Comments and Addenda

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Comments on Some Recent Muon Experiments

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Several studies have appeared in the literature based on emulsion exposures to either 10.1-GeV/c μ^+ or 15.8-GeV/c μ^- at the Brookhaven AGS in 1966. The effect of discrepancies in the values of the primary momenta of both beams on their results is discussed.

A NUMBER of stacks of nuclear emulsion were exposed to the "11-" and "17-" GeV/c muon beams produced by the Columbia-Rochester-BNL collaboration at the Brookhaven National Laboratory[1] in 1966. The values quoted by a number of authors[2–6] for the momenta of both beams after additional filtration varied by as much as 1 GeV/c. The momentum spectra of both beams as produced by the Columbia-Rochester-BNL group are shown in Figs. 1 and 2. These spectra were obtained prior to the filtration used in the emulsion exposures. Filters of 61 in. of light concrete and 60 in. of heavy concrete were used in the 11-GeV/c μ^+ and 17-GeV/c μ^- exposures, respectively. Calculations[7] indicate attenuations in the filters of 0.9-GeV/c for the μ^+ and 1.2 GeV/c for the μ^- beam. The spectral distributions shown in Figs. 1 and 2 can be shifted to lower momenta without significant change in shape. The peak momenta of 11- and 17-GeV/c muons are reduced to 10.1 and 15.8 GeV/c, respectively.

Thompson *et al.*[2] used the filtered 10.1-GeV/c μ^+ beam described above and similar electron exposures made at SLAC to compare the inelastic cross sections of leptons in nuclear emulsion. They calculated the expected ratio of electron to muon total cross sections to be 3.5, but measured it to be only 2.2. The difference in incident peak momenta between their value of 10.5 GeV/c and our estimate of 10.1 GeV/c is small when compared to the spread in the momenta of the incident muons, and certainly will not remove their reported anomaly.

Jain and Wixon[3] have used both the 10.1- and 15.8-GeV/c beams to investigate the spectra of knock-on electrons produced at small angles. The kinematical relationship between the kinetic energy of the secondary

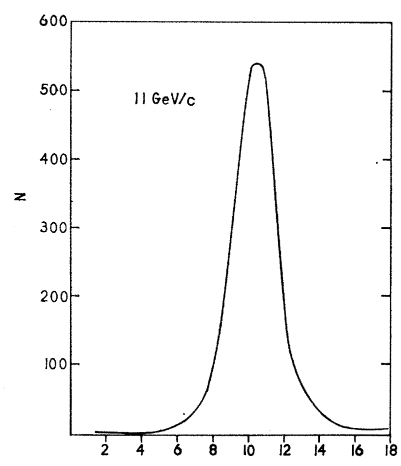

FIG. 1. Momentum spectrum of the 11-GeV/c μ^+ beam before entering the concrete filter.

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[2] R. L. Thompson, J. J. Lord, J. A. Kirk, M. E. Nelson, and R. J. Piserchio, Phys. Rev. **177**, 2022 (1969).

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[4] P. L. Jain, R. D. Malucci, and M. J. Potoczak, Phys. Rev. Letters **24**, 526 (1970).

[5] P. L. Jain, R. D. Malucci, and M. J. Potoczak, Phys. Rev. Letters **24**, 530 (1970).

[6] P. L. Jain, R. D. Malucci, and M. J. Potoczak, Nuovo Cimento Letters **3**, 684 (1970).

[7] F. R. Hickey, Ph.D. thesis, Clarkson College of Technology (unpublished).

electron and its emission angle is relatively insensitive to even a change of a GeV/c in the peak of the incident momentum spectrum. Hence, no significant change can be expected in their reported results.

Jain, Malucci, and Potoczak[4,5] have studied the inelastic interactions of muons in nuclear emulsions at both 10.1 and 15.8 GeV/c. In their calculations, they have used 14.6 instead of 15.8 GeV/c for the peak momenta of the negative muons and 10.1 GeV/c for the positive muons. The total cross sections, the angular distributions of shower particles in the c.m. frame, and the kinetic energies of the shower particles in the lab frame were presented in Ref. 4 and will not be seriously affected by correcting the incident momenta from 14.6 to 15.8 GeV/c. Further analysis in both papers, however, relies on evaluating the Lorentz invariants q^2, the square of the four-momentum transferred to the target, and W, the invariant mass of the final system not including the muon.

Each value of q^2 was calculated in the lab frame from the expression

$$q^2 = 2EE'(1-\cos\theta_L)\,, \qquad (1)$$

where E and E' are the energies of the incident and outgoing muons, respectively, and θ_L is the scattering angle of the muon. The energy of each outgoing muon E' was determined from multiple-scattering measurements of the product of its momentum and velocity.[4–6] Use of 14.6 GeV/c instead of 15.8 GeV/c for the peak incident momentum introduces a systematic underestimate of 8% in the values of q^2 calculated from Eq. (1). The effect of this correction on the shape of the q^2 distribution has already been discussed by these authors.[8]

A larger correction is introduced into the values of W, which were calculated from the invariant relationship

$$W^2 - M_p{}^2 = 2M_p(E-E') - q^2\,, \qquad (2)$$

where M_p is the proton mass. The systematic error in the first term of the right-hand side of Eq. (2), that is introduced by the use of the wrong peak momenta, would normally be considerably larger than the 8% underestimate in q^2. Even for true energy transfer, $\epsilon = E-E'$, of as large as 3 GeV, the underestimate of W due to using 14.6 GeV/c would still be 40%. This may explain the inconsistency they report between plots of average multiplicity versus q^2 for the 10.1- and 15.8-GeV/c beams [Figs. 2(a) and 2(b) of Ref. 4]. Data representing events with low and high values of W were plotted separately—designated by circles and triangles, respectively. The same value of W (1.8 GeV) was used for both beams to categorize events as low or high. They report the 10.1-GeV/c μ^+ data to be in agreement with the limiting fragmentation hypothesis of Benecke *et al.*[9] In discussing the 15.8-GeV/c μ^- data the authors point out the low ratio of events with $W \geq 1.8$ GeV to those with $W < 1.8$ GeV. The low value of this ratio can be explained by the fact that their separation at $W = 1.8$ GeV is equivalent to separating the correctly calculated values of W at 2.5 GeV. Correct separation at $W = 1.8$ GeV will most certainly result in the transfer of some events from the $W < 1.8$-GeV to the $W \geq 1.8$-GeV category. This will result in lowering the average multiplicity for most values of q^2 in Fig. 2(b) (Ref. 4), possibly yielding agreement with their 10.1-GeV/c data.

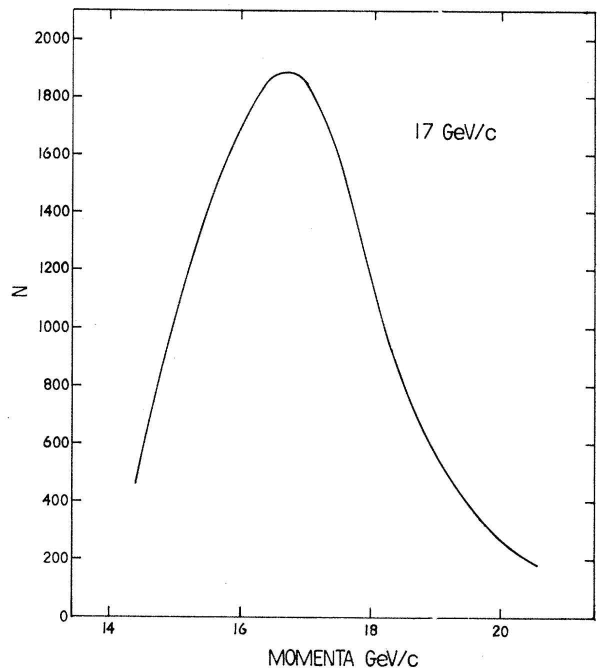

FIG. 2. Momentum spectrum of the 17-GeV/c μ^- beam before entering the concrete filter.

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