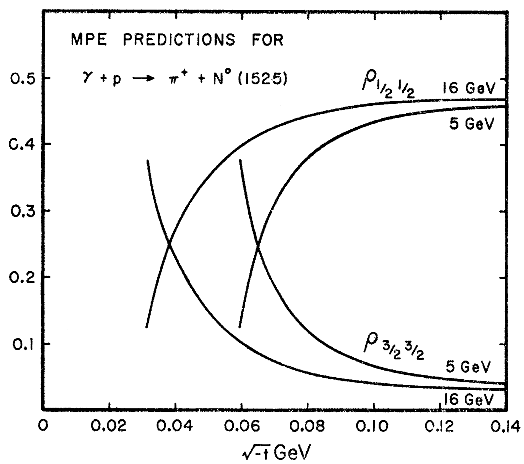

FIG. 3. MPE predictions for the spin-density matrix of the outgoing $N(1520)$.

gation has been made in the energy region of the resonance[7] to allow us to draw two qualitative conclusions. The cross section for the reaction $\gamma N \to \pi N(1520)$ is appreciably smaller than for $\gamma N \to \pi\Delta$, and there is no evidence for an anomalous energy behavior for this reaction. These features are both consistent with MPE predictions.

The predictions of the MPE amplitude for $d\sigma/dt$ in the near forward direction for unpolarized and polarized photons are given in Figs. 1 and 2. Although we anticipate accurate predictions from the MPE only for $|t|<\mu^2$, we have continued the curves to larger values of $|t|$ for completeness. Experience with $\gamma N \to \pi^{\pm} N$ and $\gamma N \to \pi^{\pm}\Delta$ suggests[1,2] that phenomenological fits for $d\sigma/dt$ could be extended to larger values of $|t|$ by the inclusion of the factor e^{3t}. The features can be readily compared with the results for $\gamma N \to \pi\Delta$ and explained by inspection of Eq. (7). The explanation of the kinematic suppression of the cross section and enhancement of the dip-bump-dip structure were both discussed in Sec. II.

The absence of evidence for $d\sigma/dt$ exhibiting a constant behavior with s rather than s^{-2} is consistent with the neglibility of the contribution from the B_9 amplitude appearing in the electric Born terms.

Experimental results for upper limits on the cross section, measurements of $d\sigma/dt$ and comparisons of experimental s-channel spin density matrix elements $\rho_{\lambda\lambda'}$ for the outgoing $N(1520)$ with the MPE predictions in Fig. 3 would provide confirmation on an ascending scale for the strength of the restrictions on the reaction required by low-t theorems.

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[7] B. Richter (private communication).

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Nonlinear Regge Trajectories*

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Various assumptions for the mass dependence of Regge trajectories are compared with the data for the presumed trajectories. We also discuss the possible observation of the asymptotic behavior of these trajectories with data which will be available in the near future.

THERE has been much interest in dynamical theories which incorporate infinitely rising Regge trajectories. The current experimental data seem to suggest an indefinite rise though Mandelstam analyticity requires that the slope of the trajectories rise no faster than the particle mass m as $m^2 \to \infty$.[1] Consistent with this requirement is the prediction[2] of a recent model that the asymptotic behavior under the condition of infinitely many coupled channels is as $(Z \ln Z)^{1/2}$, where $Z = mr_0$ and r_0 is the core radius of a strongly nonlocal interaction.

Keleman *et al.*[3] have questioned the linearity of Regge trajectories in a recent Letter (hereafter referred to as KLP). There the three-parameter formulas

$$m = A - B/(C+J)^2 \tag{1}$$

and

$$m = A - B/(C+J) \tag{2}$$

were compared to

$$m^2 = A + BJ\,, \tag{3}$$

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TABLE I. χ^2 of Eqs. (1)–(4) for the trajectories N_α, Δ, ρ, Λ, and Σ. χ^2 is determined using the same experimental data as Ref. 1, except that for the ρ trajectory the lowest mass members of the A_2 and R multiplets are used. The numbers in parentheses in the last line of the table give the results for the case where the spin-$\frac{1}{2}$ resonance is included in the Σ trajectory. $M_{\rm nl}$ is the energy at which the nonlinearity of Eq. (4) should be observed distinctly, for the parameters given in the previous three columns.

Particle (spin)	Mass (MeV)	χ^2 Eq. (3)	χ^2 Eq. (1)	χ^2 Eq. (2)	χ^2 Eq. (4)	10^6C	A (10^{-6} MeV^{-2})	B	$M_{\rm nl}$ (BeV)
N (3/2)	1515	16	2.9	2.4	16	50.3	0.86	−0.52	76.4
N (7/2)	2190				20	0.4	0.88	−0.55	7.3
N (11/2)	2650				28	0.2	0.89	−0.54	4.8
N (15/2)	3030								
Δ (3/2)	1236	5.3	19	17	5.4	32.0	0.91	0.12	59.4
Δ (7/2)	1940				5.1	8.0	0.91	0.12	29.7
Δ (11/2)	2420				6.6	2.0	0.90	0.12	14.9
Δ (15/2)	2850								
Δ (19/2)	3230								
ρ (1)	765	2.9	5.1	3.1	2.9	10.0	0.96	0.45	32.3
A_2 (2)	1269				3.2	1.0	0.96	0.45	10.3
R (3)	1630				5.2	0.2	0.96	0.46	5.1
S (4)	1929								
T (5)	2195								
U (6)	2384								
Λ (1/2)	1115.6	15	36	34	15	5.2	0.95	−0.68	23.4
Λ (3/2)	1518.8				14	1.0	0.95	−0.68	10.2
Λ (5/2)	1815				17	0.5	0.95	−0.68	6.9
Λ (7/2)	2100								
Λ (9/2)	2350								
Σ (3/2)	1382	44	1.2	1.0	44	10.0	0.90	−0.22	33.4
Σ (5/2)	1765	(151)	(8.6)	(7.0)	(91)	(10.0)	(0.90)	−(0.22)	(33.4)
Σ (7/2)	2030								
Σ (9/2)	2250								
Σ (11/2)	2455								

where J is the particle spin. It was shown that the present experimental evidence is indecisive in choosing between a form for $\mathrm{Re}\alpha(m^2)$ which represents a straight line and one which predicts a maximum mass [represented by the parameter A in Eqs. (1) and (2) for each trajectory].

However, the experimental evidence for such a maximum mass appears to be nonexistent. Moreover, when the trajectories with more than four known particles are considered, the forms for $\mathrm{Re}\alpha(m^2)$ which include a decreasing slope seem to fit the data as well as those which have an increasing slope with increasing mass. This suggests that the nearly constant slope for the trajectories in the experimental region is not inconsistent with a decreasing slope at higher energies.

These considerations lead us to suggest the formula

$$J=B+\left[\frac{1}{Am^2}+\frac{1}{C+(Z\ln Z)^{1/2}}\right]^{-1}, \qquad (4)$$

where A, B, and C are adjustable parameters. This formula is very nearly linear in the experimental region and has the asymptotic behavior suggested by Ref. 2. The interaction radius r_0 has been chosen to be one-half the pion Compton wavelength. The parameter C adjusts the position of the transition between the m^2 behavior of the experimental region and the slower asymptotic rise. For each trajectory the values of A and B do not differ much from the corresponding parameters in the straight-line formula. A and B are chosen to minimize χ^2 for various values of C and the results are summarized in Table I where a comparison with the results of KLP is also made.

Forthcoming data should be able to distinguish easily between the formulas reviewed here. The NAL accelerator will generate 10–20 BeV in the center-of-mass system for protons on protons, and this should be helpful in establishing the constancy or nonconstancy of the slope.

The last column of Table I gives $M_{\rm nl}$, the value of the particle mass at which the right-hand side of Eq. (4) becomes 10% nonlinear. For each trajectory considered (excepting the Σ), a sufficiently low value of C brings $M_{\rm nl}$ within the testable range without affecting χ^2 drastically. Thus the asymptotic behavior predicted by Eq. (4) may begin to be observed at these energies. It should further be noted that the asymptotic masses of KLP all lie below 10 BeV and so Eqs. (2) and (3) will be tested directly.