outgoing particles do not have "wee momenta" [i.e., $p=O(1\text{ GeV})$] in the c.m. system, but have momenta $\xi\sqrt{s}$ with $0<\epsilon\leqslant\xi<1$, then one has an average multiplicity $\bar{n}$ proportional to $\sim\ln\{\epsilon+[\epsilon^2+(p_\perp^2+m^2)/s]^{1/2}\}^{-1}$. By suitable choice of the parameters, $\bar{n}$ might be proportional to $\ln s$ when, say, $10<\sqrt{s}<10^5$ GeV and become constant when $\sqrt{s}\gg10^5$ GeV. This serves to illustrate the point that if one requires the integrals (1) and (2) to be divergent, then the original simplicity of the concept of limiting fragmentation will no longer hold.

Nevertheless, we note that the results (1) and (2) are by no means a serious objection to the BCYY framework in the sense that a large but finite multiplicity as $s\to\infty$ has not yet been excluded experimentally and that the BCYY framework could easily have a divergent multiplicity simply by assuming the presence of the pionization process at extreme energies.[2] At the present time, most people apparently believe that all multiplicities will become infinite[5] at infinite incident energies.

In conclusion, we would like to emphasize that the hypothesis of limiting distributions in a particular reference system is a sort of dynamical assumption in the sense that the "rate" of divergence of the average multiplicity is fixed. On the other hand, the number of kaons and pions has been measured in cosmic-ray experiments and we find, for example, in NN collisions at an energy $\sim10^3$ GeV in the laboratory the ratio K/π is quite different for the set of high-energy outgoing mesons ($K/\pi\approx0.1$) and the set of low-energy mesons ($K/\pi\approx0.7$).[6] This may not be easy to accommodate within the intuitive picture of fragmentation without pionization.

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[5] If the average multiplicity $\bar{n}$ is finite, then a dynamical explanation is needed of why it should be finite and what its upper bound should be. A finite $\bar{n}$ is unlikely from the view point of hadron democracy; and it is also apparently unlikely in the quark model or in the "parton" model of Feynman.

[6] See M. Koshiba, in *High Energy Collisions*, Ref. 4.

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Forward Photoproduction of a Charged Pion and $N(1520)$*

Robert Beck Clark

Center for Particle Theory, Department of Physics, University of Texas, Austin, Texas 78712

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The analysis of the reaction $\gamma p\to\pi^+N(1520)$ is considered and the influence of low-t theorems in the near forward direction is discussed. It is shown that present preliminary experimental results are not inconsistent with the dominance of the amplitudes whose forms near $t\approx\mu^2$ are determined by low-t theorems. Predictions are presented for $d\sigma/dt$ and the spin-density matrix $\rho_{\lambda\lambda'}$.

I. INTRODUCTION

THE success of the description of the high-energy photoproduction reactions $\gamma N\to\pi^\pm N$ and $\gamma N\to\pi^\pm\Delta$ in the near-forward direction by parts of the electric Born terms[1–3] suggests an investigation of the generality of the effect in other charged-pion photoproduction reactions. The most accessible of these is probably the reaction involving the "second nucleon resonance" $\gamma N\to\pi^\pm N(1520)$. In a recent paper by Campbell, Clark, and Horn,[3] hereafter called CCH, the success of the electric Born terms has been explained in part by low-t theorems which place reaction-dependent restrictions on the t dependence of the reaction in the near forward direction. In this note, the results obtained for $\gamma N\to\pi^\pm\Delta$ are extended to $\gamma N\to\pi^\pm N(1520)$.

II. KINEMATICAL STRUCTURE AND LOW-t THEOREMS

The kinematical details for $\gamma N\to\pi N(1520,\,J^P=\frac{3}{2}^-)$ may be generalized from those given for $\gamma N\to\pi\Delta(1238,\,J^P=\frac{3}{2}^+)$ in CCH by the appropriate inclusion of the factor $i\gamma_5$ in Eq. (7) of CCH. A set of amplitudes (B_i) is thereby defined and the application of the gauge-invariance condition yields low-t theorems for certain amplitudes which are identical in form to Eq. (11a) of CCH,

$$\lim_{t\to\mu^2}[B_4(s,t)k\cdot P+B_5(s,t)]=-\lim_{t\to\mu^2}B_2(s,t)k\cdot P=-efF_\pi. \quad (1)$$

* Supported in part by the National Science Foundation.

[1] For $\gamma N\to\pi N$, see B. Richter, in *Proceedings of the Third International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967* (Clearing House of Federal Scientific and Technical Information, Washington, D. C., 1968), p. 313; A. M. Boyarski, F. Bulos, W. Busza, R. Diebold, S. D. Eckland, G. E. Fischer, J. R. Rees, and B. Richter, Phys. Rev. Letters **20**, 300 (1968).

[2] For $\gamma N\to\pi\Delta$, see P. Stichel and M. Scholz, Nuovo Cimento **34**, 1381 (1965); A. M. Boyarski, R. Diebold, S. D. Ecklund, G. E. Fischer, Y. Murata, B. Richter, and W. S. C. Williams, Phys. Rev. Letters **22**, 148 (1969).

[3] J. A. Campbell, Robert Beck Clark, and D. Horn, Phys. Rev. D **2**, 217 (1970).

The s dependence of $B_2(s,t)$ is therefore determined exactly at $t=\mu^2$ by the expression

$$B_2(s,\mu^2)=+2efF_\pi/(s-M_i^2)\,. \tag{2}$$

The philosophy of the application of the low-t theorems is then to determine the importance of the role played by the determined amplitudes in the region of t corresponding to the near forward direction.

The complete expression for $d\sigma/dt$ in terms of the (B_i), which was obtained using Hearn's[4] REDUCE system, was used to perform the numerical calculations. An approximate form for $d\sigma/dt$ analogous to Eq. (12) in CCH and containing only the highest-order terms in s and lowest-order terms in t, should describe $d\sigma/dt$ for high energies in the near forward direction and is given by

$$\frac{d\sigma}{dt}\approx\frac{1}{192\pi}\left\{(M_i-M_f)^2\frac{2\mu^4-t(M_f^2-M_i^2)^2/M_f^2}{(t-\mu^2)^2}|B_2(s)|^2\right.$$
$$+2\sum_{i=4}^{10}s^{n(i)}\alpha_{2i}\frac{(\beta_{2i}-t)}{t-\mu^2}\operatorname{Re}[B_2(s)B_i^*(s)]$$
$$\left.+2\sum_{j=4}^{10}\sum_{i=4}^{10}s^{[n(i)+n(j)]}\alpha_{ij}\operatorname{Re}[B_i(s)B_j^*(s)]\right\}, \tag{3}$$

where 5 is excluded from the sums and the α's and β's are simple functions of the external masses. The values of the powers are $n(i)=0$ for $i=4$ and 8 and $n(i)=1$ for $i=6$, 7, 9, and 10.

The general form of $d\sigma/dt$ for $\gamma N\to\pi^\pm N(1520)$ is found to differ from the corresponding expression for $\gamma N\to\pi^\pm\Delta$ by the replacement $M_i\to -M_i$ and a change in the sign of amplitudes B_6, B_7, B_8, B_{11}, and B_{12}.[5] If the B_2 amplitude were to provide the dominant contribution for $|t|<\mu^2$, as seems to be the case in the $\gamma N\to\pi\Delta$ reaction, we would expect the following qualitative behavior when $\gamma N\to\pi N(1520)$ is compared with $\gamma N\to\pi\Delta$. The over-all cross section for $\gamma N\to\pi N(1520)$ would be suppressed due to the effect of the $(M_f-M_i)^2$ factor and the characteristic dip-bump-dip structure of $d\sigma/dt$ should appear and may even be accentuated by the difference in the value for M_f in the coefficient of t.

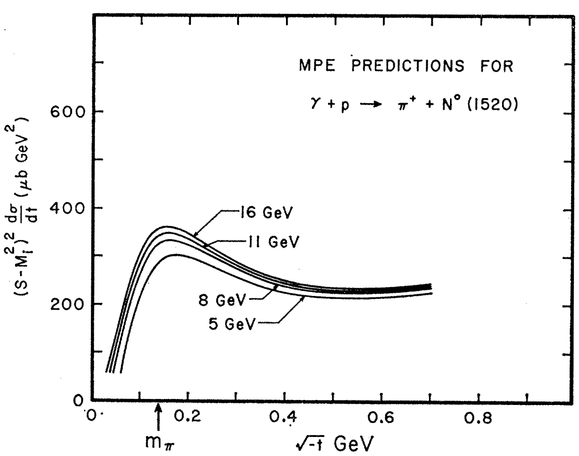

FIG. 1. MPE predictions for $d\sigma/dt$ at small momentum transfer for unpolarized photons.

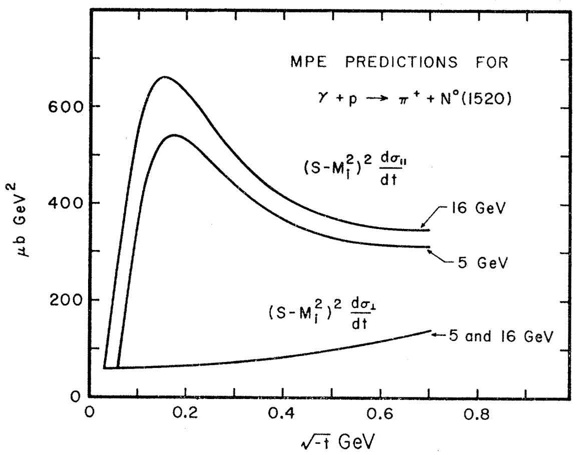

FIG. 2. MPE predictions for incident photons polarized perpendicular and parallel to the plane of production.

III. MPE PREDICTIONS FOR $\gamma p\to\pi^+N^0(1520)$

The electric Born term and MPE (minimal gauge-invariant extension of pion exchange[3]) forms of the amplitudes are equivalent to those in Eqs. (14) and (16) of CCH. The MPE amplitude, we recall, is defined as that part of the electric Born terms which is necessary to complete the pion exchange to a gauge-invariant expression and is given by

$$\begin{aligned}B_2^{\mathrm{MPE}}(s,t)&=2ef[F_i/(s-M_i^2)+F_f/(u-M_f^2)]\,,\\ B_5^{\mathrm{MPE}}(s,t)&=-efF_\pi\,.\end{aligned} \tag{4}$$

If we determine f from the average of the experimental partial decay widths,[6] $\Gamma\approx 65$ MeV, for $N^0(1520)\to p+\pi$, then MPE makes the general prediction that for $\gamma p\to\pi^+N^0(1520)$ in the forward direction,

$$(s-M_i^2)^2\left.\frac{d\sigma}{dt}\right|_{t_{\min};\,\mathrm{MPE}}=\frac{(M_i-M_f)^2e^2f^2}{24\pi} \tag{5}$$

is about 60 μb GeV^2.

Detailed experimental results for the process $\gamma N\to\pi N(1520)$ are presently unavailable. Sufficient investi-

[4] A. C. Hearn, in *Interactive Systems for Experimental Applied Mathematics*, edited by M. Klerer and J. Reinfelds (Academic, New York, 1968), pp. 79–90.

[5] This result can be obtained directly by moving the γ_5 to the right of the covariants $I_i^{\nu\mu}$ in Eq. (7) of CCH and utilizing the property that if we define $U(p_i,\lambda)=\eta\gamma_5 u(p_i,\lambda)$, where η is an inessential phase factor, and use $(\not p_i-M_i)u(p_i,\lambda)=0$, then $(\not p_i+M_i)U(p_i,\lambda)=0$, so that the sum over the initial nucleon polarizations gives $\Sigma U(p_i,\lambda)\bar U(p_i,\lambda)=(\not p_i-M_i)/2M_i$. The author is indebted to John T. Donohue for this observation.

[6] Particle Data Group, Rev. Mod. Phys. **42**, 78 (1970).

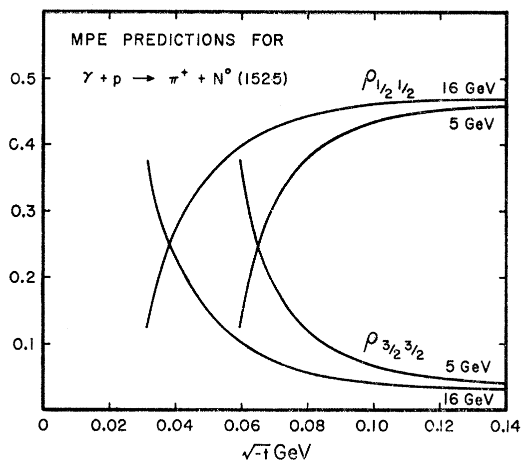

FIG. 3. MPE predictions for the spin-density matrix of the outgoing $N(1520)$.

gation has been made in the energy region of the resonance[7] to allow us to draw two qualitative conclusions. The cross section for the reaction $\gamma N \rightarrow \pi N(1520)$ is appreciably smaller than for $\gamma N \rightarrow \pi\Delta$, and there is no evidence for an anomalous energy behavior for this reaction. These features are both consistent with MPE predictions.

The predictions of the MPE amplitude for $d\sigma/dt$ in the near forward direction for unpolarized and polarized photons are given in Figs. 1 and 2. Although we anticipate accurate predictions from the MPE only for $|t| < \mu^2$, we have continued the curves to larger values of $|t|$ for completeness. Experience with $\gamma N \rightarrow \pi^\pm N$ and $\gamma N \rightarrow \pi^\pm \Delta$ suggests[1,2] that phenomenological fits for $d\sigma/dt$ could be extended to larger values of $|t|$ by the inclusion of the factor e^{3t}. The features can be readily compared with the results for $\gamma N \rightarrow \pi\Delta$ and explained by inspection of Eq. (7). The explanation of the kinematic suppression of the cross section and enhancement of the dip-bump-dip structure were both discussed in Sec. II.

The absence of evidence for $d\sigma/dt$ exhibiting a constant behavior with s rather than s^{-2} is consistent with the neglibility of the contribution from the B_9 amplitude appearing in the electric Born terms.

Experimental results for upper limits on the cross section, measurements of $d\sigma/dt$ and comparisons of experimental s-channel spin density matrix elements $\rho_{\lambda\lambda'}$ for the outgoing $N(1520)$ with the MPE predictions in Fig. 3 would provide confirmation on an ascending scale for the strength of the restrictions on the reaction required by low-t theorems.

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[7] B. Richter (private communication).

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Nonlinear Regge Trajectories*

C. T. GRANT AND EARLE L. LOMON
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 2 June 1970)

Various assumptions for the mass dependence of Regge trajectories are compared with the data for the presumed trajectories. We also discuss the possible observation of the asymptotic behavior of these trajectories with data which will be available in the near future.

THERE has been much interest in dynamical theories which incorporate infinitely rising Regge trajectories. The current experimental data seem to suggest an indefinite rise though Mandelstam analyticity requires that the slope of the trajectories rise no faster than the particle mass m as $m^2 \rightarrow \infty$.[1] Consistent with this requirement is the prediction[2] of a recent model that the asymptotic behavior under the condition of infinitely many coupled channels is as $(Z \ln Z)^{1/2}$, where $Z = mr_0$ and r_0 is the core radius of a strongly nonlocal interaction.

Keleman *et al.*[3] have questioned the linearity of Regge trajectories in a recent Letter (hereafter referred to as KLP). There the three-parameter formulas

$$m = A - B/(C+J)^2 \tag{1}$$

and

$$m = A - B/(C+J) \tag{2}$$

were compared to

$$m^2 = A + BJ\,, \tag{3}$$

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[1] R. W. Childers, Phys. Rev. Letters **21**, 868 (1968).
[2] Earle L. Lomon, Phys. Rev. D **1**, 549 (1970).
[3] P. J. Kelemen, K. Y. Lee, and W. F. Piel, Jr., Phys. Rev. Letters **23**, 998 (1969).