3.28) and determine $\cos\theta_c$ when sufficiently accurate experimental values of the correlation parameters a, A, and B are available.

Now consider the K_{l3} decays. These decays have a relatively large momentum transfer, a final muon (for $K_{\mu3}$), and a large number of experimental events. The matrix element for $K_{l3}{}^+$ is

$$\begin{aligned}M(K^+\to\pi^0\bar{l}\nu)&=\frac{G_0}{\sqrt{2}}\sin\theta_c\left\{[(P+P')^\sigma f_+(q^2)+(P-P')^\sigma f_-(q^2)]\times\left[g_{\sigma\lambda}\left(1+\frac{G_0}{\sqrt{2}}(a_0+a_1q^2)\right)+q_\sigma q_\lambda\frac{G_0}{\sqrt{2}}b_1\right]\right.\\&\left.+\frac{G_0}{\sqrt{2}}[(P+P')_\lambda\tilde{f}_+(0)+(P-P')_\lambda\tilde{f}_-(0)]\right\}\bar{u}(k)\Gamma^\lambda v(p)\,,\\q&=P'-P=-(p+k)\,.\end{aligned}\quad(3.31)$$

The subtraction constants a_0, $\tilde{f}_+(0)$, and $\tilde{f}_-(0)$ effectively renormalize the coupling constant $(G_0\sin\theta_c)$; the term a_1q^2 simulates a form-factor variation; and the term with b_1 is an effective scalar interaction. We can put an upper bound on b_1 from the experimental[15]

[13] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965).
[14] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965).
[15] C. Rubbia, in Topical Conference on Weak Interactions, CERN, 1969 (unpublished).

upper bound on the scalar coupling in K_{e3} decay. We find

$$|G_\mu b_1 m_e^2| < 6\times 10^{-4}\,. \qquad (3.32)$$

This limit is not small enough to guarantee the perturbation-theoretic requirement for, e.g., (3.22), but it is an experimental (as opposed to hypothetical) limit[16] which does justify neglect of the b_1 contributions to the parameters of μ decay and β decay. After a number of detailed calculations with the matrix element (3.31), we find that the major contributions of the second-order weak terms to the observable properties of the $K_{\mu 3}$ decays can be summarized by the following replacements in the conventional (first-order weak) parametrization:

$$\lambda_+ \to \lambda_+^{\text{eff}} = \lambda_+ + (G_\mu/\sqrt{2}) a_1 m_\pi^2\,, \qquad (3.33)$$

$$\xi \to \xi^{\text{eff}} = \xi + (G_\mu/\sqrt{2})[\xi(\tilde{a}_- - \tilde{a}_+) + b_1(m_K^2 - m_\pi^2)]\,. \qquad (3.34)$$

Here λ_+ is the parameter which occurs in the standard linear approximation for the form factor in the restricted range of q^2 available in the decay

$$f_+(q^2) = (1+\lambda_+ q^2/m_\pi^2) f_+(0) \qquad (3.35)$$

and

$$\xi = f_-(0)/f_+(0)\,. \qquad (3.36)$$

λ_+^{eff} is fairly well determined experimentally[15] to be small,

$$\lambda_+^{\text{eff}} = 0.029 \pm 0.010. \qquad (3.37)$$

Since λ_+ is expected theoretically to be small, (3.33) and (3.37) can be converted into an order-of-magnitude upper bound on a_1; in particular,

$$|0.3 G_\mu a_1 m_\mu^2| < 0.01\,. \qquad (3.38)$$

Since the parametrization is not essentially different (only the significance of the parameters is changed), the inclusion of small second-order weak contributions to the K_{l3} decays cannot resolve the present[15] discrepancy between polarization experiments which generally find $\xi \approx -1$, and branching-ratio experiments which generally find ξ small and positive (except the X_2 collaboration[17] which found $\xi \approx -0.6$ from a branching-ratio measurement).

[16] If the experimental situation with regard to the form factor $f_-(q^2)$ becomes clear, a much smaller bound on b_1 can be obtained from $K_{\mu 3}$ decays.

Included in the semileptonic processes are the $\Delta Q = 0$ decays, none of which have been seen so far. In the absence of neutral currents, these processes should be second-order weak. For the decay $\alpha \to \beta + l + \bar{l}$, we have, in place of (3.10) or (3.20),

$$M(\alpha \to \beta l \bar{l}) = \tfrac{1}{2} G_\mu^2 i \int \frac{d^4 q}{(2\pi)^4} M_{\lambda\sigma}(q, \ldots) \bar{u}(p) \Gamma^\sigma \times \frac{1}{\gamma(q-p')} \Gamma^\lambda v(p')\,, \qquad (3.39)$$

$$M_{\lambda\sigma}(q, \ldots) = i \int d^4x\, e^{iqx} \langle \beta | T^*(J_\lambda(x) J_\sigma^\dagger(0)) | \alpha \rangle_c\,. \qquad (3.40)$$

If the large-q behavior of $M_{\lambda\sigma}(q, \ldots)$ is determined by the Bjorken[6] limit for the corresponding T product, then the integral in (3.39) is quadratically divergent and the coefficient of the quadratic cutoff is determined by the equal-time commutator of J_λ with $J_\sigma^\dagger$. In this way, Ioffe and Shabalin[4] and Mohapatra, Rao, and Marshak[5] have estimated $\Lambda < 25$ BeV from the experimental upper limit for the $K_L^0 \to \mu\bar{\mu}$ branching ratio. According to the discussion following (2.11), we would replace the Λ^2 appearing in the $K_L^0 \to \mu\bar{\mu}$ calculation by

$$\Lambda^2 \to 4\pi^2 (C_0 + C_1 m_K^2)\,.$$

Unfortunately, we do not have any information about these subtraction constants from the preceding work. In particular, they may not bear any relations to the constants $\tilde{a}_0$, which come from the matrix element of a truncated product of three hadronic currents. Thus the observed small upper limit for $K_L^0 \to \mu\bar{\mu}$ does not necessarily imply correspondingly small upper limits for the $\Delta S = -\Delta Q$ and $\Delta S = 2$ processes discussed above.

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[17] T. Eichten *et al.*, Phys. Letters **27B**, 586 (1968).