

Although covariantizing seagull terms can be explicitly constructed under certain conditions,¹² such constructions generally require some prior information concerning the structure of the vertex function in question. To obtain this prior information about $M_\lambda^{(+)}$ in Eq. (6), we have examined several field-theoretic models of H_w^{pv} using the current correlation function technique of Boulware and Brown.¹² By means of this technique, we have explicitly constructed the seagull and Schwinger terms in $M_\lambda^{(+)}$ and have shown how the noncancellation of the seagull and Schwinger terms,

¹² L. S. Brown, Phys. Rev. **150**, 1338 (1966); D. G. Boulware and L. S. Brown, *ibid.* **156**, 1724 (1967); R. F. Dashen and S. Y. Lee, *ibid.* **187**, 2017 (1969); D. J. Gross and R. Jackiw, Nucl. Phys. **B14**, 269 (1969).

which is demanded by the decay of Eq. (1), actually comes about. These models were checked for consistency by demonstrating that for the analogous $pv NN\gamma$ amplitudes the corresponding seagull and Schwinger terms do indeed cancel one another, as is demanded by gauge invariance.¹¹ A more complete discussion of these points will be given elsewhere.⁶

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Test of the Dominance of Tensor-Meson Nonet Trajectories in $SU(3)$ -Symmetry Breaking*

S. Y. LEE

Department of Physics, University of California, Los Angeles, California 90024

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It is shown that the reactions of very-high-energy forward photoproduction of neutral vector mesons off nucleons furnishes a further test of the idea that $SU(3)$ -symmetry breaking is dominated by the nonet of tensor-meson trajectories at zero momentum transfer.

AS was pointed out by Okubo¹ and others,²⁻⁴ the dominance of the A_2 Regge trajectory in the electromagnetic violation of $SU(3)$ at zero momentum transfer seems to work very well for the electromagnetic mass difference of hadrons within the same supermultiplet. On the other hand, Dashen⁵ has noted that if the A_2 Regge trajectory dominates the electromagnetic violation of $SU(3)$, then other members of the nonet of tensor-meson [$A_2(1320)$, $K^*(1430)$, $f'(1500)$, $f_0(1250)$] trajectories might also dominate $SU(3)$ -symmetry breaking due to strong or weak interactions, thus suggesting the dominance of nonet tensor-meson trajectories in the $SU(3)$ -symmetry breaking at zero momentum transfer. It is very interesting to see that this idea can be further tested by considering high-energy forward photoproduction of neutral vector mesons off nucleons.

We denote the photon, proton, and neutron by γ , p , and n , respectively, and consider the reactions

$$\gamma + p \rightarrow V_0 + p, \quad (1)$$

$$\gamma + n \rightarrow V_0 + n, \quad (2)$$

where V_0 stands for any one of the neutral vector mesons (ρ_0, ϕ, ω). Let A_{V_0p} , A_{V_0n} and $d\sigma_{V_0p}/dt$, $d\sigma_{V_0n}/dt$ be the scattering amplitude and differential cross section, respectively, for (1) and (2), where t is the momentum transfer. It is easy to see that the sum of these scattering amplitudes, $A_{V_0p} + A_{V_0n}$, is a pure $I=0$ amplitude, while their difference, $A_{V_0p} - A_{V_0n}$, is a pure $I=1$ amplitude. We assume that at very high energy and in the forward direction, the sum of the amplitudes is dominated by the Pomeranchukon P and the Regge pole of P' of tensor meson f_0 and that the difference of amplitudes is dominated by the A_2 Regge pole. Denote the Pomeranchuk trajectory by $\alpha_P(t)$, the P' trajectory by $\alpha_{P'}(t)$, and the A_2 trajectory by $\alpha_{A_2}(t)$. At $t=0$, their values are approximately given⁶ by

$$\alpha_P(t=0)=1, \quad \alpha_{P'}(t=0)=0.65, \quad \alpha_{A_2}(t=0)=0.5. \quad (3)$$

We see therefore that in the forward direction the principal trajectory is the Pomeranchuk trajectory, while the P' trajectory is the secondary trajectory.

Assuming the factorization of the residue functions, we get, at $t=0$ and very high energy,

$$A_{V_0p} + A_{V_0n} = \Gamma_{\gamma V_0 P} \Gamma_{NNP} R_{PV_0N} + \Gamma_{\gamma V_0 P'} \Gamma_{NNP'} R_{P'V_0N}, \quad (4)$$

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¹ S. Okubo, Phys. Rev. Letters **18**, 256 (1967); **18**, 383 (E) (1967).

² H. Harari, Phys. Rev. Letters **17**, 1303 (1967).

³ D. Gross and H. Pagels, Phys. Rev. **172**, 1381 (1968).

⁴ J. Ball and F. Zachariasen, Phys. Rev. **177**, 2264 (1969).

⁵ R. F. Dashen, Phys. Rev. **183**, 1245 (1969).

⁶ P. D. B. Collins and E. J. Squires, *Springer Tracts in Modern Physics* (Springer, Berlin, 1968), Vol. 45.

where Γ_{ABT} is the residue function of trajectory T at the vertex $A-B-T$ and R_{TV_0N} is a function of trajectory, energy, and masses of the vector meson and nucleon. We do not need to know the explicit expression for R_{TV_0N} in our subsequent discussion. However, we need to note that at very high energy, the masses of the vector mesons and nucleons in R_{TV_0N} can be neglected so that we have

$$R_{T\rho_0N} = R_{T\omega N} = R_{T\phi N} \equiv R_T,$$

where R_T is defined by the above equation. In terms of R_T , Eq. (4) can be rewritten as

$$A_{V_0P} + A_{V_0n} = \Gamma_{\gamma V_0P} \Gamma_{NNP} R_P + \Gamma_{\gamma V_0P'} \Gamma_{NNP'} R_{P'}. \quad (5)$$

Similarly, we get

$$A_{V_0P} - A_{V_0n} = \Gamma_{\gamma V_0A_2} \Gamma_{NNA_2} R_{A_2}. \quad (6)$$

Keeping only the contribution from the principal Regge pole P and the interference term between the contributions of P and secondary Regge pole P' (in other words, neglecting quadratic terms like $|R_{P'}|^2$, $|R_{A_2}|^2$, etc.), we obtain from (5) and (6) an equation involving the differential cross sections:

$$C \left(\frac{d\sigma_{V_0P}}{dt} + \frac{d\sigma_{V_0n}}{dt} \right)_{t=0} = \Gamma_{\gamma V_0P}^2 \Gamma_{NNP}^2 |R_P|^2 + \Gamma_{\gamma V_0P} \Gamma_{NNP} \Gamma_{\gamma V_0P'} \Gamma_{NNP'} (R_P^* R_{P'} + R_P R_{P'}^*), \quad (7)$$

where $C = 32\pi s^2$.

To get useful information out of (7), we need to know the residue function $\Gamma_{\gamma V_0P}$ and $\Gamma_{\gamma V_0P'}$. Using vector-meson dominance yields

$$\Gamma_{\gamma\rho_0T} = g_{\gamma\rho_0} \Gamma_{\rho_0\rho_0T}, \quad (8)$$

$$\Gamma_{\gamma\omega T} = g_{\gamma\omega} \Gamma_{\omega\omega T} + g_{\gamma\phi} \Gamma_{\phi\omega T}, \quad (9)$$

$$\Gamma_{\gamma\phi T} = g_{\gamma\omega} \Gamma_{\omega\phi T} + g_{\gamma\phi} \Gamma_{\phi\phi T}, \quad (10)$$

where T stands for P or P' and $g_{\gamma\rho_0}$, for instance, is the γ - ρ_0 coupling constant. However, since the vector mesons can be satisfactorily classified as an $SU(3)$ nonet,^{7,8} which we designate by the 3×3 matrix V , the diagonal elements of V take the form

$$V_1^1 = (\rho_0 + \omega)/\sqrt{2}, \quad V_2^2 = (-\rho_0 + \omega)/\sqrt{2}, \quad V_3^3 = -\phi.$$

From the general $SU(3)$ -invariant interaction Lagrangian for the residues at the vertex $V-V-P$, at $t=0$, given by

$$L_{VVP} = \Gamma_{VPP} \text{Tr}(\bar{V}V),$$

we find that

$$\Gamma_{\rho_0\rho_0P} = \Gamma_{\omega\omega P} = \Gamma_{\phi\phi P} \equiv \Gamma_{VP} \quad (11)$$

and

$$\Gamma_{\phi\omega P} = \Gamma_{\omega\phi P} \equiv 0. \quad (12)$$

Combining Eqs. (8)–(12) yields

$$\Gamma_{\gamma\rho_0P} = g_{\gamma\rho_0} \Gamma_{VP}, \quad (13)$$

$$\Gamma_{\gamma\omega P} = g_{\gamma\omega} \Gamma_{VP}, \quad (14)$$

$$\Gamma_{\gamma\phi P} = g_{\gamma\phi} \Gamma_{VP}. \quad (15)$$

With the above relations, the contributions due to the Pomeranchukon P can be eliminated, and we find that the ratios of the differential cross sections depend only on the contribution due to P' , i.e.,

$$\frac{g_{\gamma\omega}^2 (d\sigma_{\rho_0P}/dt + d\sigma_{\rho_0n}/dt)_{t=0} - g_{\gamma\rho_0}^2 (d\sigma_{\omega P}/dt + d\sigma_{\omega n}/dt)_{t=0}}{g_{\gamma\phi}^2 (d\sigma_{\rho_0P}/dt + d\sigma_{\rho_0n}/dt)_{t=0} - g_{\gamma\rho_0}^2 (d\sigma_{\phi P}/dt + d\sigma_{\phi n}/dt)_{t=0}} = \frac{g_{\gamma\omega}^2 \Gamma_{\rho_0\rho_0P'} - \Gamma_{\omega\omega P'} - (g_{\gamma\phi}/g_{\gamma\omega}) \Gamma_{\phi\omega P'}}{g_{\gamma\phi}^2 \Gamma_{\omega\omega P'} - \Gamma_{\phi\phi P'} - (g_{\gamma\omega}/g_{\gamma\phi}) \Gamma_{\omega\phi P'}} \quad (16)$$

and

$$\frac{g_{\gamma\omega}^2 (d\sigma_{\rho_0P}/dt + d\sigma_{\rho_0n}/dt)_{t=0} - g_{\gamma\rho_0}^2 (d\sigma_{\omega P}/dt + d\sigma_{\omega n}/dt)_{t=0}}{g_{\gamma\phi}^2 (d\sigma_{\omega P}/dt + d\sigma_{\omega n}/dt)_{t=0} - g_{\gamma\omega}^2 (d\sigma_{\phi P}/dt + d\sigma_{\phi n}/dt)_{t=0}} = \frac{g_{\gamma\rho_0}^2 \Gamma_{\rho_0\rho_0P'} - \Gamma_{\omega\omega P'} - (g_{\gamma\phi}/g_{\gamma\omega}) \Gamma_{\phi\omega P'}}{g_{\gamma\phi}^2 \Gamma_{\omega\omega P'} - \Gamma_{\phi\phi P'} + (g_{\gamma\phi}/g_{\gamma\omega} - g_{\gamma\omega}/g_{\gamma\phi}) \Gamma_{\phi\omega P'}}, \quad (17)$$

where use is made of (7) by setting $V_0 = \rho_0, \omega$, or ϕ , and of Eqs. (8)–(10) with $T = P'$.

Now applying the assumption that at zero momentum transfer the $SU(3)$ -symmetry breaking is dominated by the nonet of tensor-meson trajectories, we can then relate the residue functions $\Gamma_{VVP'}$ to the changes of the mass-squared matrix of vector mesons resulting from the perturbation of medium strong interactions:

$$\lambda \Gamma_{\rho_0\rho_0P'} = \delta M_{33}^2, \quad (18)$$

$$\lambda \begin{pmatrix} \Gamma_{\phi\phi P'} & \Gamma_{\phi\omega P'} \\ \Gamma_{\omega\phi P'} & \Gamma_{\omega\omega P'} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \times \begin{pmatrix} \delta M_{88}^2 & \delta M_{80}^2 \\ \delta M_{08}^2 & \delta M_{00}^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1}, \quad (19)$$

where λ is a constant and θ is the ϕ - ω mixing angle. The change of mass-squared matrix can be expressed in terms of the physical masses of the vector mesons as follows:

$$\delta M_{33}^2 = -\delta M_{88}^2 = \frac{2}{3}(m_{\rho}^2 - m_{K^*}^2), \quad (20)$$

$$\delta M_{80}^2 = \delta M_{08}^2 = -[(m_8^2 - m_{\omega}^2)(m_{\phi}^2 - m_8^2)]^{1/2}, \quad (21)$$

$$\delta M_{00}^2 = 0, \quad (22)$$

where, from the Gell-Mann-Okubo mass formula,

$$m_8^2 = \frac{1}{3}(4m_{K^*}^2 - m_{\rho}^2). \quad (23)$$

⁷ S. Okubo, Phys. Rev. Letters **5**, 165 (1963).

⁸ V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

From (18)–(22), we get finally

$$\lambda\Gamma_{\rho_0\rho_0P'} = \frac{2}{3}(m_{\rho^2} - m_{K^{*2}}), \quad (24)$$

$$\lambda\Gamma_{\omega\omega P'} = \frac{2}{3}\sin^2\theta (m_{K^{*2}} - m_{\rho^2}) - \sin 2\theta [(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}, \quad (25)$$

$$\lambda\Gamma_{\phi\phi P'} = \lambda\Gamma_{\omega\phi P'} = \frac{1}{3}\sin 2\theta (m_{K^{*2}} - m_{\rho^2}) - \cos 2\theta [(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}, \quad (26)$$

$$\lambda\Gamma_{\phi\phi P'} = \frac{2}{3}\cos^2\theta (m_{K^{*2}} - m_{\rho^2}) + \sin 2\theta [(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}. \quad (27)$$

By inserting (24)–(27) into (16) and (17), we then obtain two equations independent of λ , which relate the differential cross sections of the forward photoproduction of neutral vector mesons at very high energy:

$$\frac{g_{\gamma\omega}^2(d\sigma_{\rho_0p}/dt + d\sigma_{\rho_0n}/dt)_{t=0} - g_{\gamma\rho_0}^2(d\sigma_{\omega p}/dt + d\sigma_{\omega n}/dt)_{t=0}}{g_{\gamma\phi}^2(d\sigma_{\rho_0p}/dt + d\sigma_{\rho_0n}/dt)_{t=0} - g_{\gamma\rho_0}^2(d\sigma_{\phi p}/dt + d\sigma_{\phi n}/dt)_{t=0}} = \frac{g_{\gamma\omega}^2[2(1+\sin^2\theta) + (g_{\gamma\phi}/g_{\gamma\omega})\sin 2\theta](m_{K^{*2}} - m_{\rho^2}) - 3[\sin 2\theta + (g_{\gamma\phi}/g_{\gamma\omega})\cos 2\theta][(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}}{g_{\gamma\phi}^2[2(1+\cos^2\theta) + (g_{\gamma\omega}/g_{\gamma\phi})\sin 2\theta](m_{K^{*2}} - m_{\rho^2}) + 3[\sin 2\theta - (g_{\gamma\omega}/g_{\gamma\phi})\cos 2\theta][(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}} \quad (28)$$

and

$$\frac{g_{\gamma\omega}^2\left(\frac{d\sigma_{\rho_0p}}{dt} + \frac{d\sigma_{\rho_0n}}{dt}\right)_{t=0} - g_{\gamma\rho_0}^2\left(\frac{d\sigma_{\omega p}}{dt} + \frac{d\sigma_{\omega n}}{dt}\right)_{t=0}}{g_{\gamma\phi}^2\left(\frac{d\sigma_{\omega p}}{dt} + \frac{d\sigma_{\omega n}}{dt}\right)_{t=0} - g_{\gamma\omega}^2\left(\frac{d\sigma_{\phi p}}{dt} + \frac{d\sigma_{\phi n}}{dt}\right)_{t=0}} = \frac{\left(2(1+\sin^2\theta) + \frac{g_{\gamma\phi}}{g_{\gamma\omega}}\sin 2\theta\right)(m_{K^{*2}} - m_{\rho^2}) - 3\left(\sin 2\theta + \frac{g_{\gamma\phi}}{g_{\gamma\omega}}\cos 2\theta\right)[(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}}{g_{\gamma\phi}^2\left[2\cos 2\theta + \left(\frac{g_{\gamma\omega}}{g_{\gamma\phi}} - \frac{g_{\gamma\phi}}{g_{\gamma\omega}}\right)\sin 2\theta\right](m_{K^{*2}} - m_{\rho^2}) + 3\left[2\sin 2\theta + \left(\frac{g_{\gamma\phi}}{g_{\gamma\omega}} - \frac{g_{\gamma\omega}}{g_{\gamma\phi}}\right)\cos 2\theta\right][(m_8^2 - m_{\omega^2})(m_{\phi^2} - m_8^2)]^{1/2}}. \quad (29)$$

The validity of the above two equations serves as a test of checking the idea of dominance of tensor nonet trajectories in $SU(3)$ -symmetry breaking at zero momentum transfer. However, for the time being, owing to the lack of sufficient data on the high-energy photoproduction of neutral vector mesons off neutrons, we are unable to check their validity experimentally.

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