s-Channel Helicity Conservation with t-Channel Dynamics*

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The compatibility of s-channel helicity conservation and t-channel exchanges in diffractive-type processes is examined. It is found that compatibility exists only in the infinite-energy limit. This requires specification of ratios of residue or vertex functions for t-channel helicity amplitudes. Assuming these conditions are satisfied, finite-energy corrections for ρ^0 polarized photoproduction are calculated and found to be consistent with data at 2.8 and 4.7 GeV. Predicted values of spin-density matrix elements for other diffractive processes are given for the usual t-channel analysis in unpolarized experiments.

I. INTRODUCTION

TWO-BODY elastic and inelastic processes at high energy (s) are conventionally analyzed in terms of dynamics in the crossed (t) channel, i.e., Regge-pole exchange. In particular, diffractive-type processes (both elastic and inelastic) have been assumed due to the exchange of the Pomeranchukon or other trajectories with quantum numbers of the vacuum in the crossed channel. The distinguishing characteristic of these processes has been lack of energy dependence of the cross section. Recently, it has been discovered that ρ^0 production by polarized photons seems to conserve helicity in the direct (s) channel.[1] An examination of Regge-pole-model fits to πN elastic scattering reveals that Pomeranchukon exchange terms are consistent with s-channel helicity conservation for nucleons also.[2] If this is a general feature of diffractive-type processes, then the question of compatibility with t-channel dynamics arises. For t-channel exchanges, only the residues and trajectory function are adjustable, and helicity constraints are more natural to impose in the t channel. Also, factorization imposes constraints among various reaction channels with common external particles.[3] Gilman *et al.*[2] have stated that s-channel helicity conservation and t-channel Pomeranchukon exchange can be made compatible for the sets of reactions $(\pi N,\pi\pi,NN)$ and $(\pi A_1,\pi\pi,A_1A_1)$. It is the purpose here to examine the general case of a two-body reaction with arbitrary-spin particles proceeding via factorizable Pomeranchukon exchange, and see if s-channel helicity conservation can be imposed.

II. FORMALISM

Consider the s-channel reaction $A+B\to C+D$. We take as the t channel $\bar{D}+B\to C+\bar{A}$. The t-channel helicity amplitudes are

$$F_{\lambda_C\lambda_A,\lambda_D\lambda_B}=\beta_{\lambda_C\lambda_A}\beta_{\lambda_D\lambda_B}d_{\lambda_D-\lambda_B,\lambda_C-\lambda_A}{}^{\alpha}(z)\,, \tag{1}$$

where the β's are residue functions for the Pomeranchukon coupling to the external particles, d is the rotation coefficient, α is the trajectory function, and z is the cosine of the t-channel scattering angle. In the high-energy limit ($s\to\infty$) and outside the forward scattering cone for unequal-mass particles, $|z|\to\infty$ and we use[4]

$$d_{\lambda\mu}{}^{\alpha}(z)=a_\lambda(\alpha,z)a_{-\mu}(\alpha,z)h_{\lambda\mu}(z)\,, \tag{2}$$

where

$$a_\lambda=(-i)^\lambda\left[\frac{\Gamma(2\alpha+1)(z/2)^\alpha}{\Gamma(\alpha+\lambda+1)\Gamma(\alpha-\lambda+1)}\right]^{1/2} \tag{3}$$

and

$$h_{\lambda\mu}=\left(1+\frac{1}{z}\right)^{|\lambda+\mu|/2}\left(1-\frac{1}{z}\right)^{|\lambda-\mu|/2}\xrightarrow[|z|\to\infty]{}1\,. \tag{4}$$

In this limit, the t-channel helicity amplitudes can be written in factored form:

$$F_{\lambda_C\lambda_A,\lambda_D\lambda_B}=f_{\lambda_C\lambda_A}f_{\lambda_D\lambda_B}(-1)^{\lambda_A-\lambda_C}\,, \tag{5}$$

where

$$f_{\lambda\lambda'}=\beta_{\lambda\lambda'}a_{\lambda-\lambda'}\,. \tag{6}$$

To get the s-channel helicity amplitudes $G_{\lambda_C\lambda_D,\lambda_A\lambda_B}$, we use the crossing relation of Cohen-Tannoudji *et al.*[5]

$$\begin{aligned}G_{\lambda_C\lambda_D,\lambda_A\lambda_B}=&(-1)^{\sigma_{AD}+2s_B+2s_D}e^{i\pi(\lambda_B-\lambda_C)}\\&\times\sum_{\lambda_i'}d_{\lambda_A'\lambda_A}{}^{s_A}(\chi_A)d_{\lambda_B'\lambda_B}{}^{s_B}(\chi_B)d_{\lambda_C'\lambda_C}{}^{s_C}(\chi_C)d_{\lambda_D'\lambda_D}{}^{s_D}(\chi_D)\\&\times(-1)^{\lambda_A'-\lambda_C'}f_{\lambda_C'\lambda_A'}f_{\lambda_D'\lambda_B'}\,.\end{aligned} \tag{7}$$

For s-channel helicity conservation, $G_{\lambda_C\lambda_D,\lambda_A\lambda_B}$ should be proportional to $\delta_{\lambda_A\lambda_C}\delta_{\lambda_B\lambda_D}$. The only possibility is to have the amplitude factor into parts depending only on the particles at each vertex. This factorization is apparent in the crossing relation, and has been noted before.[6] The crossing angles χ_i in general have dependence on all masses in the reaction. If one looks at the

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[1] J. Ballam *et al.*, Phys. Rev. Letters **24**, 960 (1970); **24**, 1467(E) (1970).

[2] F. J. Gilman, J. Pumplin, A. Schwimmer, and L. Stodolsky, Phys. Letters **31B**, 387 (1970).

[3] Here we restrict our consideration to factorizable Regge-pole exchange. If the diffractive process is actually a nonfactorizable mechanism such as a Regge cut, helicity conservation does not lead directly to simple relations between coupling strengths.

[4] In practice, the restriction $|z|$ large at lab energies in the multi-GeV range means momentum transfers 0.01–0.05 $(\text{GeV}/c)^2$ greater than the minimum transfer (forward scattering).

[5] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann. Phys. (N.Y.) **46**, 239 (1968).

[6] G. C. Fox and Elliot Leader, Phys. Rev. Letters **18**, 628 (1967); **18**, 766(E) (1967).

high-energy limit, however, the dependence on masses at the opposite vertex disappears. For example,

$$\cos\chi_A=-\frac{(s+m_A^2-m_B^2)(t+m_A^2-m_C^2)+2m_A^2(m_B^2+m_C^2-m_A^2-m_D^2)}{\mathcal{S}_{AB}\mathcal{T}_{CA}}\underset{s\to\infty}{\longrightarrow}-\frac{t+m_A^2-m_C^2}{\mathcal{T}_{CA}},\tag{8}$$

where

$$\mathcal{T}_{ij}^2=[t-(m_i+m_j)^2][t-(m_i-m_j)^2],\tag{9}$$

$\mathcal{S}_{ij}$ is the same with t replaced by s, and the phases are chosen such that $\mathcal{S}_{AB}$ is positive and $\mathcal{T}_{CA}$ is negative in the s-channel physical region. Similar results hold for the other crossing angles, so we may write

$$G_{\lambda_C\lambda_D,\lambda_A\lambda_B}\underset{s\to\infty}{\longrightarrow}(-1)^{\sigma_A D+2s_B+2s_D}\times e^{i\pi(\lambda_B-\lambda_C)}g_{\lambda_D\lambda_B}{}^{BD}\bar{g}_{\lambda_C\lambda_A}{}^{AC},\tag{10}$$

with

$$g_{\lambda_D\lambda_B}{}^{BD}=\sum_{\lambda_{B'}\lambda_{D'}}d_{\lambda_{B'}\lambda_B}{}^{s_B}(\chi_B)d_{\lambda_{D'}\lambda_D}{}^{s_D}(\chi_D)f_{\lambda_{D'}\lambda_{B'}}\tag{11}$$

and $\bar{g}$ the same with an additional factor of $(-1)^{\lambda_{A'}-\lambda_{C'}}$. Now helicity conservation can be imposed at each vertex by adjusting the ratios of various $f_{\lambda\lambda'}$.[7]

The apparent difference in the restriction on the f's at opposite vertices due to the different factors in g and $\bar{g}$ is in fact not present. This factor is cancelled due to the dependence of crossing angles on the ordering of the particles in the s and t channels. For example, if $A=B$ and $C=D$, then $\chi_A=-\pi-\chi_B$ and $\chi_C=\pi-\chi_D$. We then use $d_{\lambda\mu}{}^J(\pi-\chi)=(-1)^{J+\lambda}d_{\lambda,-\mu}{}^J(\chi)$ along with other symmetry relations of the d functions, and the parity relation[8]

$$f_{-\lambda_C-\lambda_A}=\sigma_{\bar{A}}\sigma_C(-1)^{\lambda_A+\lambda_C}f_{\lambda_C\lambda_A},\tag{12}$$

where σ is the J parity [parity times $(-1)^S$ for bosons or $(-1)^{S-1/2}$ for fermions]. All of these factors combine to cancel the original $(-1)^{\lambda_{A'}-\lambda_{C'}}$ in $\bar{g}$, giving the result

$$G_{\lambda_D\mu_D,\lambda_B\mu_B}\underset{s\to\infty}{\longrightarrow}(\text{phase factor})\times g_{\lambda_D\lambda_B}{}^{BD}g_{\mu_D\mu_B}{}^{BD}.\tag{13}$$

Thus, the restrictions on the f's for s-channel helicity conservation are identical at each vertex.

III. RESTRICTIONS ON VERTEX FUNCTIONS

In principle, all reactions in which isospin, strangeness, and G parity do not change at each vertex could proceed via Pomeranchukon exchange. In practice, however, an additional restriction seems to apply (as determined by the cross sections which do not decrease with energy).[9] It may be stated as $\Delta P=(-1)^{\Delta J}$, or equivalently, $\sigma_A=\sigma_C$, $\sigma_B=\sigma_D$. In terms of helicity amplitudes, this means that for mesons, a $0\to0$ transition is allowed (in terms of t-channel helicity). We restrict ourselves to these cases.

The restriction of s-channel helicity conservation gives a set of linear homogeneous equations for the f's. The number of equations equals the number of independent ratios of helicity-flip to non-helicity-flip vertex functions in the t channel, so that all of these ratios are determined. For the cases of interest there is only one independent non-helicity-flip vertex function, so that spin-density matrix elements for the final particles are completely determined. The results for some common cases are listed below.

(a) NN vertex:

$$g_{\frac{1}{2}-\frac{1}{2}}=0\Rightarrow\frac{f_{\frac{1}{2}-\frac{1}{2}}}{f_{\frac{1}{2}\frac{1}{2}}}=\left(\frac{-t}{4M^2}\right)^{1/2}.$$

This restriction when formulated in terms of invariant amplitudes has been noted to be satisfied in πN elastic scattering.[2]

(b) PV vertex (P is a 0^- meson with mass μ; V is a 1^+ meson with mass m, e.g., πA_1, KQ):

$$g_1=0\Rightarrow\frac{f_1}{f_0}=-\frac{(-2t/m^2)^{1/2}}{1+(t-\mu^2)/m^2}.$$

(c) PT vertex (T is a 2^- meson with mass m):

$$g_1=g_2=0,$$

$$\frac{f_1}{f_0}=-\sqrt{3}\frac{(-2t/m^2)^{1/2}[1+(t-\mu^2)/m^2]}{[1+(t-\mu^2)/m^2]^2+2t/m^2},$$

$$\frac{f_2}{f_0}=(\sqrt{\tfrac{3}{2}})\frac{(-2t/m^2)}{[1+(t-\mu^2)/m^2]^2+2t/m^2}.$$

(d) NN_{11}^* vertex (N is a nucleon; N_{11}^* is a $\frac{1}{2}^+$ resonance with mass M^*):

$$g_{\frac{1}{2}-\frac{1}{2}}=0,$$

$$\frac{f_{\frac{1}{2}-\frac{1}{2}}}{f_{\frac{1}{2}\frac{1}{2}}}=\frac{2x^{1/2}[(T+\gamma^2-\gamma^{-2})(\gamma-\gamma^{-1})+x(\gamma+\gamma^{-1})]}{4x-x^2+(T+\gamma^2-\gamma^{-2})^2},$$

where $x=-t/MM^*$, $\gamma^2=M^*/M$, and $T=-[x^2+2x(\gamma^2+\gamma^{-2})+(\gamma^2-\gamma^{-2})^2]^{1/2}$.

(e) NN_{13}^* vertex (N_{13}^* is a $\frac{3}{2}^-$ resonance with

[7] Note that only the relative t dependence of the f's is adjustable, so that the high-energy limit is essential to eliminate s dependence from the crossing angles as well as to gain independence from the other particle masses. In other words, exact s-channel helicity conservation is possible only in the infinite-energy limit.

[8] We use the phase conventions of Ref. 5.

[9] D. R. O. Morrison, Phys. Rev. 165, 1699 (1968).

mass M^*):

$$g_{\frac{1}{2}-\frac{1}{2}}=g_{\frac{3}{2}\frac{1}{2}}=g_{\frac{3}{2}-\frac{1}{2}}=0\,,$$

$$\frac{f_{\frac{1}{2}-\frac{1}{2}}}{f_{\frac{1}{2}\frac{1}{2}}}=\frac{\sin(\chi^{+}+\chi_D)+\sin\chi^{+}\cos\chi_D}{\cos(\chi^{+}+\chi_D)+\cos\chi^{+}\cos\chi_D}\,,$$

$$\frac{f_{\frac{3}{2}\frac{1}{2}}}{f_{\frac{1}{2}\frac{1}{2}}}=-\tfrac{1}{2}\sqrt{3}\frac{\cos\chi^{+}\sin\chi_D}{\cos(\chi^{+}+\chi_D)+\cos\chi^{+}\cos\chi_D}\,,$$

$$\frac{f_{\frac{3}{2}-\frac{1}{2}}}{f_{\frac{1}{2}\frac{1}{2}}}=-\tfrac{1}{2}\sqrt{3}\frac{\sin\chi^{+}\sin\chi_D}{\cos(\chi^{+}+\chi_D)+\cos\chi^{+}\cos\chi_D}\,,$$

$$\chi^{+}=\tfrac{1}{2}(\chi_B+\chi_D)\,,$$

$$\cos\chi_D=-\frac{t+M^{*2}-M^2}{\mathcal{T}_{NN^*}}\,,\quad \cos\chi_B=\frac{t+M^2-M^{*2}}{\mathcal{T}_{NN^*}}\,,$$

$$\sin\chi_D=\frac{-2M^*(-t)^{1/2}}{\mathcal{T}_{NN^*}}\,,\quad \sin\chi_B=\frac{2M(-t)^{1/2}}{\mathcal{T}_{NN^*}}\,.$$

(f) γV vertex (γ is the photon; V is a spin-1 [10] meson of odd charge conjugation and mass m):

$$g_{01}=g_{-11}=0\,,$$

$$\frac{f_{01}}{f_{11}}=\left(\frac{-2t}{m^2}\right)^{1/2},$$

$$\frac{f_{-11}}{f_{11}}=-\frac{t}{m^2}\,.$$

(g) γT vertex (T is a spin-2 meson of odd charge conjugation and mass m):

$$g_{01}=g_{-11}=g_{-21}=g_{21}=0\,,$$

$$\frac{f_{01}}{f_{11}}=\frac{(6x)^{1/2}(x-1)}{3x-1}\,,$$

$$\frac{f_{-11}}{f_{11}}=\frac{x(x-1)(x-3)}{3x-1}\,,$$

$$\frac{f_{-21}}{f_{11}}=-\frac{2x^{3/2}}{3x-1}\,,$$

$$\frac{f_{21}}{f_{11}}=\frac{2x^{1/2}}{3x-1}\,,$$

$$x=-t/m^2\,.$$

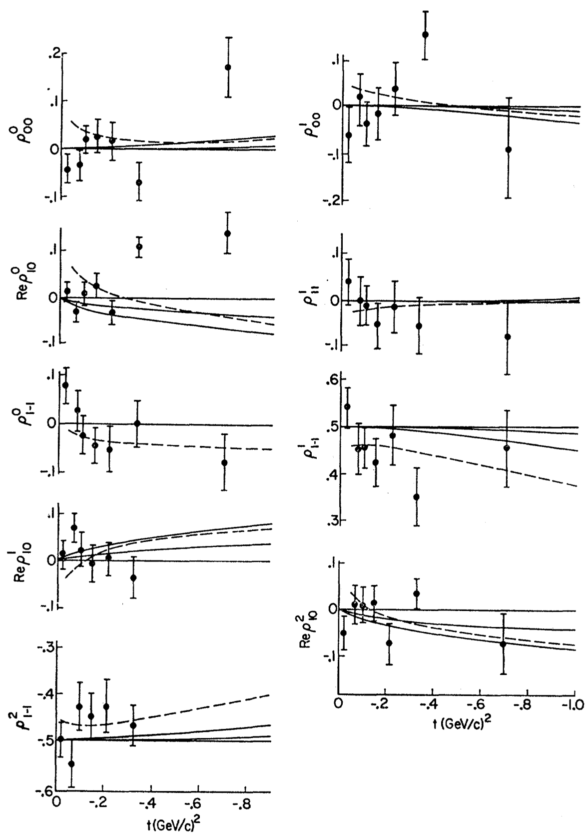

FIG. 1. Spin-density matrix elements in the s-channel frame for the reaction $\gamma p\rightarrow\rho^0 p$.

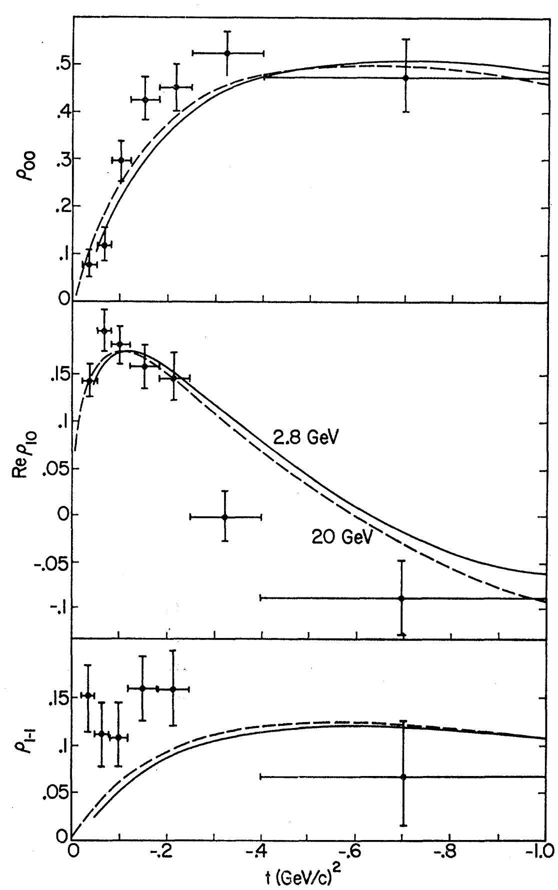

FIG. 2. Spin-density matrix elements in the t-channel frame for the reaction $\gamma p\rightarrow\rho^0 p$.

[10] Since the helicity crossing matrix is diagonal for massless particles, no parity relations need be used in the restrictions on vertex functions involving photons. The relations are valid for mesons of either parity in the γV and γT cases.

IV. s-CHANNEL HELICITY CONSERVATION IN $\gamma p \to \rho^0 p$

In ρ^0 production by polarized photons, the analysis of ρ^0 decay angular distributions in the s-channel frame at 2.8 and 4.7 GeV shows consistency with s-channel helicity conservation.[1] Here we assume the appropriate couplings for helicity conservation at infinite energy, and then use the actual s dependence of crossing angles and t-channel amplitudes to calculate finite-energy corrections. Figure 1 shows the predicted values of the spin-density matrix elements at 2.8 GeV, 4.7 GeV, and infinite energy, as well as the measured values at 2.8 GeV. The predictions at infinite energy are $\rho_{1-1}{}^2 = -0.5$, $\rho_{1-1}{}^1 = 0.5$, and all other elements$=0$. It is seen that the predicted energy variation falls within the error bars for small momentum transfer. Thus, the results are consistent with a factorizable Pomeranchukon exchange whose residue functions obey conditions for exact helicity conservation at infinite energy. The dashed curves are the predictions if only the $\gamma\rho^0$ vertex obeys helicity conservation, and the NN vertex ratio is 100 times that predicted for helicity conservation.

V. t-CHANNEL PREDICTIONS FOR UNPOLARIZED EXPERIMENTS

As mentioned previously, the assumption of s-channel helicity conservation completely determines the spin-density matrix elements in the cases considered here. These elements have been calculated in the t-channel frame for some typical reactions. Residue function ratios for s-channel helicity conservation at infinite energy were used, but exact energy dependence of the total amplitude was included [the function $h_{\lambda\mu}$ in Eq. (4)]. In this case the energy variation is much less than that in the s channel, since there is no additional energy dependence from the crossing angles.

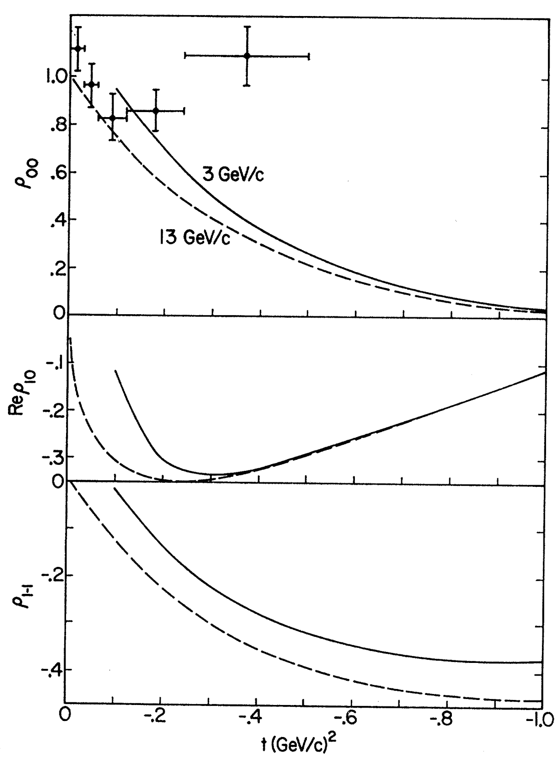

FIG. 3. Spin-density matrix elements in the t-channel frame for the reaction $KN \to QN$.

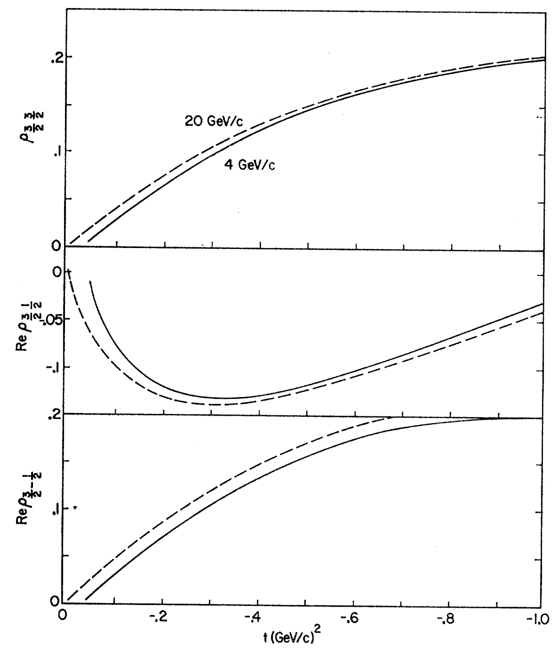

FIG. 4. Spin-density matrix elements in the t-channel frame for the reaction $NN \to N^*(1520)N$.

Figure 2 shows the ρ^0 photoproduction again, but this time in the t-channel frame with only the elements measurable in an unpolarized experiment. There is some discrepancy in the predicted and measured values for ρ_{1-1}, indicating that this element may be more sensitive to s-channel helicity conservation with factorizable t-channel exchange than the s-channel elements themselves.

Figure 3 shows predictions for $KN \to QN$, as well as measured ρ_{00} values at 13 GeV/c.[11] There is an indication that t-channel helicity conservation rather than s-channel holds, but this awaits more data at large momentum transfer and extraction of the off-diagonal elements. Similar predictions hold for the reaction $\pi N \to A_1 N$.

Figure 4 shows predictions for the reaction $NN \to N^*(1520)N$. Note that the values are quite different than those for the $N^*(1236)$ production in πN or NN reactions, which presumably goes by ρ or π exchange.

[11] M. S. Farber, T. Ferbel, P. F. Slattery, and H. Yuta, Phys. Rev. D 1, 78 (1970).