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Nature of Intrinsic Breakdown of Chiral Symmetry in Terms of Physical Fields

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A method for considering the intrinsic breakdown of chiral $SU(3)\times SU(3)$ symmetry is presented. The method is principally based on the knowledge of the physical masses of the particles whose fields transform linearly under the symmetry group. The exact Hamiltonian of asymptotic fields for the parity couples $(0^+,0^-)$ and $(1^+,1^-)$ is considered, and the nature of the resulting transformations is described. It is shown that the transformation does differ appreciably from the simple $(3,3^*)$ form, which has been recently assumed. It is found that the representations which include subgroups of $SU(3)$ beyond the singlet and octet are restricted by the physical masses but cannot all simultaneously be made to vanish. It is shown that a universal condition on the fields considered exists, which requires the vanishing of the commutator between the axial charge and the divergence of the $SU(3)$ vector generator for those specific cases in which $\Delta I>1$ and/or $\Delta Y>1$. This condition provides mass formulas among the scalar and pseudoscalar particles and among the vector and axial-vector particles which are in good agreement with experiment.

INTRODUCTION

In this paper we propose an approach to the problem of symmetry breaking in chiral $SU(3)\times SU(3)$ which is mainly based on the knowledge of the physical masses of the particles contained in the representations of the group. We work with the Hamiltonian expressed in terms of the physical (asymptotic) in-fields, since it is exactly known once the physical masses of the particles considered are experimentally determined. Our basic assumption is that these fields transform linearly under the $SU(3)\times SU(3)$ generators. This assumption, its basis, and its limitations are discussed in Sec. I.

We consider the effects of breaking of the symmetry from two different (but closely related) points of view. One consists in the determination of the transformation properties of the Hamiltonian itself, and the mass relations that make these transformations as simple as possible. We find that it is impossible to make the Hamiltonian transform simply as a $(3,3^*)\oplus(3^*,3)$ representation, as suggested by Gell-Mann, Oakes, and Renner.[1] In fact, $(3,3^*)$ seems to contribute relatively little in our scheme [cf. Eqs. (37) and (38)]. This is the content of Sec. II, where we also discuss the implications of spontaneous breakdown in our model. The second approach consists of investigating the mass constraints necessary for the vanishing of "exotic" (ΔI or $\Delta Y>1$) commutators; this requirement is slightly less restrictive than the one on the Hamiltonian, and is discussed in Sec. III. It is shown that it can be satisfied for commutators between vector charges and the time derivatives of axial charges, but not for commutators involving axial charges and their derivatives.

Finally, we compare our model to the model of asymptotic symmetry and explain the common features and disagreements between the two models in Sec. IV.

I. TRANSFORMATION OF IN-FIELDS

As was mentioned in the Introduction, the basic assumption of our model is the linear transformation property of physical in-fields under the chiral $SU(3)\times SU(3)$ group. In this section we shall give arguments for the plausibility of this assumption.

The linear transformation of asymptotic fields seems very reasonable in the limit of exact symmetry (i.e., time-independent generators), where it implies linear transformations for the state vectors.[2] It can easily be seen, however, that it cannot hold exactly in the physical world where the symmetry is broken; if it were correct, the generators would be bilinear in the physical fields, and several unsatisfactory results would follow. For example, if we define the form factors for K_{l3} decay as

$$(2q_0 2p_0)^{1/2}\langle\pi(q)\,|\,V_\mu\,|\,K(p)\rangle = f_+(t)(p+q)_\mu + f_-(t)(p-q)_\mu\,, \quad (1)$$

then the requirement that $T\equiv\int d^3x\, V_4(x)$ be bilinear in the fields implies

$$f_-(t)=0\,. \quad (2)$$

[Notice that this result is independent of the chiral nature of the group; it only depends on the vector $SU(3)$ generator.[3]]

The question therefore of the importance of nonlinear effects has to be examined more carefully. We shall show below that, for the matrix elements of commutators between generators and their time derivatives that are of interest to us, *the assumption of linear transformation of in-fields corresponds to the approximation where the second- (not first) and higher-order effects of symmetry-breaking interactions are disregarded.*[4] In what follows, $O(\epsilon^n)$ stands for the nth- or higher-order effects of the symmetry-breaking interaction.

[1] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[2] H. Umezawa, University of Wisconsin-Milwaukee technical report (unpublished); J. Lopuszánski, Dubna report, 1969 (unpublished), and references contained therein.

[3] It is interesting that $f_-(t)$ does not contribute to (1) in the limit $|\mathbf{p}|$, $|\mathbf{q}|\to\infty$. All the arguments of this paper can also be obtained under the assumption that linear transformations hold only for physical fields of infinite momentum. The connection of this scheme with asymptotic symmetry will be discussed elsewhere. A different approach to this problem, in which the $\lim|p|,|q|\to 0$, has recently been discussed by J. Rest and D. Welling, University of Wisconsin-Milwaukee Report No. 4867-71-1 (unpublished).

[4] This is a generalization of the Ademollo-Gatto theorem. M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).

Let us denote the $SU(3)\times SU(3)$ generators by G_α collectively. These generators in general are time dependent, and we are interested in the contribution of equal-time commutators of the form

$$[G_\alpha,\dot{G}_\beta]. \tag{3}$$

In particular, for applications, the matrix elements of (3) between single-particle states are most important, and we limit our discussion to such terms.

Denoting the annihilation operators of in-fields ϕ_i by $a_i(\mathbf{k})$, where $\mathbf{k}$ is the three-momentum, we can write their transformation as

$$[G_\alpha,a_i(\mathbf{k})]=i\sum_j u_{\alpha ij}(\mathbf{k})a_j(\mathbf{k})+\delta u_{\alpha i}(\mathbf{k}). \tag{4}$$

Here the right-hand side of (4) is a linear combination of normal products of a and $a^\dagger$: The first term picks up all the terms *linear* in a's (but not in $a^\dagger$'s) and the remainder is denoted by δu. *The δu manifest the first- and higher-order effects of symmetry-breaking interactions: $\delta u=O(\epsilon)$.*

By inspecting the vacuum expectation value of the Jacobi identity for $(a_i,G_\alpha,a_j^\dagger)$, we find a constraint,

$$u_{\alpha ij}^*(\mathbf{k})=-u_{\alpha ji}(\mathbf{k}). \tag{5}$$

Using this, we shall first prove that the contribution of δu [in (4)] to $a^\dagger a$ terms in $[G_\alpha,G_\beta]$ is of $O(\epsilon^2)$. To this end, let us recall that the Hamiltonian expressed in terms of the in-field creation and annihilation operators is of the form (see Sec. II)

$$H=\sum_i\int d^3k\,\omega_i(\mathbf{k})a_i^\dagger(\mathbf{k})a_i(\mathbf{k}). \tag{6}$$

Then (4) together with (5) leads to

$$\begin{aligned}(1/i)\dot{G}_\beta&=[G_\beta,H]\\&=w_\beta+\delta w_\beta,\end{aligned} \tag{7}$$

where

$$w_\beta=i\sum_{i,j}\int d^3k[\omega_i(\mathbf{k})-\omega_j(\mathbf{k})]u_{\beta ij}(\mathbf{k})a_i^\dagger(\mathbf{k})a_j(\mathbf{k}) \tag{8}$$

and

$$\delta w_\beta=\sum_i\int d^3k\,\omega_i(\mathbf{k})[a_i^\dagger(\mathbf{k})\delta u_{\beta i}(\mathbf{k})-\delta u_{\beta i}^\dagger(\mathbf{k})a_i(\mathbf{k})]. \tag{9}$$

It is an easy matter to show that $a^\dagger\delta u$ and $\delta u^\dagger a$ do not contain any $a^\dagger a$ terms when they are written as a linear combination of normal products. This means that δw_β does not have any $a^\dagger a$ terms, while w_β contains $a^\dagger a$ terms only. By a similar computation, we find that

$$\begin{aligned}[G_\alpha,w_\beta]=-\sum_j\sum_l\sum_i\int d^3k[\omega_i(\mathbf{k})-\omega_j(\mathbf{k})]\\ \times[u_{\beta ij}(\mathbf{k})u_{\alpha il}^*(\mathbf{k})a_l^\dagger(\mathbf{k})a_j(\mathbf{k})+\text{H.c.}]+\delta w_{\alpha\beta},\end{aligned} \tag{10}$$

where H.c. is the Hermitian conjugate of its preceding term and $\delta w_{\alpha\beta}$ contains $\delta u^\dagger a$ and $a^\dagger\delta u$ terms only. We are interested in $\delta w_{\alpha\beta}$ which does not contain any $a^\dagger a$ terms.

Since δw_β does not contain any $a^\dagger a$ terms, only the δu term in (4) creates $a^\dagger a$ terms in $[G_\alpha,\delta w_\beta]$. This means that the $a^\dagger a$ terms in $[G_\alpha,\delta w_\beta]$ are of $O(\epsilon^2)$ because δw_β itself is of $O(\epsilon)$. We have thus proved that $a^\dagger a$ terms in $[G_\alpha,\dot{G}_\beta]$ are given by

$$\begin{aligned}-i\sum_j\sum_l\sum_i\int d^3k[\omega_i(\mathbf{k})-\omega_j(\mathbf{k})]\\ \times[u_{\beta ij}(\mathbf{k})u_{\alpha il}^*(\mathbf{k})a_l^\dagger(\mathbf{k})a_j(k)+\text{H.c.}]+O(\epsilon^2),\end{aligned} \tag{11}$$

where $O(\epsilon^2)$ comes from $[G_\alpha,\delta w_\beta]$.

Let us now note that $u_{\alpha ij}(\mathbf{k})$ becomes independent of $\mathbf{k}$ in the limit $\delta u\to 0$. Therefore, we can write

$$u_{\alpha ij}(k)=u_{\alpha ij}{}^0+O(\epsilon), \tag{12}$$

where $u_{\alpha ij}{}^0$ is independent of $\mathbf{k}$.[5]

Since the mass differences are of $O(\epsilon)$, we can write

$$\begin{aligned}\omega_i(k)-\omega_j(k)&=\frac{m_i^2-m_j^2}{\omega_i(\mathbf{k})+\omega_j(\mathbf{k})}\\&=\frac{m_i^2-m_j^2}{2(\mathbf{k}^2+\bar{m}^2)^{1/2}},\end{aligned} \tag{13}$$

where $\bar{m}$ is the average mass among the linear multiplet of the in-fields ϕ_i.

We thus find that the $a^\dagger a$ contribution to (3) is proportional to the term

$$\sum_i(m_i^2-m_j^2)u_{\beta ij}{}^0u_{\alpha il}{}^{0*}, \tag{14}$$

in the approximation where we neglect the effects of $O(\epsilon^2)$.

In the following, we are mainly interested in the linear transformation of in-fields ϕ_i rather than the creation operators

$$[G_\alpha,\phi_i]=i\sum_j u_{\alpha ij}{}^0\phi_j. \tag{15}$$

This does not mean that $\delta u=0$ [cf. (4)], but rather that δu is linear in $a^\dagger$:

$$\delta u_{\alpha i}(\mathbf{k})=v_{\alpha ij}(\mathbf{k})a_j^\dagger(\mathbf{k}). \tag{16}$$

As we have seen, existence of δu modifies (14) only by terms of $O(\epsilon^2)$. This proves that *the linear transformation (15) applied to (3) corresponds to the approximation where symmetry-breaking effects of $O(\epsilon^2)$ are disregarded.*

We can further show that, when the ϕ_i's are bosons, use of the linear transformation (15) together with the condition (3) exactly leads to the form (14). On the other hand, when the ϕ_i's are fermions, (3) and (15) give us a result which is different from (14) by terms of

[5] The $u_{\alpha ij}{}^0$'s contain the mixing angle in general.

$O(\epsilon^2)$. To show this, we note that $v(\mathbf{k})$ in (16) has the following asymptotic form[6]:

$$v_{\alpha ij}(\mathbf{k}) \propto |\mathbf{k}|^{-n} \quad \text{for } |\mathbf{k}|\to\infty\,, \tag{17}$$

where

$$n=\begin{cases}2 \text{ for boson}\\ 1 \text{ for fermion}.\end{cases} \tag{18}$$

Since $\delta u(\mathbf{k})$ vanishes at infinite $|\mathbf{k}|$ according to (17), we have

$$u_{\alpha ij}(\mathbf{k})=u_{\alpha ij}^{0}(\mathbf{k}) \quad \text{for } |\mathbf{k}|\to\infty\,. \tag{19}$$

It can be seen from (9), (16), and (17) that $a^\dagger(k)a(k)$ terms in $[G_\alpha,\delta w_\beta]$ are proportional to k^{-2n+1} for large k (i.e., k^{-3} for boson and k^{-1} for fermion), while $a^\dagger(k)a(k)$ terms in (10) can be written as

$$\frac{1}{2k}\sum_{j,l}\sum_{i}\left[(m_i^2-m_j^2)u_{\beta ij}{}^{0}u_{\alpha il}{}^{0*})a_l{}^\dagger(k)a_j(k)\right] \quad \text{for } |\mathbf{k}|\to\infty\,, \tag{20}$$

which is of order k^{-1}. [Remember that δw_β in (10) does not contain any $a^\dagger a$ term.] Noting that the quantity inside the bracket is just equal to the left-hand side of (14), we see that the contribution of $a^\dagger(k)a(k)$ terms with large $|\mathbf{k}|$ in $[G_\alpha,\dot{G}_\beta]$ is proportional to (14), when ϕ_i's are bosons. On the other hand, as is shown later, $(1/i)[G_\alpha,\dot{G}_\beta]$ computed under the assumption of the linear transformation (15) takes the form

$$\sum_{l,j} f_{lj}{}^{\alpha\beta}\int d^3x\, \phi_l(x)\phi_j(x)\,.$$

Identifying the $a^\dagger(k)a(k)$ terms with large $|\mathbf{k}|$ in this quantity with (20), we find that $f_{lj}{}^{\alpha\beta}$ must agree with the quantity in the bracket in (20). Since (3) now is proportional to $f_{lj}{}^{\alpha\beta}=0$, we conclude that *use of the linear transformation (15) in the computation (3) leads exactly to (14)*. When the ϕ_i's are fermions, (15) together with (3) leads to a result which differs from (14) by terms of $O(\epsilon^2)$. It is worthwhile to note that δu of the form (16) contributes to $[G_\alpha,\dot{G}_\beta]$ by the so-called Z diagrams. The above argument shows that the Z diagrams with infinite $|\mathbf{k}|$ do not contribute to $[G_\alpha,\dot{G}_\beta]$ when ϕ_i's are bosons, while they do contribute when ϕ_i's are fermions.

It is also interesting to note that, since our argument concentrated on terms of the type $a^\dagger a$ in $[G_\alpha,\dot{G}_\beta]$, it does not apply to the matrix element (1) which gets first-order contributions in the symmetry breaking. Therefore, a large value of $f_-(t)$ does not necessarily imply that terms of $O(\epsilon^2)$ are also large.

II. HAMILTONIAN APPROACH

A. Scalar-Pseudoscalar System

The strong interaction Hamiltonian for the scalar-pseudoscalar system, when expressed in terms of the physical (asymptotic) scalar and pseudoscalar fields, is of the form

$$H=(\text{kinetic energy})+\tfrac{1}{2}m_i^2P_iP_i+\tfrac{1}{2}M_i^2S_iS_i\,, \tag{21}$$

where (m_i,P_i) and (M_i,S_i) are the masses and fields of the pseudoscalar and scalar mesons, respectively. Summation over repeated indices is implied.

As mentioned in the previous section, *our basic assumption is that these fields transform linearly under the $SU(3)\times SU(3)$ generators.*

We also showed that the effects of nonlinearity in the physical field transformation are one order higher in the symmetry breaking when compared with the effects of mass splitting; therefore, we believe our assumption to be reasonable to first order in the symmetry breaking in the perturbative approach commonly used in discussing such phenomena.

The question still remains as to which representation to choose for the scalar-pseudoscalar system. Given the existence of nine pseudoscalar mesons, the natural choice would be either $(3,3^*)+(3^*,3)$ or $(8,1)+(1,8)+(1,1)$. However, charge-conjugation considerations exclude the latter possibility, and we are led to the following commutation relations:

$$\begin{aligned}&[T_A,S_i]=if_{Aij}S_j\,, && [T_A,P_j]=if_{Ajk}P_k\,,\\ &[X_A,S_j]=-id_{Ajk}P_k\,, && [X_A,P_j]=id_{Ajk}S_k\,,\end{aligned} \tag{22a}$$

where T_A, X_A are the vector and axial-vector generators, respectively, and where the indices A run from 1 to 8, while i and j run from 0 to 8. At the end of this section, the possibility of spontaneous breaking of the symmetry will be considered; in that case, a c-number term would be added to the right-hand side of the first and last relation in (22a). We would like to emphasize that (22a) is the only choice consistent with the existence of only nine pseudoscalar mesons.

With the commutation relations (22a) it is straightforward to calculate the transformation properties of the Hamiltonian given by (21). Despite the presence of time derivatives and the fact that the generators are time dependent, it can be shown that the kinetic-energy term is invariant under any group transformation, provided the transformation is linear. The only noninvariance therefore results from the mass term in (21), and can be expressed in terms of the physical masses m_i and M_i. This is the purpose of this section.

We assume both nonets to be mixed, and define the mixing angles in the following way:

$$\begin{aligned}&\eta=\cos\phi\, P_8+\sin\phi\, P_0\,, && \eta'=-\sin\phi\, P_8+\cos\phi\, P_0\,,\\ &\sigma=\cos\theta\, S_8+\sin\theta\, S_0\,, && \sigma'=-\sin\theta\, S_8+\cos\theta\, S_0\,.\end{aligned} \tag{22b}$$

To fix θ, we define $M_\sigma > M_{\sigma'}$.

The transformations imposed by (22b) require (21) to be of the form

[6] This can be seen by inspecting the structure of the generators G_α, which generate the linear transformation (15).

$$\begin{aligned}H=H_0&+\{A_{33}[(d,\bar d)+(u,\bar u)]+B_{33}(s,\bar s)+\text{c.c.}\}\\&+\{A_{66}[(M_{11},M^{11})+(M_{22},M^{22})+(M_{12},M^{12})]\\&+B_{66}[(M_{13},M^{13})+(M_{23},M^{23})]\\&+C_{66}(M_{33},M^{33})+\text{c.c.}\}\\&+\{A_{88}[(M_1{}^1,M_1{}^1)+(M_2{}^2,M_2{}^2)]\\&+B_{88}[(M_1{}^1,M_2{}^2)+(M_2{}^2,M_1{}^1)]\\&+(-B_{88}+A_{88})[(M_2{}^1,M_1{}^2)+(M_1{}^2,M_2{}^1)]\\&+(A_{88}+B_{88})[2(M_3{}^3,M_3{}^3)-(M_3{}^3,M_1{}^1)\\&-(M_1{}^1,M_3{}^3)-(M_3{}^3,M_2{}^2)-(M_2{}^2,M_3{}^3)]\\&+C_{88}[(M_1{}^3,M_3{}^1)+(M_2{}^3,M_3{}^2)\\&+(M_3{}^1,M_1{}^3)+(M_3{}^2,M_2{}^3)]\}\\&+\{A_{18}[(2M_3{}^3-M_1{}^1-M_2{}^2,M_\alpha{}^\alpha)+\text{c.c.}]\}\,.\end{aligned}\quad(23)$$

We shall use the following notation: (u,d,s) stand for the members of the quark triplet; $M_{\alpha\beta}$ ($M^{\alpha\beta}$) stand for the members of the 6 (6*) representation of $SU(3)$. Therefore $M_{\alpha\beta}=M_{\beta\alpha}$. The indices run from 1 to 3. $M_\beta{}^\alpha$ stands for the members of the $SU(3)$ nonet, and $M_\alpha{}^\alpha$ for the corresponding trace. The first (second) term in the parenthesis (A,B) refers to the transformation property under the left (right) $SU(3)$, of the chiral $SU(3)\times SU(3)$. Finally, c.c. refers to interchange of A and B in the parenthesis (A,B). H_0, the invariant part of the Hamiltonian, has the form

$$H_0=(\text{kinetic-energy term})+A_{00}(M_\alpha{}^\alpha,M_\beta{}^\beta)\,.\quad(24)$$

The coefficients appearing in (23) and (24) are expressed in terms of the scalar and pseudoscalar masses and the mixing angles as follows:

$$2A_{00}=\tfrac19(8M_\kappa{}^2+6M_\delta{}^2+2M_\sigma{}^2+2M_{\sigma'}{}^2)+(S\to P)\,,\quad(25\text{a})$$

$$2A_{33}=M_\kappa{}^2+\tfrac13(M_\sigma{}^2-M_{\sigma'}{}^2)\times[\cos2\theta-(1/2\sqrt2)\sin2\theta]-(S\to P)\,,\quad(25\text{b})$$

$$2B_{33}=\tfrac32M_\delta{}^2-\tfrac14(M_\sigma{}^2+M_{\sigma'}{}^2)+\tfrac1{12}(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)-(S\to P)\,,\quad(25\text{c})$$

$$2A_{66}=\tfrac12M_\delta{}^2+\tfrac14(M_\sigma{}^2+M_{\sigma'}{}^2)-\tfrac1{12}(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)-(S\to P)\,,\quad(25\text{d})$$

$$2B_{66}=M_\kappa{}^2-\tfrac13(M_\sigma{}^2-M_{\sigma'}{}^2)\times[\cos2\theta-(1/2\sqrt2)\sin2\theta]-(S\to P)\,,\quad(25\text{e})$$

$$2C_{66}=\tfrac12(M_\sigma{}^2+M_{\sigma'}{}^2)+\tfrac16(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)-(S\to P)\,,\quad(25\text{f})$$

$$2A_{88}=(1/18)[-8M_\kappa{}^2-6M_\delta{}^2+7(M_\sigma{}^2+M_{\sigma'}{}^2)-(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)]+(S\to P)\,,\quad(25\text{g})$$

$$2B_{88}=(1/18)[-8M_\kappa{}^2+12M_\delta{}^2-2(M_\sigma{}^2+M_{\sigma'}{}^2)+2(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)]+(S\to P)\,,\quad(25\text{h})$$

$$2C_{88}=-\tfrac23(M_\sigma{}^2-M_{\sigma'}{}^2)\times[\cos2\theta-(1/2\sqrt2)\sin2\theta]+(S\to P)\,,\quad(25\text{i})$$

$$2A_{18}=(1/18)[4M_\kappa{}^2-6M_\delta{}^2+(M_\sigma{}^2+M_{\sigma'}{}^2)+(M_\sigma{}^2-M_{\sigma'}{}^2)\times(\cos2\theta+2\sqrt2\sin2\theta)]+(S\to P)\,,\quad(25\text{j})$$

where $(S\to P)$ implies the expression preceding it with scalar masses and mixing angle replaced by their pseudoscalar counterparts.

Of the different terms appearing in (23), those in the first curly bracket transform according to the (3,3*) + (3*,3) representation, those in the second according to (6,6*) + (6*,6), those in the third according to (8,8), and those in the fourth according to (8,1) + (1,8). The only other possible representations in the product $[(3,3^*)+(3^*,3)]\otimes[(3^*,3)+(3,3^*)]$, namely, (6,3) and (3*,6*), are forbidden by Bose statistics in our case.

The next step is to find the maximum simplification possible for the Hamiltonian as expressed by (23). This simplification can proceed in two directions: We try to choose the scalar masses (which we treat as unknown parameters) in such a way that the transformation properties of H under $SU(3)\times SU(3)$ become as simple as possible, or we can see if, besides the simplicity of the $SU(3)\times SU(3)$ transformation, we are able to make the ordinary $SU(3)$ transformation properties of the different terms in (23) particularly simple. Since (3,3*) and (8,1) are the only representations in (23) that do not contain representations of $SU(3)$ higher than the octet, we try to make the (6,6*) and (8,8) terms singlet under $SU(3)$, so that all the time derivatives of the axial generators would transform like singlets or octets. The results are as follows.

(1) Requiring that H transform as a mixture of singlet and octet under $SU(3)$, i.e., assuming that the scalar and pseudoscalar masses satisfy the Gell-Mann–Okubo mass formula with mixing, the following constraints are imposed on the coefficients (25a)–(25j):

$$A_{66}-2B_{66}+C_{66}=0\,,\quad(26)$$

$$5A_{88}+4B_{88}-8C_{88}=0\,.\quad(27)$$

(2) It is very unlikely that (6,6*) and (6*,6) are absent from the Hamiltonian. For this to happen we must have

$$M_\delta{}^2+M_\sigma{}^2+M_{\sigma'}{}^2=m_\pi{}^2+m_\eta{}^2+m_{\eta'}{}^2\,,$$

which is incompatible with the belief that the mass of the scalar mesons should be around 1 BeV.

(3) The (6,6*) and (6*,6) contribution to H can be made an $SU(3)$ singlet, however, provided the following condition holds:

$$2(M_\delta{}^2-M_\kappa{}^2-m_\pi{}^2+m_K{}^2)-(3/\sqrt2)(M_\sigma{}^2-M_{\sigma'}{}^2)\times\sin2\theta+(3/\sqrt2)(m_\eta{}^2-m_{\eta'}{}^2)\sin2\phi=0\,.\quad(28)$$

(4) The (8,8) contribution to the Hamiltonian can be made an $SU(3)$ singlet if

$$-4(M_\kappa{}^2+m_K{}^2-M_\delta{}^2-m_\pi{}^2)+\tfrac32\sqrt2(M_\sigma{}^2-M_{\sigma'}{}^2)\times\sin2\theta+\tfrac32\sqrt2(m_\eta{}^2-m_{\eta'}{}^2)\sin2\phi=0\,.\quad(29)$$

If both (28) and (29) hold simultaneously, the time derivative of the axial generators will be a mixture of singlet and octet under $SU(3)$.

(5) *It is impossible to eliminate simultaneously the entire (8,8) and (6,6*)⊕(6*,6) contributions in H, for any choice of the scalar masses.* Therefore, the assumptions of Ref. (1) are not borne out in our model.

(6) It is impossible simultaneously to eliminate $(6,6^*)\oplus(6^*,6)$ completely and to make the (8,8) contribution singlet under $SU(3)$. This result is also independent of the choice of scalar masses.

(7) The contribution of (8,8) disappears completely and $(6,6^*)\oplus(6^*,6)$ becomes an $SU(3)$ singlet if the following set of relations is satisfied:

$$M_\delta^2-M_\kappa^2=\tfrac{1}{3}(m_K^2-m_\pi^2)-(1/\sqrt{2})(m_\eta^2-m_{\eta'}^2)\sin 2\phi\,, \tag{30}$$

$$2M_\delta^2-(M_\sigma^2+M_{\sigma'}^2)=-\tfrac{4}{3}m_K^2-\tfrac{2}{3}m_\pi^2+(m_\eta^2+m_{\eta'}^2)-\sqrt{2}(m_\eta^2-m_{\eta'}^2)\sin 2\phi\,, \tag{31}$$

$$\tan 2\theta=\frac{8\sqrt{2}(m_K^2-m_\pi^2)+3(m_\eta^2-m_{\eta'}^2)\sin 2\phi}{-20m_K^2+2m_\pi^2+9(m_\eta^2+m_{\eta'}^2)+(6/\sqrt{2})(m_\eta^2-m_{\eta'}^2)\sin 2\phi}\,. \tag{32}$$

Substituting the masses of the pseudoscalar mesons in these relations, we obtain the two sets of solutions:

$$\begin{aligned}&\theta=9.4^\circ\,,\quad M_\delta^2-M_\kappa^2=0.24\,,\quad 2M_\delta^2-(M_\sigma^2+M_{\sigma'}^2)=1.21\,;\\ &\theta=11.7^\circ\,,\quad M_\delta^2-M_\kappa^2=-0.08\,,\quad 2M_\delta^2-(M_\sigma^2+M_{\sigma'}^2)=0.57\,.\end{aligned} \tag{33}$$

The upper values correspond to positive mixing angle ($\phi=10.9°$) for the pseudoscalar mesons, the lower to negative. If we introduce the value of the σ mass in these relations,

$$M_\sigma^2=1.14\ \mathrm{BeV}^2\,, \tag{34}$$

we obtain the following values for the other masses:

$$\begin{aligned}&M_\delta^2=1.44\ \mathrm{BeV}^2\,,\quad M_{\sigma'}^2=0.53\ \mathrm{BeV}^2\,,\quad M_\kappa^2=1.20\ \mathrm{BeV}^2\,;\\ &M_\delta^2=0.99\ \mathrm{BeV}^2\,,\quad M_{\sigma'}^2=0.27\ \mathrm{BeV}^2\,,\quad M_\kappa^2=1.07\ \mathrm{BeV}^2\,.\end{aligned} \tag{35}$$

We believe that the second solution ($\phi<0$) is the one most likely to be true, for the following reasons.

(i) The mass of the δ meson, which is reasonably well known, is quite close to the one obtained here ($M_\delta^2=0.93$ BeV^2).

(ii) The scalar mixing angle is approximately equal in magnitude and opposite in sign from the pseudoscalar one.

It is also interesting that the mass of the σ' falls halfway between the masses of the $\sigma(410)$ and the $\sigma(750)$ reported in the literature. We want to point out too that *mixing is essential for this situation to arise.* Thus the mixing effects are very important, even though the mixing angles are quite small.

With the choice (34) and (35) for the scalar masses, the coefficients (25a)–(25j) take the values

$$A_{00}=1.21\ \mathrm{BeV}^2\,, \tag{36a}$$

$$A_{33}=0.64\ \mathrm{BeV}^2\,, \tag{36b}$$

$$B_{33}=0.78\ \mathrm{BeV}^2\,, \tag{36c}$$

$$C_{66}=B_{66}=A_{66}=0.19\ \mathrm{BeV}^2\,, \tag{36d}$$

$$A_{18}=0.1\ \mathrm{BeV}^2\,, \tag{36e}$$

$$A_{88}=B_{88}=C_{88}=0\,. \tag{36f}$$

A problem arises when we try to find the relative importance of the different representations in H, because of the arbitrary relative normalization of inequivalent $SU(3)\times SU(3)$ representations. We believe the best approach is to separate the octet and singlet parts of H under $SU(3)$ and to compare the different contributions to each part separately. The contributions to the octet part can be calculated in a straightforward way from (36a)–(36f):

$$H(\text{octet})=0.09[(3,3^*)\oplus(3^*,3)]+0.42[(8,1)\oplus(1,8)]\,. \tag{37}$$

For the singlet part the extra complication of the kinetic energy term arises; we evaluate it at low momentum and approximate it with the mean mass of scalars and pseudoscalars. Then

$$H(\text{singlet})=8.01[(1,1)]+1.19[(3,3^*)\oplus(3^*,3)]+0.34[(6,6^*)\oplus(6^*,6)]\,. \tag{38}$$

It is interesting that the violation of symmetry in (38) is of the order of 15%, very close to the magnitude of $SU(3)$ violation.

B. Vector–Axial-Vector System

The Hamiltonian for this system will have the form

$$H=(\text{kinetic energy})+\tfrac{1}{2}m_i^2V_iV_i+\tfrac{1}{2}M_i^2A_iA_i\,, \tag{39}$$

where V_i (A_i) and m_i (M_i) stand for the physical vector (axial-vector) fields and their respective masses.

Again arguments based on charge conjugation and dimensionality imply that the V_i's and A_i's transform according to the $(8,1)\oplus(1,8)\oplus(1,1)$ representation of $SU(3)\times SU(3)$:

$$
\begin{aligned}
[T_A,V_{i,\mu}]&=if_{Aij}V_{j,\mu}\,,\\
[X_A,V_{i,\mu}]&=if_{Aij}A_{j,\mu}\,,\\
[T_A,A_{j,\mu}]&=if_{Aij}A_{j,\mu}\,,\\
[X_A,A_{i,\mu}]&=if_{Aij}V_{j,\mu}\,.
\end{aligned}
\tag{40}
$$

Then the representations appearing in (39) are

$$(1,1),\ (8,1),\ (1,8),\ (8,8),\ (27,1),\ (1,27)\,, \tag{41}$$

where use was made of Bose symmetry to eliminate antisymmetric representations. We would like to point out the absence of $(3,3^*)$ and $(3^*,3)$ in (41). If we assume that the vector and axial-vector masses satisfy the Gell-Mann–Okubo formula (with mixing), the last two representations are eliminated and H takes the form

$$
\begin{aligned}
H=H_0+\{&A_{88}[(M_1{}^1,M_1{}^1)+(M_2{}^2,M_2{}^2)]\\
&+B_{88}[(M_1{}^1,M_2{}^2)+(M_2{}^2,M_1{}^1)]+(A_{88}-B_{88})\\
&\times[(M_2{}^1,M_1{}^2)+(M_1{}^2,M_2{}^1)]+(A_{88}+B_{88})\\
&\times[2(M_3{}^3,M_3{}^3)-(M_3{}^3,M_1{}^1)-(M_1{}^1,M_3{}^3)\\
&-(M_3{}^3,M_2{}^2)-(M_2{}^2,M_3{}^3)]+C_{88}[(M_1{}^3,M_3{}^1)\\
&+(M_2{}^3,M_3{}^2)+(M_3{}^1,M_1{}^3)+(M_3{}^2,M_2{}^3)]\}\\
&+\{A_{18}[(2M_3{}^3-M_1{}^1-M_2{}^2,\ M_\alpha{}^\alpha)+\text{c.c.}]\}\,,
\end{aligned}
\tag{42}
$$

$$H_0=(\text{kinetic energy})+A_{00}(M_\alpha{}^\alpha,M_\beta{}^\beta)\,. \tag{43}$$

We define mixing angles θ and ϕ as follows:

$$
\begin{aligned}
\omega&=\cos\theta\,\omega_8+\sin\theta\,\omega_1\,,\\
\phi&=-\sin\theta\,\omega_8+\cos\theta\,\omega_1\,,\\
E&=\cos\phi\,E_8+\sin\phi\,E_1\,,\\
D&=-\sin\phi\,E_8+\cos\phi\,E_1\,,
\end{aligned}
$$

where E and D are the two ($I=0$, $Y=0$) physical axial-vector fields.

The results of the computation yield:

$$
\begin{aligned}
2A_{00}=-(1/6\sqrt{2})[&3(M_A{}^2+m_\rho{}^2)+4(M_{K_A}{}^2+m_{K^*}{}^2)\\
&+(M_E{}^2\cos^2\phi+M_D{}^2\sin^2\phi+m_\omega{}^2\cos^2\phi\\
&+m_\phi{}^2\sin^2\theta)-4\sqrt{2}(m_\omega{}^2\sin^2\theta+m_\phi{}^2\cos^2\theta\\
&+M_E{}^2\sin^2\phi+M_D{}^2\cos^2\phi)]\,,
\end{aligned}
\tag{44a}
$$

$$2A_{88}=(4/9)[(m_{K^*}{}^2-M_{K_A}{}^2)+2(m_\rho{}^2-M_A{}^2)]\,, \tag{44b}$$

$$2C_{88}=-2(m_{K^*}{}^2-M_{K_A}{}^2)\,, \tag{44c}$$

$$
\begin{aligned}
2A_{18}=(1/3\sqrt{10})[&3(M_{A_1}{}^2+M_\rho{}^2)-2(M_{K_A}{}^2+M_{K^*}{}^2)\\
&-(M_E{}^2\cos^2\phi+M_D{}^2\sin^2\phi+m_\omega{}^2\cos^2\theta\\
&+m_\phi{}^2\sin^2\theta)+2(\sqrt{5})(m_\omega{}^2-m_\phi{}^2)\sin 2\theta\\
&+2(\sqrt{5})(M_E{}^2-M_D{}^2)\sin 2\phi]\,.
\end{aligned}
\tag{44d}
$$

The Gell-Mann–Okubo mass formula for both nonets has been assumed in the above formulas.

If we want to make the (8,8) contribution an $SU(3)$ singlet (for the same reasons as in the preceding section), the following relation must be satisfied:

$$M_{K_A}{}^2-m_{K^*}{}^2=M_A{}^2-m_\rho{}^2\,. \tag{45}$$

None of the three K_A's listed in the tables [$K_A(1230)$, $K_A(1320)$, and $K_A(1800)$] satisfies this relation. It is possible however, that mixing among the different particles occurs and the formula is satisfied.[7] Notice that it is not possible to eliminate (8,8) completely, unless the two nonets are degenerate ($m_{K^*}{}^2=M_{K_A}{}^2$ and $m_\rho{}^2=M_A{}^2$).

C. Spontaneous Breakdown

Until now, we have considered the breakdown of chiral $SU(3)\times SU(3)$ symmetry by means of the mechanism of mass splitting and linear transformation of in-fields. There exists, however, another type of symmetry breaking which is required in this scheme, viz., spontaneous breakdown. Its appearance is manifested by the fact that single mesons decay via the hadronic axial current to the vacuum. This necessitates the presence of a term linear in the corresponding physical fields in the axial generators.

We thus conclude that the physical fields must obey inhomogeneous commutation relations.[8] Therefore, we modify (22a) by replacing the scalar nonet S_j by S_j+C_j, where the C_j's are c-numbers. We further choose those C_j's so as not to violate isospin or hypercharge conservation, i.e., $C_j=\delta_{j0}C_0+\delta_{j8}C_8$, with C_0 and C_8 to be determined.

With this modification, our Hamiltonian (21) acquires the additional terms

$$H'=-2M_i{}^2C_iS_i'+C_i{}^2M_i{}^2\,,$$

where the S_i''s are operators transforming linearly under the group.

Clearly, if the Hamiltonian is to be invariant under $SU(3)$, (where only C_8 appears), $M_\delta{}^2$ must be zero, and since all scalar masses must be equal (no intrinsic breakdown), this implies $M_i{}^2=0$ for all i (Goldstone-Nambu-Umezawa bosons). Invariance under $SU(3)\times SU(3)$ (both C_0 and C_8 present) similarly necessitates all scalar and pseudoscalar masses to be zero.

This noninvariance of the Hamiltonian is also seen in the time derivatives of the corresponding generators, which acquire the additional terms

$$-2f_{Ai8}C_8M_i{}^2S_i \quad\text{and}\quad -m_i{}^2(d_{Ai0}C_0P_i+d_{Ai8}C_8P_i)$$

for vector and axial-vector generators, respectively.

We can compute the value of C_0 and C_8 in terms of the decay constants f_π, f_K.[9] The vacuum expectation value of the inhomogeneous commutation relations yields

$$f_\pi=\frac{\sqrt{2}C_0+C_8}{\sqrt{3}}\,,\quad f_K=\frac{2\sqrt{2}C_0-C_8}{2\sqrt{3}}\,.$$

[7] S. Matsuda and S. Oneda, Phys. Rev. **174**, 1992 (1968); Nucl. Phys. **B9**, 55 (1969); S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967).

[8] L. Bessler, T. Muta, H. Umezawa, and D. Welling, Phys. Rev. D **2**, 349 (1970).

[9] We define f_π and f_K through $\langle 0|A_{\mu,\pi}(0)|\pi(k)\rangle=ik_\mu f_\pi/[(2\pi)^3k_0]^{1/2}$ and $\langle 0|A_{\mu,k}(0)|K(k)\rangle=ik_\mu f_K/[(2\pi)^3k_0]^{1/2}$.

This produces $C_8/C_0=-0.2$, if $f_K/f_\pi=1.25$. We see that if we have no spontaneous breakdown of SU_3, then $f_\pi=f_K$, whereas no spontaneous breakdown of $SU_3 \times SU_3$ requires $f_\pi=f_K=0$.

Since the physical masses are not zero, the Hamiltonian cannot be made invariant in the presence of C_0 and C_8. This noninvariance makes the divergences of the currents nonzero [partial conservation of vector current (PCVC) or partial conservation of axial-vector current (PCAC)].

It is our contention that the c-numbers remain when the physical masses are turned on. It remains for further analysis to uncover the extent to which C_0 and C_8 are affected, if at all, by the presence of intrinsic breakdown. For example, we wish to know whether the deviation of f_π and f_K from equality is predominantly due to mass splitting. This investigation, together with the possible relation of spontaneous breakdown to the model of Gell-Mann, Oakes, and Renner, will appear elsewhere.

III. COMMUTATOR APPROACH

In the previous section, our guiding criterion has been the maximum simplicity of the transformation properties of the Hamiltonian. An alternative criterion would be the simplicity of the algebra of divergences. In particular, we shall be interested in discovering the conditions under which those commutators between generators and their time derivatives, whose quantum numbers are incompatible with an $SU(3)$ singlet or octet, can be made to vanish. We shall call such commutators with quantum numbers $I>1$ and or $Y>1$, "exotic."

The vanishing of such commutators is the starting point of the Gell-Mann–Oakes–Renner approach. There is no compelling reason why these commutators should vanish, although the absence of exotic boson resonances and the field-current identity approach point in that direction. Using the same idea, Matsuda and Oneda[7] were able to obtain many results compatible with experiment, by going to the infinite-momentum limit and assuming asymptotic $SU(3)$ symmetry.

In Sec. II we have discovered the existence in H' of the representations $(6,6^*)$ and $(8,8)$, each of which can contain elements with exotic quantum numbers; therefore, all exotic commutators cannot simultaneously vanish. There is a close connection between the vanishing of the exotic commutators and the elimination of the different terms in the Hamiltonian discussed in the previous section. In particular, for exotic commutators we have the following.

(i) $[T_A,\dot{T}_B]$. Since T_A is an $SU(3)$ generator, these commutators will belong to an $SU(3)$ octet if and only if $\dot{T}$ is an octet operator. This then implies that H must transform as a combination of singlet and octet under $SU(3)$. Therefore, *the Gell-Mann–Okubo mass formula guarantees that all exotic commutators of the type* $[T,\dot{T}]$ *are zero.*

(ii) $[T_A,\dot{X}_B]$. Since X_A is an octet operator, $\dot{X}_A$ will belong to an octet if and only if the $(6,6)$ and $(8,8)$ contributions to H are purely singlet in $SU(3)$. (These are the only representations in H, for the cases considered, that contain $SU(3)$ representations higher than the octet.)

(iii) $[X_A,\dot{X}_B]$. Since X_A is not a generator of the (vector) $SU(3)$, the only way to make these commutators zero is to eliminate completely the $(6,6^*)$ and $(8,8)$ contributions from H.

We have seen in Sec. II that all these conditions cannot be satisfied simultaneously. Our solution (30)–(33) and (45), however, automatically satisfies the conditions in (i) and (ii) above. However, this is not the only possible solution, since in deriving (30)–(33) we assumed the (8,8) contribution was zero, which is not necessary for (ii).

We require the vanishing of the following commutators:

$$[X_{\pi^+},\dot{T}_{K^+}]=[T_{K^+},\dot{X}_{\pi^+}]=[T_{K^+},\dot{X}_{K^+}]=0\,, \quad (46)$$

where the subscripts denote the quantum numbers of the operators $(I=2,\ Y=1)$ and $(I=1,\ Y=2)$, respectively.

Using (21) and (22), we obtain

$$i\dot{T}_A\equiv[T_A,H]=+if_{48j}(\Delta m^2P_0P_j+\Delta M^2S_0S_j) +\tfrac{1}{2}if_{Aij}[(m_i^2-m_j^2)P_iP_j+(M_i^2-M_j^2)S_iS_j]\,, \quad (47)$$

where

$$\Delta m^2=\tfrac{1}{2}\sin 2\phi\,(m_\eta^2-m_{\eta'}^2)\,,$$
$$\Delta M^2=\tfrac{1}{2}\sin 2\theta\,(M_\sigma^2-M_{\sigma'}^2)\,.$$

We further obtain

$$i[X_A,\dot{T}_B]=[(m_i^2-m_j^2)P_iS_k-(M_i^2-M_j^2)P_kS_i] \times f_{Bji}d_{Ajk}+(\Delta M^2P_kS_j-\Delta m^2S_kP_j)f_{B8j}d_{A0k} +(\Delta M^2P_kS_0-\Delta m^2P_0S_k)f_{B8j}d_{Aik}\,. \quad (48)$$

Then if $[X_{K^+},\dot{T}_{K^+}]=0$, we must have

$$(m_\pi^2-m_K^2)+3\Delta m^2/\sqrt{2}=(M_\delta^2-M_\kappa^2)+3\Delta M^2/\sqrt{2}\,, \quad (49)$$

which is identical to (28). For $[X_{\pi^+},T_{K^+}]=0$, we must have,

$$(m_\pi^2-m_K^2)+\tfrac{1}{3}(M_\delta^2-M_\kappa^2)=\sqrt{2}\Delta M^2 \quad (50)$$

and

$$(M_\delta^2-M_\kappa^2)+\tfrac{1}{3}(m_\pi^2-m_K^2)=\sqrt{2}\Delta m^2\,, \quad (51)$$

where (51) is identical to (30). We note that (49), (50), and (51) are not linearly independent, but substitution of (50) and (51) into (49) yields the Gell-Mann–Okubo mass formula.

In considering the vector–axial-vector fields $(1^-,1^+)$, we obtain, using (29) and (30),

$$i\dot{T}_A=\tfrac{1}{2}if_{Aij}[(m_i^2-m_j^2)V_iV_j+(M_i^2-M_j^2)A_iA_j] +if_{A8j}(\Delta m^2V_0V_j+\Delta M^2A_0A_j) \quad (52)$$

and

$$i[X_A,\dot{T}_B]=[(m_i^2-m_j^2)V_kV_j + (M_i^2-M_j^2)A_iV_k]f_{Bji}f_{Ajk} + (\Delta M^2V_kA_0+\Delta m^2V_0A_k)f_{B8j}f_{Akj}. \quad (53)$$

For $[X_{\pi^+},\dot{T}_{\pi^+}]$ we obtain just the Gell-Mann–Okubo formula, since H' contains only (8,8) in this case and thus just one possible 27-plet in $SU(3)$. The exotic commutator $[X_{\pi^+},\dot{T}_{K^+}]=0$ requires

$$(m_{K^*}^2-m_\rho^2)+(M_A^2-M_{K_A}^2)=0, \quad (54)$$

which is just (45).

Thus, in the commutator approach, we do observe a universal condition, viz., the vanishing of the exotic commutator $[\dot{T},X]$, at least for the systems considered in this paper. It is however impossible to make all the exotic commutators of $[X,\dot{X}]$ zero; the reason for this is not clear.

IV. DISCUSSION AND CONCLUSIONS

In the preceding sections, we showed that the assumption of linear transformations for the physical in-fields yields very specific information about the nature of $SU(3)\times SU(3)$ breaking. It is interesting that our results are very similar to those obtained from asymptotic symmetry arguments by Oneda *et al.*[6] They also find that it is possible to achieve vanishing of matrix elements of exotic commutators of the type $[T_A,\dot{X}_B]$ in the limit of infinite momentum, but that this is not true for $[X_A,\dot{X}_B]$ commutators. The reason for the similar results seems to lie in the fact that, in the asymptotic limit, they saturate the commutators with one-particle states; this assumption is closely related to linear transformation for the in-fields.

The similarity breaks down, however, if one considers matrix elements of these commutators between baryon states; in this case, the results obtained by our method are very different from those of Oneda *et al.*[6] It is significant that for such matrix elements, Z-diagrams are equally important in the infinite-momentum limit as one-particle contributions; in fact, adding the Z-diagram contributions to the results of asymptotic symmetry reproduces exactly our results. We believe that this is a general property of fermion states. The baryons present some delicate and interesting problems in our model; we have investigated them and the details will be presented in a separate paper.

Another general area of investigation is related to the problem of mixing. In fact, mixing between different physical fields appears naturally in the framework of linear transformations. In this paper, the problem was treated in a very elementary way; we only considered mixing between fields in the same irreducible representation. We already found, however, that in the case of vector mesons the vanishing of exotic commutators requires mixing of fields beyond the same representation. A more general problem relates to the possibility and limitations of mixing between fields of different spin. Such a mixing would give greater flexibility to our model and is the subject of a separate investigation in progress.

In conclusion, we found that our model naturally specifies the transformation properties of the Hamiltonian once the experimental masses of the various fields are known. These are more complicated than those postulated by Gell-Mann, Oakes, and Renner; however, some of the features of their scheme, e.g., the vanishing of exotic commutators of the type $[T_A,\dot{X}_B]$, seem to hold true in our case too.

However, our model predicts that the form of the breaking is not universal (as assumed by the authors of Ref. 1), but depends on the nature of the particles considered. This is clear from the different transformation law discovered for the pseudoscalar-scalar and the vector–axial-vector multiplets, and the fact that, owing to the nature of asymptotic fields, the scalar-pseudoscalar part of the Hamiltonian will only contribute to matrix elements involving the corresponding particles, and the same will be true of the other terms in the hadronic Hamiltonian. Therefore, adding extra terms to our Hamiltonian corresponding to other types of particles will not affect the properties of σ terms for scalar-pseudoscalar matrix elements which were found in our paper. This property of the breaking, though not present in the simple case of $SU(3)$, is not novel; it is well known, for instance, that the breaking of $SU(6)$ symmetry has different forms for baryon and meson states.[10]

ACKNOWLEDGMENT

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[10] See, e.g., H. J. Lipkin, in *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), pp. 416–419.