leads to the interesting possibility that the majority of the common baryon mass comes from this source. In particular, in keeping with our nonet structure assumption, if we now assume that $\langle B|s^0|B\rangle$ is related to $\langle B|s^8|B\rangle_D$ by the generalized d coefficient, we find $\langle B_i|s^0|B_j\rangle \sim \delta_{ij}\times 400$ MeV, so that the j^0j^0 part of our model Eq. (1) (which is equivalent to a gluon model with a large gluon mass) could constitute the $SU(3)\times SU(3)$-invariant part of the Hamiltonian. Note that the invariant part should, according to recent calculations, produce the major part of the baryon masses.[13]

Summing up, we should emphasize that our conclusions depend on several assumptions, as stated above, as well as on the $g_V{}^2$ determination by Abarbanel *et al.* [Eq. (7)]. It is our belief that the result obtained here, pointing out the importance of H^V of Eq. (1) in fixing the hadron spectrum, might be correct and should be further investigated.

[13] A recent estimate [J. K. Kim and F. von Hippel, Phys. Rev. Letters 22, 740 (1969)] done in the framework of the Gell-Mann–Oakes–Renner model [Phys. Rev. 175, 2195 (1968)] for $SU(3)\times SU(3)$ breaking has indicated that the $SU(3)\times SU(3)$-violating Hamiltonian accounts only for a small portion ($\approx$170 MeV) of the common mass of the baryon octet.

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Nishijima's Model with a Fifth Interaction

AMIR SCHORR
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel

AND

YUVAL NE'EMAN*†
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel
and
Center for Particle Theory, University of Texas, Austin, Texas

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Nishijima's model for CP violation introduces a divergencelike term in the strong Hamiltonian and attributes the nonleptonic weak interaction to the second-order contribution due to that term (CP violation arises in third order). By using a unitary scalar density plus the fifth interaction for the original strong Hamiltonian, we induce in Nishijima's weak Hamiltonian a current-current structure, somewhat similar to the conventional one. Moreover, it becomes equivalent to tadpole terms for $|\Delta S|=1$. The $\Delta S=0$ weak transitions are somewhat modified. We get the correct $|\Delta I|=\frac{3}{2}$ admixture from a counter term which relegates the $|\Delta S|=2$ contributions to fourth order (i.e., second-order weak interactions). We find $|\eta_{00}|\sim|\eta_{+-}|$ to within 10%.

INTRODUCTION

In the preceding article[1] we investigated the possibility that the "fifth interaction" represented by the Hamiltonian

$$H^V(x)=g_V{}^2 j_\mu{}^V(x) j^{\mu V}(x)\,,$$

with (1)

$$j_V{}^\mu(x)=j_0{}^\mu(x)\cos\phi_V+j_8{}^\mu(x)\sin\phi_V\,,$$

describes the strong interactions, including the breaking of $SU(3)$, for the baryon octet $(\frac{1}{2})^+$. We checked consistency with the elastic scattering amplitudes for baryons, and with the nonleptonic decay amplitudes of hyperons.

We used the strong assumption of symmetric-nonet conserved vector current (SNCVC), equating the reduced matrix elements of $j_0{}^\mu j_{\mu 8}$ and $j_{i\mu}j_l{}^\mu$ [i, $1=1\cdots 8$, according to Gell-Mann's notation for the $SU(3)$ octet components]. However, the resulting difference between D^w/F^w and D^s/F^s [the ratios in nonleptonic parity-violating decays of hyperons, and in the $SU(3)$-breaking part of the Hamiltonian describing the baryon octet masses, respectively] has yet to be accounted for.

In this paper we construct an explicit Hamiltonian for nonleptonic weak interactions, using the Nishijima model.[2] We thus investigate the possibility of a universal D/F and the correlation between the fifth-interaction parameters and the nonleptonic decays.

NISHIJIMA MODEL

Nishijima has suggested adding to $H_0(x)$, which is the Hamiltonian describing the strong and semistrong

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[1] Y. Ne'eman, S. Nussinov, and A. Schorr, Phys. Rev. D 1, 229 (1970).

[2] K. Nishijima, Progr. Theoret. Phys. (Kyoto) 41, 739 (1969).

interaction, a term $f\partial_\mu K^\mu$ ($f\simeq 2.8\times 10^{-3}$):

$$K_\mu = j_\mu^{3(+)}\cos^2\theta_C + \tfrac{1}{2}(j_\mu^{3(+)} + \tfrac{1}{2}\sqrt{3}\, j_\mu^{8(+)})\sin^2\theta_C - j_\mu^{6(+)}\sin\theta_C\cos\theta_C\,, \quad (2)$$

where $j_\mu^{i(+)} = j_\mu^i + j_\mu^{5i}$ and θ_C = the Cabibbo angle. $\int d^3x\, K_0(x)$ is the third component in the $SU(2)_{\text{weak}}$ space of a vector whose other two components are $\int d^3x\, K_0^1(x)$ and $\int d^3x\, K_0^2(x)$, K^1 and $K^2 = K^{1\dagger}$ are the currents appearing in the weak leptonic interaction. $\partial_\mu K^\mu$ represents the divergence of the current K_μ when only the interaction described by $H_0(x)$ is "present" (since a real divergence of a current—if constituting a part of a Hamiltonian—does not contribute anything to any process).

Nishijima pointed out that the term $f\partial_\mu K^\mu$, which breaks CP symmetry, does not contribute anything in first order. In second order it gives the weak interaction, and in third order the CP-nonconserving Hamiltonian:

$$\begin{aligned} S_{A,B}^{(1)} &= -if\int d^4x \langle B,\text{out}|\,\partial_\mu K^\mu\,|A,\text{in}\rangle \\ &= (2\pi)^2 f (P_A - P_B)_\mu \delta(P_A - P_B)_\mu \\ &\quad\times \langle B,\text{out}|K^\mu|A,\text{in}\rangle = 0\,. \end{aligned} \quad (3)$$

It should be remarked that A and B represent eigenstates of $H_0(x)$, which decay in the presence of $f\partial_\mu K^\mu$.

In second order we obtain a term conserving CP:

$$\begin{aligned} S_{B,A}^{(2)} &= \tfrac{1}{2}f^2\int d^4x \left[\int_{x^0=y^0} d^3y\, K^0(y), \partial_\mu K^\mu(x)\right] \\ &\equiv \int d^4x\, H^{\text{NL}}(x)\,. \end{aligned} \quad (4)$$

CHOICE OF STRONG HAMILTONIAN

In his original article, Nishijima provides an example in which $H_0 = u_0$, where u_0 is actually a tadpole present in the representation $(\bar{3},3)+(3,\bar{3})$ of $SU(3)\times SU(3)$ (cf. Gell-Mann, Oakes, and Renner[3]). For this model the vector currents satisfy

$$\partial_\mu j_i^\mu = -i\left[H(x,t), \int j_i^0(y,t)d^3y\right] = 0\,, \quad (5)$$

while for axial vectors, $\partial_\mu A_i^{\ \mu} = (\sqrt{\tfrac{2}{3}})V^i$.

Such a model does not incorporate the breaking of $SU(3)$ in $H_0(x)$ and, moreover, gives rise to parity-violating decays for H^{NL} only. If a term $-cu_8(x)$ is added to $H_0(x)$ as in the Gell-Mann–Oakes–Renner model, one obtains parity-conserving terms in H^{NL}, but all terms with $|\Delta S| = 1$ come out to be proportional to divergences of currents, so that no parity-conserving decay is obtained that does not conserve strangeness. Generally, it is possible to "rotate" any model representing H^{NL} and H_0 in the above representation of $SU(3)\times SU(3)$ in such a way as to conserve parity, strangeness, and Q_{em}.[4] As a result, we consider a better choice for H_0 to be

$$H^0 = g_V^2 j_\mu^V j_\mu^{V\dagger} + lu_0 \equiv hj_0j_0 + a(j_0j_8 + j_8j_0) + bj_8j_8 + lu_0\,, \quad (6)$$

which conserves the symmetry $SU(2)$.

The H^{NL} thus obtained cannot be "rotated away."

RESULTING NONLEPTONIC WEAK INTERACTION

We use Nishijima's approximation for K^μ:

$$K_\mu \simeq j_\mu^{3(+)} + \beta j_\mu^{6(+)} = \bar{K}_\mu\,, \quad \beta\simeq -2/7$$

$$Q^{\bar{K}} = \int d^3x\, \bar{K}^0(x) = Q_3^{(+)} + \beta Q_6^{(+)}\,, \quad (7)$$

$$\partial_\mu \bar{K}^\mu = -i[H_0(x), Q^{\bar{K}}(x_0)] = \beta f_{867}(aj_0j_7^{(+)} + j_7^{(+)}j_0) + b(j_8j_7^{(+)} + j_7^{(+)}j_8) + (\sqrt{\tfrac{2}{3}})l(v_3 + \beta v_6)\,.$$

As mentioned, in first order $f\partial_\mu\bar{K}^\mu$ does not contribute to transition amplitudes. Let us construct H^{NL} according to

$$\begin{aligned} S^{(2)} &= \tfrac{1}{2}f^2\int d^4x [Q_{\bar{K}}(x_0), \partial_\mu\bar{K}^\mu] \\ &= \int d^4x H^{\text{NL}}(x) - iH^{\text{NL}} \\ &= f^2\beta f_{867}f_{376}[a(j_0j_6^{(+)} + j_6^{(+)}j_0) + b(j_8j_6^{(+)} + j_6^{(+)}j_8) \\ &\quad -\beta a(j_0j_3^{(+)} + j_3^{(+)}j_0) - \sqrt{3}a\beta(j_0j_8^{(+)} + j_8^{(+)}j_0) \\ &\quad + b\sqrt{3}\beta(j_8j_8^{(+)} + j_8^{(+)}j_8) + \beta b(j_8j_3^{(+)} + j_3^{(+)}j_8)] \\ &\quad -\tfrac{1}{2}b\beta^2f^2f_{867}^2 j_7^{(+)}j_7^{(+)} - (\tfrac{1}{2}\sqrt{\tfrac{2}{3}})\beta f^2 lu_6\,. \end{aligned} \quad (8)$$

The first, second, and last terms have $|\Delta S| = 1$, the third to sixth have $|\Delta S| = 0$, and the seventh describes a transition with $|\Delta S| = 2$. It is again possible to use the principle that a divergence does not contribute in a transition between two states possessing identical four-momenta:

$$\partial_\mu j_7 = ab_{876}(j_0j_6 + j_6j_0) + bf_{876}(j_8j_6 + j_6j_8)\,, \quad (9)$$

so that we can omit these terms from the above expression. Also we have

$$\partial_\mu j_7^{\mu 5} = f_{876}[a(j_0j_6^5 + j_6^5j_0) + b(j_8j_6^5 + j_6^5j_8)] + (\sqrt{\tfrac{2}{3}})lv_7\,, \quad (10)$$

so that the $|\Delta S| = 1$ part of the Hamiltonian can be

[3] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[4] N. Cabibbo and C. Maiani, Phys. Rev. D **1**, 707 (1970); S. Nussinov and G. Preparata, Phys. Rev. **175**, 2180 (1968).

written as

$$H^{\mathrm{NL}}(|\Delta S|=1)=\frac{i}{\sqrt{6}}f^2\beta l(u_6-v_7), \qquad (11)$$

which implies $l\sim\frac{1}{2}$, and where we have omitted the term proportional to $\partial_\mu j_7{}^{\mu(+)}$. It is not possible to rotate H_0+H^{NL} in such a way as to conserve S, since instead of u_6, v_7 we obtain terms of the form $j_0 j_7{}^{(+)}$, $j_0 j_6{}^{(+)}$.

We get a poor result for the $|\Delta S|=2$ amplitude A_2:

$$\frac{A_2}{A_1}=\frac{3\sqrt{6}}{8}\frac{b\beta}{l}\sim\frac{1}{150}\text{ to }\frac{1}{300}, \qquad (12)$$

where A_1 is the $|\Delta S|=1$ amplitude. The experimental factor is much smaller; it is obtained by the Okun-Pontecorvo argument based on the (K_L,K_S) mass difference.

The only way to improve this result is to add to H_0 another term of the current×current form, so as to cancel in second order the $j_7{}^{(+)}j_7{}^{(+)}$ term. The simplest such expression seems to be

$$H_0'=-3bj_3j_3. \qquad (13)$$

For

$$b=g_V{}^2\sin^2\phi_V \qquad (14)$$

and $\sin\phi_V=\frac{1}{9}$ to $\frac{1}{12}$, this correction to the fifth interaction would bring about a $<1\%$ admixture of $|\Delta I|=2$ in the strong-interaction Hamiltonian (6). This tadpole-like term might contribute to the $(\pi^\pm,\pi^0)$ mass difference, etc.

As a result we get, in second order in f, a part with $\Delta I=\frac{3}{2}$, with an amplitude $(1/14)A_{1/2}$, which is close to the experimental value ($\sim 6\%$).

EFFECTS OF SOFT-PION APPROXIMATION

It is at this point that the relation between D^s/F^s and D^w/F^w can be clarified. The NL parity-violating decays are described by

$$H_{\mathrm{p.v.}}{}^{\mathrm{NL}}(|\Delta S|=1)=i(\tfrac{1}{4}\sqrt{3})f^2\beta a(j_0j_6{}^5+j_6{}^5j_0). \qquad (15)$$

By using partial conservation of axial-vector current (PCAC), one obtains[5,6]

$$\frac{c}{\mu^2}\langle B^a\pi^b(k)|H_{\mathrm{p.v.}}{}^{\mathrm{NL}}|B^c\rangle_{k\to0}$$

$$=\langle B^a|\left[\int d^3x\, j_b{}^{05},H_{\mathrm{p.v.}}{}^{\mathrm{NL}}\right]|B^c\rangle, \qquad (16)$$

$$\partial_\mu A^{\mu b}(x)=c\phi^b(x),\quad c=2m_N\mu^2/g_{\pi NN}(-G_A/G_V),$$

where the last term is present in the j_0j_i octet and is thus given by the same F^sD^s as the fifth interaction.

Indeed we have

$$\frac{1}{2}\frac{\sqrt{3}}{4}f^2\begin{pmatrix}D^s\\F^s\end{pmatrix}=0.42\times10^{-6}\begin{pmatrix}65\\-220\end{pmatrix}=10^{-5}\begin{pmatrix}2.73\\-9.24\end{pmatrix}. \qquad (17)$$

These values should be compared with the experimental ones, derived through the PCAC[1,7] approximation:

$$\begin{pmatrix}D^w\\F^w\end{pmatrix}=10^{-5}\begin{pmatrix}3.2\\-7.2\end{pmatrix}.$$

The correspondence is good as far as orders of magnitude are concerned, but there is a sizable difference in the actual ratios D^s/F^s and D^w/F^w. This difference we attribute to the error in PCAC. Consider a matrix element satisfying

$$\langle A(p)|\partial_\mu j_l{}^\mu|\pi_i B(p)\rangle=0, \qquad (18)$$

where

$$\langle A(p)|j_l{}^\mu|\pi_i B(p)\rangle\neq0\quad(f_{ijl}\neq0) \qquad (19)$$

and the state πB has the same four-momentum p as A has, so that the first matrix element vanishes as a result of kinematics.

In this case we get by PCAC for soft pions

$$\langle A|[Q_i{}^5,\partial_\mu j_l{}^\mu]|B\rangle\neq0, \qquad (20)$$

i.e., the zero obtained from the kinematics is changed in the above approximation.

In our case the $|\Delta S|=1$ part of the parity-violating Hamiltonian is effectively proportional to v_7 only, according to (11). However, when applying PCAC in (13) we are using the entire $(j_0j_6{}^5+j_6{}^5j_0)$ expression, including the contribution of $\partial_\mu j_7{}^{\mu5}$, which should have vanished as in (18). As in (20), we do get through PCAC a nonvanishing contribution from the latter term, which is proportional to

$$[Q_i{}^5,v_7]=id_{i7k}u_k,$$

involving the $\begin{pmatrix}D_u\\F_u\end{pmatrix}$ of the u_K multiplet's matrix elements between baryons.

CP VIOLATION

In the third order of the S-matrix expansion we encounter CP violation:

$$S^{(3)}=\left(-\frac{if}{6}\right)^3\int d^4xd^4yd^4z$$

$$\times T(\partial_\mu\bar{K}^\mu(x)\partial_\nu\bar{K}^\nu(y)\partial_\gamma\bar{K}^\gamma(z)). \qquad (21)$$

Let us just compute the $|\Delta S|=1$ term. By a simple technique, omitting terms of magnitude β^3, we obtain

$$H^{(3)}=\tfrac{1}{2}f^3[Q^{\bar{K}}[Q^{\bar{K}},\partial_\mu\bar{K}^\mu]]$$

$$=f[Q^{\bar{K}},H^{(2)}(x)], \qquad (22)$$

[5] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965).

[6] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966).

[7] Y. Hara, Progr. Theoret. Phys. (Kyoto) **37**, 710 (1967).

$$H^3(|\Delta S|=1)=\frac{f^3\beta}{2\sqrt{6}}l(v_6+u_7)+(h\partial_\mu j_0{}^{\mu(+)})$$
$$=f^2\frac{\beta l}{2\sqrt{6}}u_7-f^2\frac{\beta\sqrt{3}a}{8}(j_0j_7{}^5+j_7{}^5j_0)+(h\partial_\mu j_6{}^\mu)\,, \quad (23)$$

where the last terms—in brackets—do not affect the decay.

It is now possible to calculate various ratios for the amplitudes of CP-breaking interactions, from strong and weak interaction parameters.

It is important to notice that the addition of the H_0' counterterm (13) ensures that there is no $|\Delta S|=2$ interaction even in third order; such a term appears here only in fourth order, in agreement with the Okun-Pontecorvo argument.

As an immediate result, we also see that in third order, the $|\Delta I|=\frac{3}{2}$ amplitude is just 7% of the $|\Delta I|=\frac{1}{2}$ amplitude, so that the parameters $|\eta_{00}|$ and $|\eta_{+-}|$ should be equal to within $\sim$10%.

Finally we emphasize that the SNCVC assumption made in Ref. 1—namely, that the octet matrix elements of

$$\{j_ij_l\}_8,\ \{j_i{}^5j_l{}^5\}_8,\ \{j_0j_i\}\quad (i,l=1\cdots 8)$$

are equal between states belonging to the lowest baryon octet—enabled us to obtain[1] the value $\sim\frac{1}{9}-\frac{1}{12}$ for $\sin\phi_V$. This explains our decision to retain this value, in spite of the fact that we used in this article a form of $H^{\rm NL}$ apparently somewhat different from the one used in Ref. 1, where $H^{\rm NL}\propto\{j_c,j_c{}^\dagger\}$ (j_c is the Cabibbo current).

Note that the only value depending on $\sin\phi_V$ which we get here, $A_{3/2}/A_{1/2}$, is in very good agreement with experiment.

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Vector-Meson Exchange Contributions to the $K_1\rightarrow\gamma\gamma$ Decay Rate*

ZENAIDA EBORA SANTOS UY
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11790
(Received 17 August 1970)

By assuming that (1) the decay $K_1\rightarrow\gamma\gamma$ is saturated by the 2π intermediate state, (2) the $K\pi\pi$ vertex is of the first-order weak interaction, and (3) the $\pi\pi\rightarrow\gamma\gamma$ process goes by exchanges of ρ, ω, and ϕ, the contributions to the decay rate of $K_1\rightarrow\gamma\gamma$ from the three exchanges are calculated. It is found that the contributions do not alter in any significant way the result of the perturbation calculation using the Born term for the $\pi\pi\rightarrow 2\gamma$ reaction.

I. INTRODUCTION

At present there is no experimental information on the decay of the short-lived neutral kaon into two γ rays. In view of the possibility of observing effects due to CP violation in the decays $K\rightarrow 2\gamma$, it is important to know the expected decay rate of $K_S{}^0\rightarrow 2\gamma$. Previous calculations[1,2] have used the unitarity relation for $K_S{}^0\rightarrow 2\gamma$ and assumed the two-pion state to be the dominant one. The decay rate for $K_S{}^0\rightarrow 2\pi$ is known (we will in future write $K_1\rightarrow 2\pi$ and assume CP conservation), so the model-dependent part of this calculation is the estimate of the $2\pi\rightarrow 2\gamma$ amplitude, for which the Born approximation has been used. In this paper we improve this estimate by including the vector-meson exchanges. We find that they make a rather small correction to the results already published.[1,2]

In Sec. II we review briefly the results of the previous calculations and define our notation. Section III contains the details of the vector-meson exchange diagrams and our conclusions are given in Sec. IV.

II. PERTURBATION CALCULATION OF RATE OF $K_1\rightarrow\pi\pi\rightarrow\gamma\gamma$

In the decay mode $K_1\rightarrow\gamma\gamma$, the Lorentz-invariant matrix element $\mathfrak{M}$ that satisfies gauge invariance and Bose statistics can be written as[1]

$$\mathfrak{M}=\tfrac{1}{2}\mathcal{H}(s)F_{\mu\nu}{}^{(1)}F^{\mu\nu(2)}$$
$$=\mathcal{H}(s)[(\epsilon_1\cdot\epsilon_2)(k_1\cdot k_2)-(\epsilon_1\cdot k_2)(\epsilon_2\cdot k_1)]\,. \quad (1)$$

The superscripts refer to the two photons; the ϵ's and k's are their polarization and momentum four-vectors, respectively. $\mathcal{H}(s)$ in the rest frame of K_1 is just a function of the squared mass $s=M_K{}^2$ of the K_1 and is represented by the box in Fig. 1.

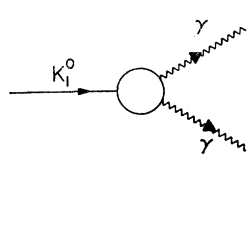

FIG. 1. Feynman diagram describing $K_1\rightarrow\gamma\gamma$.

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[1] V. Barger, Nuovo Cimento **32**, 128 (1964); H. Stern, *ibid.* **51A**, 197 (1967).

[2] B. R. Martin and E. de Rafael, Nucl. Phys. **B8**, 131 (1968).