Possible CP Violation in $K^{\pm} \to \pi^{\pm}\gamma\gamma$

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A comparison of the decays $K^{\pm} \to \pi^{\pm}\gamma\gamma$ for the 2γ invariant mass near the π^0 mass may show a relatively large asymmetry which is indicative of CP violation. This is shown in a simple phenomenological analysis of these decays. Model estimates of the magnitude of this effect are also given.

I. INTRODUCTION

It is well known that any inequality in the partial decay rate for a particle and an antiparticle into corresponding charge-conjugate final states is an indication of CP rather than CPT nonconservation, so long as that final state can be mixed with some other final state (up to some suitable order in the mixing interaction.[1,2] In particular, a difference in the partial decay rates[3] of

$$K^+ \to \pi^+\gamma\gamma \tag{1}$$

and

$$K^- \to \pi^-\gamma\gamma\,, \tag{2}$$

or a difference in the charged-pion spectrum in these decays, would provide clear evidence for the violation of CP.

The decays (1) and (2) can proceed via (i) the process

$$K^{\pm} \to \pi^{\pm}\pi^0\,, \tag{3}$$

which violates the nonleptonic $|\Delta I| = \frac{1}{2}$ rule,[4] with subsequent decay of the π^0

$$\pi^0 \to \gamma\gamma\,, \tag{4}$$

and (ii) other mechanisms that can generate this decay for the two-photon mass different from the neutral pion mass.[5–7] The interference of these amplitudes in the region near the neutral pion pole would make a comparison of the decays (1) and (2) a sensitive test for any significant CP violation. In particular, models can be constructed which could give rise to an asymmetry of the order of 10% in the energy spectrum of the charged pion.

Sufficient conditions for an effect of this magnitude to exist and to be measurable are (a) the $I=2$ S-wave $\pi^{\pm}$-π^0 phase shift at the kaon mass has to be appreciable, (b) the contributions of (i) and (ii) above should be comparable over a sizable portion of phase space, and (c) a large CP violation should exist in these amplitudes.

The first two conditions can be readily satisfied.[8] The $I=2$ S-wave $\pi^{\pm}$-π^0 phase shift δ_2 at the kaon mass has been determined to be approximately $-10°$,[9] and the contributions from (i) and (ii) above are expected to be roughly comparable on the basis of several models of (ii). The third condition, a large violation of CP, is postulated. If $\eta_{+-} \neq \eta_{00}$ (the standard parameters for CP violation in the neutral kaon system), and it is the only source of CP violation, then an asymmetry of the order of one-tenth[10] the above estimate (or less) would be expected.

In Sec. II, we consider the general transition matrix element for decays (1) and (2) and briefly discuss the relative importance of various contributions to this matrix element. The differential transition rate from

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[1] G. Lüders and B. Zumino, Phys. Rev. **106**, 385 (1957).

[2] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966); G. Costa and P. K. Kabir, Phys. Rev. Letters **18**, 429 (1967); N. Christ, Phys. Rev. **159**, 1292 (1967); see also Ref. 8.

[3] For present experimental limits on the observation of reactions (1) and (2), see M. Chen *et al.*, Phys. Rev. Letters **20**, 73 (1968); J. H. Klems, R. H. Hildebrand, and R. Stiening, *ibid.* **25**, 473 (1970).

[4] M. Gell-Mann and A. Pais, in *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics*, edited by E. H. Bellamy and R. G. Moorhouse (Pergamon, London, 1954).

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[7] G. Oppo and S. Oneda, Phys. Rev. **160**, 1397 (1967); see also, Y. Fujii, Ref. 6.

[8] As background, a test of CP invariance was proposed some time ago (see Ref. 2) which was to search for differences in the decays (a) $K^+ \to \pi^+\pi^0\gamma$ and (b) $K^- \to \pi^-\pi^0\gamma$. The decays (a) and (b) appeared particularly attractive owing to the presence of an inner-bremsstrahlung (IB) term and of a structure-dependent (SD) term in the decay amplitude. It was argued that, because of the suppression of decay (3) by the nonleptonic $|\Delta I| = \frac{1}{2}$ rule with resulting suppression of the IB term, the SD contribution can be relatively large, and its interference (the electric dipole component only) with the IB contribution can give rise to a significant asymmetry in the decays (a) and (b) should a large CP violation exist. However, experiments that search for the SD term have failed to demonstrate its existence; see, e.g., Maltsev *et al.*, Yadern Fiz. **10**, 1195 (1969) [Sov. J. Nucl. Phys. **10**, 678 (1970)]. [A somewhat different analysis by S. Barshay, Phys. Rev. Letters **18**, 515 (1967), nevertheless, requires the existence of a SD term.] Thus the proposed test of CP in the decays (a) and (b) would not be effective.

[9] S. Treiman, Comments Nucl. and Particle Phys. **2**, 68 (1968), and references contained therein.

[10] Relative to a_0, the $|\Delta I| = \frac{1}{2}$ amplitude, a_2 can only have a small CP-violating phase $\phi \lesssim 0.1$. See J. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

the term expected to give the largest contribution is also given. Then a simple parametrization of these decays and numerical estimates of the partial rates and of the decay asymmetry are given in Sec. III.

II. TRANSITION MATRIX ELEMENT AND MODELS

We consider the general transition matrix element for decays (1) and (2). It can be written as

$$\langle\pi^{\pm}\gamma\gamma|T|K^{\pm}\rangle = M^{\pm}e_{\mu\nu\lambda\rho}\epsilon_{\mu}\epsilon_{\nu}'k_{\lambda}k_{\rho}'+N^{\pm}(k\cdot k'\epsilon\cdot\epsilon'-k\cdot\epsilon'k'\cdot\epsilon) + (\text{terms with more than two momentum factors}), \quad (5)$$

where $e_{\mu\nu\lambda\rho}$ is the four-dimensional totally antisymmetric tensor, k_{α} (k_{α}') and ϵ_{β} (ϵ_{β}') are the momentum and polarization vectors for the first (second) photon.

The first (second) term is parity violating (conserving). The parity-violating term has been estimated in two models.[5,6] In these models, its contribution comes from an intermediate pseudoscalar meson which decays electromagnetically into two photons. These estimates are based upon the η^0 pole model[5] and upon an off-mass-shell extrapolation of decay (3).[6] These estimates give a sizable rate for the decays (1) and (2).

The parity-conserving term and the terms with more than two momentum factors can be generated,[11] for instance, in a vector-meson-pole model.[7] Numerical estimates based upon this model give significantly smaller rates[7] than the pseudoscalar-meson models above. The parity-conserving term can also be generated via a scalar boson in the intermediate state that decays electromagnetically into the two photons.[12] Its contribution is also expected to be small.[12] Because we are interested only in the decay spectrum, there will be no interference effects between these parity-conserving contributions and the dominant parity-violating mechanism above. For simplicity then, we drop all terms except $M^{\pm}$ in the remaining analysis.

From Eq. (5), the differential transition rate can be calculated in a straightforward manner. We assume that the nonvanishing amplitude $M^{\pm}$ is a (sensitive) function only of $-q^2=-(k+k')^2$, the squared mass of the two-photon system. With this assumption, one phase-space integration can be carried out. One gets

$$\frac{d\Gamma^{\pm}(K^{\pm}\to\pi^{\pm}\gamma\gamma)}{dE_{\pi^{\pm}}}=M_k(E_{\pi^{\pm}}{}^2-m_{\pi^{\pm}}{}^2)^{1/2}\times(E_0-E_{\pi^{\pm}})^2\frac{|M^{\pm}|^2}{(4\pi)^3}, \quad (6)$$

where $E_{\pi^{\pm}}$ is the energy of the charged pion in the charged-kaon rest frame, and E_0 is the maximum energy of the charged pion in the decay.

[11] The internal-bremsstrahlung terms vanish owing to gauge invariance. See Fujii, Ref. 6.

[12] I. Lapidus, Nuovo Cimento **46A**, 668 (1966); for comments on this model, see Ref. 5.

III. SIMPLE PARAMETRIZATION AND NUMERICAL ESTIMATES

We consider $M^{\pm}$ only in the region where $\sqrt{(-q^2)}$, the invariant two-photon mass, is near the neutral pion mass. We parametrize $M^{\pm}$ in this region as

$$M^{\pm}=\frac{\langle\pi^{\pm}\pi^0|T|K^{\pm}\rangle a(\pi^0\to2\gamma)}{q^2+m_{\pi^0}{}^2-i\epsilon}+S^{\pm}, \quad (7)$$

where $\epsilon=m_{\pi^0}\Gamma(\pi^0\to2\gamma)$, $\langle\pi^{\pm}\pi^0|T|K^{\pm}\rangle$ is the physical transition matrix element for decay (3), and $a(\pi^0\to2\gamma)$ is the amplitude for decay (4) and is related to that transition matrix element by

$$\langle\gamma\gamma|T|\pi^0\rangle=a(\pi^0\to2\gamma)e_{\mu\nu\lambda\rho}\epsilon_{\mu}\epsilon_{\nu}'k_{\lambda}k_{\rho}' \quad (8)$$

and to the decay width by

$$\Gamma(\pi^0\to2\gamma)=|a(\pi^0\to2\gamma)|^2m_{\pi^0}{}^3/64\pi. \quad (9)$$

The contribution to $M^{\pm}$ from the physical pion pole has been explicitly separated out. The remainder, given by $S^{\pm}$, is expected to be a relatively smooth function of q^2 in this region.

The matrix element for $K^+\to\pi^+\pi^0$ is generally written as

$$\langle\pi^+\pi^0|T|K^+\rangle=a_2e^{i\delta_2}, \quad (10)$$

where δ_2 is the $I=2$ S-wave π^+-π^0 phase shift at the kaon mass. From CPT invariance the matrix element for $K^-\to\pi^-\pi^0$ is then given by

$$\langle\pi^-\pi^0|T|K^-\rangle=a_2{}^*e^{i\delta_2}, \quad (11)$$

where we have neglected electromagnetic final-state effects. Furthermore, a_2 is real if CP invariance holds. With the neglect of high-order $[>O(e^2)]$ electromagnetic final-state interactions,[13] we get from CPT that ($S^+\equiv S$)

$$S^-=S^*. \quad (12)$$

Again CP invariance requires S to be real.

With the above phase information, it is convenient to cast Eq. (7) into the following form:

$$M^{\pm}=|a_2a(\pi^0\to2\gamma)|\left[\frac{m_{\pi^0}{}^2e^{i(\delta_2\pm\phi)}}{q^2+m_{\pi^0}{}^2-i\epsilon}+\xi\right]\frac{1}{m_{\pi^0}{}^2}, \quad (13)$$

where ϕ is the *over-all* CP-violating phase ($\phi=0$, π if CP is conserved), and ξ is related to S by

$$\xi=Sm_{\pi^0}{}^2|a_2a(\pi^0\to2\gamma)|^{-1}. \quad (14)$$

The magnitude of ξ has been estimated in the η^0 pole model by Fäldt *et al.*[5] The η^0 pole was considered to be important, since the virtual transition $K^{\pm}\to\pi^{\pm}\eta^0$ satisfies the nonleptonic $|\Delta I|=\frac{1}{2}$ rule. Also, η^0 decay into two photons has a width larger than expected from simple $SU(3)$ considerations.[14] The magnitude of ξ in

[13] The high-order electromagnetic final-state interaction will introduce a phase of order α to S, e.g., the decay width of the η^0 in the η^0-pole model. This phase can always be absorbed into δ_2.

[14] From $SU(3)$, $a(\eta^0\to2\gamma)=(1/\sqrt{3})a(\pi^0\to2\gamma)$, which implies $\Gamma(\eta^0\to2\gamma)\simeq150$ eV. Experimentally, $\Gamma(\eta^0\to2\gamma)\simeq1$ keV.

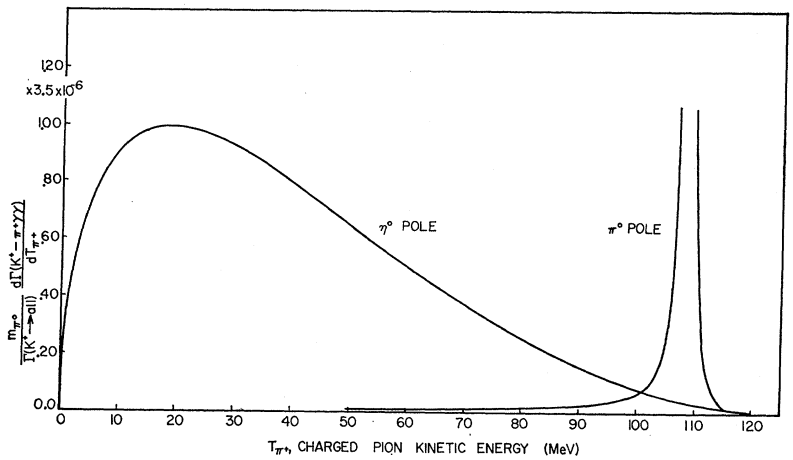

FIG. 1. Charged pion decay spectrum for reactions (1) and (2). The contributions from the η^0 pole and from the physical π^0 pole are shown separately.

the physical region of decays (1) and (2) is found to be approximately equal to 3 in this model.

The charged pion spectrum and the partial width of decays (1) and (2) were included by Fäldt *et al.*[5] We also give the charged pion spectrum due separately to the η^0 pole and to the π^0 pole for these decays. The results are plotted in Fig. 1. The branching ratio, integrated for the charged pion kinetic energy up to 70 MeV (contributed only by the η^0 pole), is[5]

$$\mathrm{BR}=\frac{\Gamma(K^\pm\to\pi^\pm\gamma\gamma,\ T_{\pi^+}<70\ \mathrm{MeV})}{\Gamma(K^\pm\to\mathrm{all})}\simeq 12\times10^{-7}. \quad (15)$$

The magnitude of ξ has also been estimated from partial conservation of axial-vector current (PCAC) and current-algebra considerations.[6] In this approach one considers the off-mass-shell amplitude in reaction (3). It was demonstrated by Fujii and others[6] that this off-mass-shell effect may be large. In fact, it was estimated by Fujii that $|\xi|\simeq 20$. For $|\xi|$ of this magnitude, the resulting branching ratio of Eq. (15) would be[6]

$$\mathrm{BR}\simeq 300\times10^{-7}. \quad (16)$$

With such a branching ratio, decays (1) and (2) could be observed in the near future.

However, it was pointed out recently by Lee[15] that the "soft-pion theorem" does not imply the nonleptonic $|\Delta I|=\frac{1}{2}$ rule in kaon decay. As a result the actual off-mass-shell amplitude may not necessarily be the one used by Fujii, and thus $|\xi|$ is not determined. The off-mass-shell extrapolation and therefore $|\xi|$ remain model dependent. We take $|\xi|$ to lie in the range $1\leqslant|\xi|\leqslant 20$.

As already noted the violation of CP is indicated by the presence of an asymmetry in the decay spectrum between reactions (1) and (2). The asymmetry is

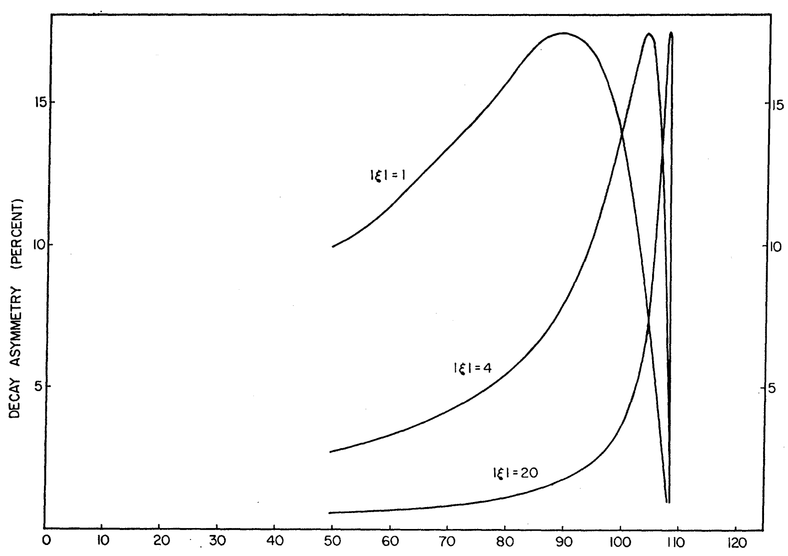

FIG. 2. Numerical estimates of the asymmetry in the charged pion decay spectrum of reactions (1) and (2). The asymmetry arises from interference of the amplitude from the physical π^0 pole with a smooth amplitude generated by another mechanism. Maximal CP violation is assumed for these estimates.

[15] T. D. Lee, Columbia University report (unpublished).

defined

$$A=\left(\frac{d\Gamma^+}{dE_+}-\frac{d\Gamma^-}{dE_-}\right)\Big/\left(\frac{d\Gamma^+}{dE_+}+\frac{d\Gamma^-}{dE_-}\right). \qquad (17)$$

We give numerical estimates of the magnitude of A using $\delta_2=-10°$ and assuming maximal *CP* violation, i.e., $\phi=\frac{1}{2}\pi$. ξ is taken to be a constant and its magnitude is chosen to be 1, 4, and 20. The magnitude of A is given in Fig. 2 as a function of the charged-pion kinetic energy in the region of the neutral-pion pole. As can be seen, the magnitude of A can be relatively large over a sizable portion of the charged pion spectrum. In particular, on the basis of the η^0-pole model ($\xi\simeq 3$), the magnitude of A averages out to be $\approx 10\%$ over the region shown. The sign of A, however, remains undetermined since the sign of ξ and ϕ are not known. For nonmaximal *CP* violation, one has to multiply the asymmetries in Fig. 2 by $\sin\phi$. For $\phi\ll 1$, the resulting asymmetry as expected, would be quite small.

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Bethe-Salpeter Solution for Nucleon-Nucleon Scattering with Pion Exchange in the $^1S_0$ and $^3P_0$ States*

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The Bethe-Salpeter equation is solved for nucleon-nucleon scattering in the ladder approximation with pion exchange for the $^1S_0$ and $^3P_0$ states with no approximations beyond use of a finite mesh. It is found that solutions exist without the need for any cutoff so long as the coupling constant $g^2/4\pi$ is between about -4 and $+7$ for the $^1S_0$ state and for $g^2/4\pi$ less than about $+4$ for the $^3P_0$ state. These results are in qualitative agreement with the predictions of Mandelstam. It is found that the Padé approximant as applied to the coefficients of the iteration solution provides an efficient alternative to the method of matrix inversion for solving the equations for a given mesh.

I. INTRODUCTION

The Bethe-Salpeter (BS) equation[1] for nucleon-nucleon (NN) scattering is of physical interest for several reasons. It is a relativistic generalization of the Schrödinger equation and sums a limited set of graphs from field theory depending on the kernel. In fact, our original motivation for considering this problem was directed toward finding a means of generating the amplitudes for the individual ladder graphs. We will consider the ladder series with pion exchange for the $J=0$ case in this calculation.

The realistic BS equation[2] is difficult to deal with since it is generally in the form of coupled singular integral equations. Several authors[3] have considered various aspects of the spin-$\frac{1}{2}$-on-spin-$\frac{1}{2}$ problem. For the NN case, Gourdin[4] took advantage of the four-dimensional symmetry and expanded the wave function in hyperspherical harmonics. The result was an infinite set of scalar coupled equations in one continuous variable which were coupled in the quantum number associated with the fourth dimension. A solution was then obtained by taking only the lowest value for the quantum number. However, the effect of this approximation above the elastic threshold is not clear. Ito *et al.*[5] and Murota *et al.*[6] have approached this problem somewhat differently. They have used the kernel-subtraction method of Levine *et al.*[7] to reduce the singularity of the kernel and taken the approximation of neglecting negative-energy intermediate states. This leads to a single integral equation in two continuous variables plus an auxiliary integral equation in one variable. These equations are solved by matrix inversion with reasonable accuracy. However, the approximation of neglecting negative-energy states is probably poor since the pseudoscalar γ_5 interaction tends to strongly couple positive- and negative-energy states. Therefore in this calculation we will employ the methods of Murota *et al.*, but we will include all intermediate states.

Fortunately we are not completely ignorant as to what to expect for solutions to the BS equation.

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