Narrow-Resonance Duality and the Conspiracy versus Absorption Dispute in Photoproduction*

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On the basis of narrow-resonance duality, we show that the finite-energy sum rules for forward photoproduction should be dominated by the resonant or nonresonant part, depending on whether the dominant Regge singularity is a conspiring pole or an absorptive cut. The physical situation clearly favors a conspiracy solution, although not necessarily the one involving conspiracy for the physical pion. Other applications of this approach are outlined.

MANY attempts have been made in the recent past to construct a Veneziano representation for the forward photoproduction of charged pions, and to infer from that the reaction mechanism responsible for the forward peak in this process.[1–4] The situation, however, has remained confusing, so much so that many of these works end up attributing the peak to contradictory reaction mechanisms. It is hoped that the present analysis helps to clarify this situation and suggests a general method for estimating the relative importance of the Regge-pole and Regge-cut contributions, on the basis of narrow-resonance duality or, equivalently, the general Veneziano representation.[5]

The hypothesis of narrow-resonance duality postulates an equivalence between the resonance contributions in one channel and the Regge-pole contributions in the crossed channel. This means that in the narrow-resonance limit, the scattering amplitude should be saturated by poles only (infinite sets of them, of course) in the complex angular momentum plane. However, the physical scattering amplitude has finite-width resonances and also a nonresonant background, which it must have in order to satisfy unitarity. But it is hoped that the physical amplitude would be described adequately by a unitarized narrow-resonance model, in which the nonresonant part would come from the iterations of the resonance-pole terms.[6] On the other hand, in a variety of models like those with absorption and rescattering, the cuts in the angular momentum plane are built out of the iterations of the Regge-pole terms—the iteration procedure again following from unitarity in some approximate form. Thus, the equivalence between the resonance and the Regge-pole contributions suggests an equivalence between the Regge cuts and the nonresonant background. This latter equivalence may be only qualitative in view of the model dependence of the iteration procedures involved. But the first equivalence should be more or less exact if one believes that the narrow-resonance model (general Veneziano representation) represents a first approximation to the physical scattering amplitude. Thus, in the presence of nonresonant background, the resonant part of the physical amplitude should be given by the pole terms in the crossed-channel angular momentum plane.

A recent analysis of the ρ-pole and ρ-Pomeranchukon cut contributions to π-π scattering by Drago[7] suggests an alternative mechanism, where the direct-channel resonances are built up by the leading Regge-pole and cut contributions of the crossed channel—the cut playing the role played by the infinite array of secondary poles in the Veneziano representation. This evidently runs contrary to the narrow-resonance duality hypothesis. In particular, the resonance poles of this model would vanish in the narrow-resonance limit, since the absorptive cuts—and perhaps any *moving* cuts in the angular momentum plane—must vanish in the meromorphic limit. It is encouraging, therefore, from the viewpoint of narrow-resonance duality, that a recent analysis[8] of the angular momentum co ntent of the bump structures observed by Drago makes their identification with the ρ and f resonances very unlikely. We shall assume in the following that the direct-channel resonances build up the Regge-*pole* contributions of the crossed channel through the general Veneziano representation.

We shall write the narrow-resonance dual amplitude as an infinite sum of Veneziano terms. Of course, the convergence of this infinite series requires the additional assumption of planarity. This is irrelevant for our purpose, however, since both planar and nonplanar[9] dual amplitudes satisfy the finite-energy sum rule (FESR), which is all that we shall need.

The following combination of the invariant photoproduction amplitudes[10] couples only to the trajectories

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[1] M. Ahmad, Fayyazuddin, and Riazuddin, Phys. Rev. Letters **23**, 504 (1969).
[2] I. Bender and H. J. Rothe, Nuovo Cimento Letters **2**, 477 (1969).
[3] F. Drago, Nuovo Cimento Letters **2**, 712 (1969).
[4] J. P. Ader, M. Capdeville, and Ph. Salin (unpublished).
[5] G. Veneziano, Nuovo Cimento **57A**, 190 (1968). By general we mean the Veneziano representation containing an infinite set of terms.
[6] For two specific approaches, see K. Kikkawa, B. Sakita and M. Virasoro, Phys. Rev. **184**, 1701 (1969); and C. Lovelace, Nucl. Phys. **B12**, 253 (1969).
[7] F. Drago, Phys. Rev. Letters **24**, 622 (1970).
[8] J. P. Holden and R. L. Thews, Phys. Rev. D **1**, 1332 (1970).
[9] M. A. Virasoro, Phys. Rev. **177**, 2309 (1969).
[10] J. Ball, W. Frazer, and M. Jacob, Phys. Rev. Letters **20**, 518 (1968).

with the quantum numbers of pion:

$$f^{(-)}=A_1^{(-)}+tA_2^{(-)}. \tag{1}$$

The general Veneziano representation for this amplitude has the form

$$\begin{aligned}f_R=&-eg\alpha'^2t\frac{\Gamma(-\alpha_\pi(t))\Gamma(\frac{1}{2}-\alpha_N(s))}{\Gamma(\frac{3}{2}-\alpha_\pi(t)-\alpha_N(s))}\\&+\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{l\geqslant n+1,m}^{\leqslant n+m}\left[A_l^{m,n}\frac{\Gamma(m-\alpha_\pi(t))\Gamma(\frac{1}{2}+n-\alpha_N(s))}{\Gamma(\frac{1}{2}+l-\alpha_\pi(t)-\alpha_N(s))}+B_l^{m,n}\frac{\Gamma(m-\alpha_\pi(t))\Gamma(\frac{1}{2}+n-\alpha_\Delta(s))}{\Gamma(\frac{1}{2}+l-\alpha_\pi(t)-\alpha_\Delta(s))}\right]\\&+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{l\geqslant n,m}^{\leqslant n+m}\left[C_l^{m,n}\frac{\Gamma(\frac{1}{2}+m-\alpha_N(u))\Gamma(\frac{1}{2}+n-\alpha_N(s))}{\Gamma(1+l-\alpha_N(u)-\alpha_N(s))}+D_l^{m,n}\frac{\Gamma(\frac{1}{2}+m-\alpha_\Delta(u))\Gamma(\frac{1}{2}+n-\alpha_\Delta(s))}{\Gamma(1+l-\alpha_\Delta(u)-\alpha_\Delta(s))}\right.\\&\left.+E_l^{m,n}\frac{\Gamma(\frac{1}{2}+m-\alpha_\Delta(u))\Gamma(\frac{1}{2}+n-\alpha_N(s))}{\Gamma(1+l-\alpha_\Delta(u)-\alpha_N(s))}\right],\end{aligned} \tag{2}$$

where the gauge-invariant pion pole comes from the first term with α' as the universal trajectory slope. The N_α and N_γ must have a common trajectory in a dual model,[11] but Δ can be given a specific signature by suitable constraints on the coefficients. There are additional terms coming from the other baryon and pion-like trajectories.

Of course, f_R would not still describe the physical amplitude since it cannot accommodate any nonresonant contribution in the s channel or Regge cuts in the t channel. In particular, it would not contain the forward peak of the physical amplitude if the peak arose from the absorptive Regge cuts. But we expect f_R to describe adequately the resonant part of the physical amplitude f. We must have, for instance,

$$\sum_{m=1}^{\infty}A_m^{m,0}+\sum_{m=0}^{\infty}(C_m^{m,0}+E_m^{m,0})=-\tfrac{1}{2}eg\alpha' \tag{3}$$

in order to reproduce the nucleon Born term. Contrary to an earlier claim,[2] however, this does not imply a non-vanishing pion residue at $t=0$, as can be seen by extracting the leading t-channel Regge contribution from Eq. (2). It is, in fact, possible within the framework of planar duality to have a finite nucleon residue at $t=0$, in spite of all the t-channel Regge contributions vanishing there.

Thus, the above representation for f_R is sufficiently general to accommodate either a conspiring or an evasive pion. The two cases can, however, be distinguished by studying the finite-energy sum rule for f_R. Assuming the leading pion Regge pole to dominate f_R beyond $\nu=N$, we have

$$\frac{1}{\pi}\int_0^N \mathrm{Im} f_R(t)d\nu=\frac{1}{\pi}\frac{\beta_\pi(t)}{\alpha_\pi(t)}N^{\alpha_\pi(t)}+C(N,t)\,, \tag{4}$$

where

$$\nu=(s-u)/4m$$

and $C(N,t)$ represents the contribution of daughters and other low-lying poles. The scale factor has been set equal to unity for simplicity of notation. Since f_R is supposed to represent the resonant part of the physical amplitude f, the left-hand integral is built up by the residues of the actual resonances lying below $\nu=N$. Of course, in view of the finite widths of the resonances, there is some ambiguity involved in extrapolating the resonance contributions from the pole points. The sensitivity of the integral to such ambiguities can be estimated, however, by comparing it with the integral evaluated in the narrow-resonance approximation.

In the case of an evasive pion the dominant term on the right-hand side vanishes at $t=0$. This would imply a major cancellation between the dominant resonance contributions to the left-hand integral[12] at $t=0$. If, on the other hand, the pion chooses conspiracy, the dominant term on the right-hand side would increase steadily as $(-t)$ decreases to zero. This would imply the dominant resonance contributions to the integral to be additive at $t=0$.

Table I shows the individual resonance contributions to the above integral, based on Walker's experimental parameters[13] for the resonances below 1.2 GeV incident momentum. The first row gives the resonance contributions, assuming the specific Breit-Wigner forms of Walker's parametrization (taken from Ref. 14). In the second row these contributions are evaluated in the narrow-resonance limit of the Breit-Wigner terms, for the purpose of comparison. In either case we see that

[11] D. P. Roy, Phys. Rev. Letters 23, 1417 (1969).

[12] An analogous case is the B^- amplitude in πN scattering, where a cancellation between the dominant resonance contributions seems to be associated with the vanishing of the ρ residue at or near $t\sim-0.5$ GeV^2. See R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).

[13] R. L. Walker, Phys. Rev. 182, 1729 (1969).

[14] D. P. Roy and S. Y. Chu, Phys. Rev. 171, 1762 (1968).

TABLE I. Individual resonance contributions, in GeV^{-1}, to the FESR $(1/\pi)\int \mathrm{Im}f(0)d\nu$, with the resonance parameters of Ref. 13. (I) Resonance and background contributions obtained with the specific Breit-Wigner parametrization of Ref. 13. (II) Resonance contributions in the narrow-resonance approximation.

Res. mass	N (940)	N^* (1236)	N^* (1471)	N^* (1519)	N^* (1561)	N^* (1652)	N^* (1672)	Background
I	1.05	0.331	0.025	0.017	−0.127	−0.002	−0.006	−0.026
II	1.05	0.399	0.032	0.022	−0.106	−0.002	−0.007	···

the dominant resonance terms add up at $t=0$. By considering the nucleon-pole contribution to the integral

$$-\frac{\alpha'}{4M}eg\frac{\mu^2+t}{\alpha_\pi(t)},$$

and the fact that the remainder *adds* to the nucleon contribution at $t=0$ and varies slowly with t, one deduces two general features of the resulting pole residue. Firstly, the residue at $t=0$ is more than half the value at the physical pion pole and, secondly, it vanishes below $t=-\mu^2$, due to the nucleon term changing sign—both the features agreeing with the conspiracy fit to high-energy photoproduction.[10] One also sees from Table I that the nonresonant contribution to the integral is, indeed, negligible compared to the resonant part. If the former is associated with Regge cuts even qualitatively, then this implies that the cut contributions are rather small for forward photoproduction.

It may appear drastic to have assumed in the above FESR that the leading Regge pole dominates over the lower-lying ones at a rather moderate value of N $(=1.2\ \mathrm{GeV})$. However, Jackson and Quigg[15] have shown that the net FESR (including resonant and nonresonant parts) for $N=1.2$ GeV, should indeed be dominated by Regge singularities, in the region $\alpha(0)\sim 0$, since the former accounts quantitively for the high-energy forward cross section. Then the narrow-resonance duality hypothesis and the fact that this FESR is dominated by the resonance contributions alone, suggest the dominant Regge singularity to be a pole (or poles) with $\alpha(0)\sim 0$. If, instead, the dominant Regge singularity were a branch point at $\alpha(0)\sim 0$, we would expect again the same magnitude of the FESR, but built mostly from the nonresonant background, following a cancellation between the nucleon and the higher-resonance terms.

It should be emphasized, however, that the present analysis cannot distinguish between the simplest conspiracy solution involving a conspiring pion and the more sophisticated solutions, e.g., the one involving an evasive physical pion and a conspiring pion-like trajectory overlapping on it.[16] Although Eq. (2) was formally written for the physical pion, one can easily add to it other pionlike contributions and then the subsequent considerations go through for the latter solution as well. One can perhaps invoke considerations like the factorization argument of Le Bellac[17] against the simple conspiracy model to distinguish between these solutions. But this is clearly outside the scope of the present work. All one can say on the basis of the narrow-resonance duality hypothesis is that the resonance domination of forward photoproduction sum rules favors the class of solutions involving a conspiring pole (or poles) in the complex l plane with pion quantum numbers, over those involving cuts.

Such a separation of the Regge-pole and Regge-cut effects, by studying the relative contributions of the resonant and nonresonant parts to the FESR, will, of course, be useful in a wider context. In πN scattering, for instance, one may be able to decide if the dip zero in B^- and the crossover zero in the A^- came from a vanishing of the pole part (either through a zero in the ρ residue or through a cancellation by lower-lying trajectories) or through a pole-cut cancellation. There seems to be some evidence of a cancellation among the leading resonance terms for B^- around the dip region,[12] which would disfavor the strong-cut interpretation of the dip as a pole-cut cancellation effect. For these situations, however, a more thorough analysis would be necessary, since these involve a cancellation among large terms. In particular, the point of cancellation is expected to be sensitive to the experimental resonance parameters and also to the ambiguities associated with the finite widths of the physical resonances.

[15] J. D. Jackson and C. Quigg, Phys. Letters **29B**, 236 (1969).

[16] G. Fox [quoted by G. F. Chew, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 410].

[17] M. Le Bellac, Phys. Letters **25B**, 524 (1967).