Van Hove Model and Nonleptonic Hyperon Decays*

D. N. GOSWAMI, J. SCHECHTER, AND Y. UEDA
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 20 November 1970)

An attempt is made to construct a pole model for nonleptonic decays where exchanged particles of all spins are included. Summation over the different spins is carried out by means of the Van Hove model, assuming that the Regge-pole term is important. It is found that a number of trajectories do not contribute for a very simple, essentially kinematical reason. A discussion of some consequences of this approach is given.

I. INTRODUCTION

MANY models of nonleptonic decays require the use of pole diagrams.[1] The three types of Feynman diagrams, corresponding to poles in the s, t, and u channels, are shown in Fig. 1.

Ordinarily models are made with only a few low-lying particles or resonances included. This is not unreasonable, but the question remains as to what happens when *all* particles are included. Since it is currently believed that particles occur in Regge families of ever-increasing spin, it is interesting to try to take the sum of all diagrams with exchanged poles belonging to one Regge family. One way of doing this is to convert the sum over discrete spin exchanges to an integral over a continuous spin variable by means of the Watson-Sommerfeld transformation. The integral is then evaluated in the complex angular momentum plane by contour integration, to give a (Regge) pole term plus background integral or cut term. This procedure is known as the Van Hove model[2] and has recently been used by many authors[3–6] to clarify some perplexing problems in the theory of high-energy scattering. The advantage of this approach over the older one is that by starting out with Feynman amplitudes, we have a reasonable guarantee of getting the right "kinematical" structure. This was strikingly illustrated in the calculation[6] of Carlitz and Kislinger, who showed that fermion trajectories could easily be "Reggeized" without introducing parity doublets.

Here we shall apply the Van Hove model to nonleptonic hyperon decay. We shall, furthermore, only pick up the pole contribution in the complex angular momentum plane. For the case of high-energy scattering, it is expected that the pole term dominates over the background integral. On the other hand, for nonleptonic decays, which are essentially low-energy processes, we have no guide to what is expected. Nevertheless, since the Regge-pole contribution has such a distinctive form, it is a very logical thing to look at for a first start on this problem. Furthermore, this contribution enters in the "duality" approach to nonleptonic decays, which has been recently discussed. We shall see that our results may have important consequences for this approach.

We find that, in the usual zero-width limit for the resonance exchanges, a considerable number of trajectories do *not* contribute to the Regge-pole part of the decay amplitude. This result would simplify the construction of models if it were assumed that the Regge poles were important. The reason certain trajectories are prevented from contributing is essentially a kinematical one and a simple rule may be set up to determine whether or not a given trajectory will contribute. For this, it is convenient to refer to Fig. 2, which is a summary of all three parts of Fig. 1. In Fig. 2, M_1, M_2, and M_3 may stand for the masses of any of the external particles involved. Let $\alpha(M^2)$ be the trajectory function for the exchanged Regge family. Then if M_3 corresponds to a pion line, the trajectories which give zero contribution are those satisfying

$$\alpha(M_3^2)>0. \tag{1.1}$$

On the other hand, if M_3 is a spin-$\frac{1}{2}$ fermion line, the trajectories which give zero contribution satisfy

$$\alpha(M_3^2)>\begin{cases}+\frac{1}{2}\\ -\frac{3}{2}\end{cases}. \tag{1.2}$$

In (1.2) the $+\frac{1}{2}$ $(-\frac{3}{2})$ on the right-hand side corresponds to s-wave (p-wave) decays with $\frac{3}{2}^+, \frac{7}{2}^+, \ldots$ or $\frac{1}{2}^-, \frac{5}{2}^-, \ldots$ exchanged trajectories and p-wave (s-wave) decays with $\frac{3}{2}^-, \frac{7}{2}^-, \ldots$ or $\frac{1}{2}^+, \frac{5}{2}^+, \ldots$ exchanged trajectories. Although the formal justification of these rules in the Van Hove model requires a fair amount of alegbraic work, the basic reasoning leading to rule (1.1), for example, can be easily seen by referring to the heuristic argument given in Sec. III. Equations (1.1) and (1.2) have some amusing consequences. Reference to the usual Chew-Frautschi plots for particles immediately shows that, for example,

(i) most s-channel trajectories do not contribute;
(ii) the K^* trajectory does not contribute to s-wave (parity-violating) decays;
(iii) the nucleon trajectory may contribute (in the u channel) to p-wave but *not* to s-wave decays.

* Work supported by the U. S. Atomic Energy Commission.
[1] Many references are given in R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Interscience, New York, 1969).
[2] L. Van Hove, Phys. Letters **24B**, 183 (1967); also R. Feynman (unpublished).
[3] L. Durand III, Phys. Rev. **154**, 1537 (1967).
[4] R. L. Sugar and J. D. Sullivan, Phys. Rev. **166**, 1515 (1968).
[5] R. Blankenbecler and R. L. Sugar, Phys. Rev. **168**, 1597 (1968).
[6] R. Carlitz and M. Kislinger, Phys. Rev. Letters **24**, 186 (1970).

In Sec. II the discussion of the t-channel poles leading to the criterion (1.1) will be presented. A simple rough argument leading to the same result will be given in Sec. III. Sections IV and V will deal with the u- and s-channel poles, respectively. Finally Sec. VI will contain a discussion of some consequences of this model.

II. t-CHANNEL POLES

First let us consider the amplitude which represents the diagram of Fig. 1(b) with a boson of spin J and mass M_J proceeding from the strong vertex to the weak vertex. After constructing this amplitude, we shall attempt to sum over J.

Formally, the decay amplitude can be written as the $q\to 0$ limit of a scattering amplitude, where q is the 4-momentum of a boson coming in to the weak vertex. Our results are actually independent of whether or not we regard the decay as the limit of a scattering, but we shall adopt the scattering picture so that some of the formulas may be useful in other connections. Furthermore our results only depend on the "Lorentz" kinematics and not the $SU(3)$ character of the weak vertex.

An easy way to see the connection between the decay and the scattering process is to proceed as follows. Each decay is described by a matrix element $\langle N_f\pi|H_w|N_i\rangle$. Replace H_w in this by an effective operator constructed out of spin-zero meson fields. This operator is required to be Hermitian, have $CP=+1$, and to have the same parity properties as the decay we wish to describe (i.e., $P=+1$ for p-wave decays and $P=-1$ for s-wave decays). Thus if we assume H_w to have a "λ_6" $SU(3)$ transformation property (which would follow from a

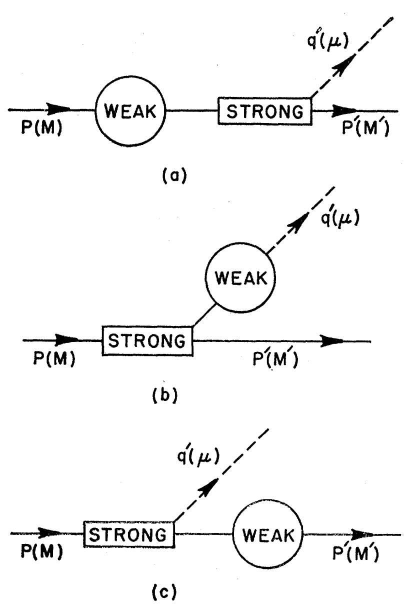

FIG. 1. Pole diagrams for nonleptonic hyperon decay. A baryon of 4-momentum p and mass M decays into another baryon of 4-momentum p' and mass M' and a pion of 4-momentum q' and mass μ. The s, t, and u diagrams are respectively (a), (b), and (c).

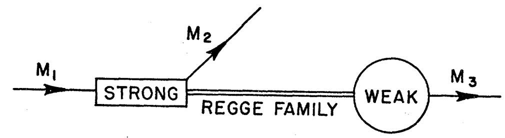

FIG. 2. Generic decay diagram for exchange of an entire Regge family.

current-current weak interaction), the effective operator for p-wave decays should be proportional to $(\kappa_0+\bar{\kappa}_0)$, where κ_0 is a $C=1$ scalar meson, and the effective operator for s-wave decays should be proportional to $(K_0'+\bar{K}_0')$, where K_0' is a $C=-1$ pseudoscalar meson. On the other hand, if we allow H_w to have a "λ_7" $SU(3)$ transformation property, we would get $i(K_0-\bar{K}_0)$ for s waves and $i(\kappa_0'-\bar{\kappa}_0')$ for p waves, where κ_0' is a $C=-1$ scalar meson. Now, once H_w has been replaced by an effective-field operator, the connection of our matrix element to the scattering amplitude with the given particle incoming and a pion outgoing is made formally apparent by the LSZ (Lehmann-Symanzik-Zimmermann) reduction formula. Note that we must supply an extra factor of i for the λ_7 case. Further note that [in the $SU(3)$ limit] the s-wave decay is *not* formally similar to ordinary meson-baryon scattering when the current-current (λ_6) picture is adopted.

For the scattering $K_0+N_i\to\pi+N_f$ with momentum conservation, $q_\mu+p_\mu=q_\mu'+p_\mu'$, we define the matrix element T_J^{scatt} for exchange of a particle with spin J, in terms of the S matrix, as

$$S_J=1-i(2\pi)^4\delta^{(4)}(q+p-q'-p')\times\left(\frac{M}{p_0}\frac{M'}{p_0'}\frac{1}{2q_0}\frac{1}{2q_0'}\right)^{1/2}\bar{u}(p')T_J^{\text{scatt}}u(p).$$

The decay amplitude is $i[\lim_{q\to 0}(T_J^{\text{scatt}})]$ since the above scattering amplitude corresponds to s-wave decay with a λ_7 interaction. For our present purpose it is sufficient to consider this example.

We construct T_J^{scatt} as the product of the spin-J propagator and the two vertex functions. Our formulas for this case are slight generalizations of those of Blankenbecler and Sugar.[5] The spin-J propagator is taken as

$$\frac{-i}{M_J^2+\Delta^2}G_{\mu1\cdots\mu J;\nu1\cdots\nu J}(\Delta)\,, \tag{2.1}$$

where

$$G_{\mu1\cdots\mu J;\nu1\cdots\nu J}(\Delta)=\sum_{\substack{\text{permutations of}\\ \{\mu\}\text{ and }\{\nu\}}}\ \sum_{r=0}^{J/2,(J-1)/2}(-1)^r\frac{(2J-2r)!}{r!(J-2r)!(J-r)!(2J)!}\times\theta_{\mu_1\mu_2}\cdots\theta_{\mu_{2r-1}\mu_{2r}}\theta_{\nu_1\nu_2}\cdots\theta_{\nu_{2r-1}\nu_{2r}}\prod_{i=2r+1}^{J}\theta_{\mu_i\nu_i}\,, \tag{2.2}$$

$$\theta_{\mu\nu}=\delta_{\mu\nu}+\Delta_\mu\Delta_\nu/M_J^2\,, \tag{2.3}$$

and

$$\Delta_\mu=p_\mu-p_\mu'\,.$$

The "weak" vertex, for the case of an incoming K meson and outgoing π meson, is described by the effective Lagrangian

$$g_{\pi KJ}C_J\bar{\phi}_{\alpha_1\cdots\alpha_J}(K\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_J}\pi)+\text{H.c.}, \tag{2.4}$$

where $g_{\pi KJ}$ is a coupling constant, $\overleftrightarrow{\partial}_\alpha=\overrightarrow{\partial}_\alpha-\overleftarrow{\partial}_\alpha$, $C_J=1$ for even J and i for odd J, and $\bar{\phi}_{\alpha_1\cdots\alpha_J}=(-1)^{N_4}\phi_{\alpha_1\cdots\alpha_J}{}^\dagger$ (N_4=number of fours in $\{\alpha_1\cdots\alpha_J\}$). In writing (2.4) we assumed that $\phi_{\alpha_1\cdots\alpha_J}$ had $C=P=(-1)^J$.

The strong vertex for coupling the initial and final baryons to the spin-J field is described by an effective Lagrangian with two independent coupling constants:

$$C_Jg_{NNJ}{}^{(1)}(\bar{N}_f\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_J}N_i)\phi_{\alpha_1\cdots\alpha_J}$$
$$+C_Jg_{NNJ}{}^{(2)}(\bar{N}_f\gamma_{\alpha_1}\overleftrightarrow{\partial}_{\alpha_2}\cdots\overleftrightarrow{\partial}_{\alpha_J}N_i)\phi_{\alpha_1\cdots\alpha_J}+\text{H.c.} \tag{2.5}$$

From (2.1)–(2.5), we then have

$$T_J{}^{\text{scatt}}=\frac{-C_J{}^2}{M_J{}^2-t}g_{\pi KJ}(g_{NNJ}{}^{(1)}P_{\mu_1}\cdots P_{\mu_J}$$
$$-ig_{NNJ}{}^{(2)}\gamma_{\mu_1}P_{\mu_2}\cdots P_{\mu_J})$$
$$\times G_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(\Delta)Q_{\nu_1}\cdots Q_{\nu_J}, \tag{2.6}$$

where $Q=q+q'$, $P=p+p'$, and $t=-\Delta^2$. Using (2.2) explicitly gives

$$T_J{}^{\text{scatt}}=\frac{-C_J{}^2}{M_J{}^2-t}g_{\pi KJ}(J)!(J-1)!$$
$$\sum_{r=1}^{(J-1)/2,J/2}\left[\frac{(2J-2r)!(-1)^r}{r!(J-2r)!(J-r)!(2J)!}\right](Q\cdot\theta\cdot Q)^r$$
$$\times\{Jg_{NNJ}{}^{(1)}(Q\cdot\theta\cdot P)^{J-2r}(P\cdot\theta\cdot P)^r$$
$$-ig_{NNJ}{}^{(2)}(Q\cdot\theta\cdot P)^{J-2r-1}(P\cdot\theta\cdot P)^{r-1}$$
$$\times[2r(\gamma\cdot\theta\cdot P)(Q\cdot\theta\cdot P)+(J-2r)(P\cdot\theta\cdot P)(\gamma\cdot\theta\cdot Q)]\}, \tag{2.7}$$

where $Q\cdot\theta\cdot P=Q_\mu\theta_{\mu\nu}P_\nu$, etc. This may be rewritten in terms of Legendre polynomials by introducing

$$\bar{Z}=(Q\cdot\theta\cdot P)/[(P\cdot\theta\cdot P)(Q\cdot\theta\cdot Q)]^{1/2} \tag{2.8}$$

and noting

$$P_J(\bar{Z})=\sum_{r=0}^{J/2,(J-1)/2}(-1)^r$$
$$\times\frac{(2J-2r)!}{r!(J-2r)!(J-r)!2^J}\bar{Z}^{J-2r}, \tag{2.9}$$

$$P_J'(\bar{Z})=(d/d\bar{Z})P_J(\bar{Z}). \tag{2.10}$$

Then (2.7) becomes

$$T_J{}^{\text{scatt}}=\frac{-C_J{}^2}{M_J{}^2-t}g_{\pi KJ}\frac{2^JJ!(J-1)!}{(2J)!}(P\cdot\theta\cdot P)^{J/2}$$
$$\times(Q\cdot\theta\cdot Q)^{J/2}\{Jg_{NNJ}{}^{(1)}P_J(\bar{Z})-ig_{NNJ}{}^{(2)}$$
$$\times[(\gamma\cdot\theta\cdot P)(P\cdot\theta\cdot P)^{-1}(JP_J(\bar{Z})-\bar{Z}P_J'(\bar{Z}))$$
$$+(\gamma\cdot\theta\cdot Q)(Q\cdot\theta\cdot Q)^{-1/2}(P\cdot\theta\cdot P)^{-1/2}P_J'(\bar{Z})]\}. \tag{2.11}$$

As written, (2.11) describes the general case of unequal-mass meson-baryon scattering. We are interested in the $q\rightarrow 0$ limit appropriate to the nonleptonic decay situation. In this limit, the various kinematical factors become

$$\begin{aligned}
&(Q\cdot\theta\cdot Q)\rightarrow -\mu^2[1-(\mu/M_J)^2],\\
&(Q\cdot\theta\cdot P)\rightarrow (M'^2-M^2)[1-(\mu/M_J)^2],\\
&(\gamma\cdot\theta\cdot Q)\rightarrow i(M-M')[1-(\mu/M_J)^2],\\
&(P\cdot\theta\cdot P)\rightarrow \mu^2-2(M^2+M'^2)+(M'^2-M^2)^2/M_J{}^2,\\
&(\gamma\cdot\theta\cdot P)\rightarrow i(M+M')[1-(M-M')^2/M_J{}^2],\\
&\quad t\rightarrow\mu^2.
\end{aligned} \tag{2.12}$$

Next we attempt to sum (2.11) over all exchanged particles with spin J belonging to a given Regge family, by using the Watson-Sommerfeld transformation. We assume that all objects in (2.11), i.e., $g_{\pi KJ}$, $g_{NNJ}{}^{(1)}$, $g_{NNJ}{}^{(2)}$, $M_J{}^2$, and $P_J(\bar{Z})$, are to be considered functions of complex J rather than objects defined for positive integral J only. The extension of $P_J(\bar{Z})$ to complex J by means of its defintion in terms of the hypergeometric function is well known. Thus we have for the entire amplitude

$$\sum_J\tfrac{1}{2}[1\pm(-1)^J]T_J$$
$$=\frac{-i}{4}\oint_c\frac{1\pm e^{-i\pi J}}{\sin\pi J}(-1)^JT_JdJ, \tag{2.13}$$

where c is the U-shaped contour (traversed counterclockwise) which encloses the positive real axis, the $+$ and $-$ signs hold for even and odd spin families, respectively, and

$$T_J=i\lim_{q\rightarrow 0}T_J{}^{\text{scatt}},$$

suitably continued. The contour c in (2.13) is then deformed in the usual way, so that (2.13) can be rewritten in terms of a Regge pole plus a background integral. Here we will only pick up the Regge-pole contribution, since this is easily evaluated and turns out to have interesting features. Whether or not the Regge-pole contribution is an important one is a question we shall not answer here.

From (2.11), we see that the position of the Regge pole in the J plane is determined by the vanishing of the $(M_J{}^2-t)$ denominator, so that if $J=\alpha$ at the pole then

$M_\alpha^2-t=0$. The result is

$$\frac{\pi}{2}\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\frac{1}{(\partial M_J^2/\partial J)_{J=\alpha}} \times\lim_{J\to\alpha}[(M_J^2-t)(-1)^J T_J]. \quad (2.14)$$

In writing (2.14) we have assumed with previous authors[2–5] that the coefficient of $(M_J^2-t)^{-1}$ in (2.11) has good analytic properties in J. This statement may be considered as a possible definition of the Van Hove model. [Actually it is not necessarily true in the case of unequal-mass scattering or in the decay case since a term like $(P\cdot\theta\cdot P)^{J/2}$ gives rise to a function of the form J^J.]

Essentially, the quantity of interest is the coefficient of $(M_J^2-t)^{-1}$ in (2.11) evaluated for $M_J^2=\mu^2$. From (2.12) we see that many of the kinematical factors will vanish in this limit. Equation (2.8) shows that $\bar{Z}\to 0$ in this limit. However, $P_J(0)$ and $P_J'(0)$ are, in general finite.[7] Thus near $J=\alpha$ the leading term of the coefficient of $(M_J^2-t)^{-1}$ in (2.11) will behave like

$$[1-(\mu/M_J)^2]^{\alpha/2}, \quad (2.15)$$

where α is the Regge trajectory function evaluated at μ^2.

It is clear that (2.15) will vanish if $\alpha>0$, giving the criterion stated previously as (1.1). If $\alpha<0$, (2.15) will diverge. We interpret this as meaning that the given Regge trajectory *can* contribute in this case. However, a means for the evaluation of the amplitude must be specified. One possible way would be to replace the denominator $(M_J^2-t)^{-1}$ by $(M_J^2+i\gamma_J-t)^{-1}$ and to "Reggeize" by considering both M_J^2 and γ_J to be functions of the complex variable J. Then the vanishing of the denominator would occur for M_J^2 off the real axis and (2.15) would no longer diverge. We shall not go into more details since we think that, at this stage, the qualitative criterion given in (1.1) of whether or not a trajectory will give a Regge-pole contribution to the amplitude in the zero-width limit is of primary interest.

Since the preceding analysis is a bit complicated, it may well be wondered if there is not a simpler way to see what is going on. In Sec. III we shall present a heuristic argument to show that the criterion obtained follows from kinematics and a property of the spin-J propagator.

III. HEURISTIC ARGUMENT

Consider, as in Sec. II, the t-channel pole diagram of Fig. 1(b) with the exchange of a resonance of spin J and mass M_J. The Feynman amplitude is given by the product of the spin-J propagator and the strong and weak vertices. It is thus proportional to

$$\frac{1}{M_J^2+q'^2}(p+p')_{\mu_1}\cdots(p+p')_{\mu_J} \times G_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(q',M_J^2)q_{\nu_1}'\cdots q_{\nu_J}', \quad (3.1)$$

where $G_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(q',M_J^2)$ is a function satisfying

$$\lim_{-M_J^2\to q'^2} q_{\nu_k}'G_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_k\cdots\nu_J}(q',M_J^2)=0. \quad (3.2)$$

To proceed as in the Van Hove model, we sum (3.1) over all J's belonging to the Regge family. We assume that a continuation of (3.1) to complex J can be made so that $M_J^2\to M^2(J)$, $g_J\to g(J)$, etc., and then convert the sum to a contour integration in the complex J plane by means of the Watson-Sommerfeld transformation. The position of the Regge pole in the complex J plane is determined from the vanishing of $[M^2(J)+q'^2]$ in (3.1). Thus the Regge-pole contribution to the amplitude is proportional to the coefficient of $[M^2(J)+q'^2]^{-1}$ in (3.1) evaluated at $-M^2(J)=q'^2$. However, this is expected to vanish by (3.2).

The reason the above argument is only heuristic is, of course, that the method of continuing (3.1) to complex J has not been specified. Note that the above argument would not hold if the value of J at the pole was zero (since no factors of q' exist when we use the spin-zero propagator), so we can roughly see why (1.1) must be satisfied. An exactly analogous argument may be given for the s- and u-channel poles.

IV. u-CHANNEL POLES

The relevant diagram here is Fig. 1(c) with a meson of 4-momentum q incoming at the weak vertex. We shall first construct the scattering amplitude for the exchange of a spin $J=l+\frac{1}{2}$ (l=integer) fermion.

The strong vertex is described by the effective Lagrangian:

$$g_l\bar{\psi}_{\alpha_1\cdots\alpha_l}\begin{Bmatrix}1\\ i\gamma_5\end{Bmatrix}(N_i\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_l}\pi^\dagger)+\text{H.c.}, \quad (4.1)$$

where g_l is a coupling constant, $\psi_{\alpha_1\cdots\alpha_l}$ is the spin-$(l+\frac{1}{2})$ field operator, and we should use

1 for $\frac{3}{2}^+, \frac{7}{2}^+, \ldots$ or $\frac{1}{2}^-, \frac{5}{2}^-, \ldots$ exchanges

and

$i\gamma_5$ for $\frac{3}{2}^-, \frac{7}{2}^-, \ldots$ or $\frac{1}{2}^+, \frac{5}{2}^+, \ldots$ exchanges.

The weak vertex is described by the effective Lagrangian

$$h_l(\bar{N}_f\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_l}\phi)\begin{Bmatrix}1\\ i\gamma_5\end{Bmatrix}\psi_{\alpha_1\cdots\alpha_l}+\text{H.c.}, \quad (4.2)$$

[7] We have

$$P_\nu(0)=(\sqrt{\pi})/\Gamma(\tfrac{1}{2}(1-\nu))\Gamma(\tfrac{1}{2}(2+\nu)),$$
$$P_\nu'(0)=-2(\sqrt{\pi})/\Gamma(\tfrac{1}{2}(1+\nu))\Gamma(-\tfrac{1}{2}\nu),$$

and

$$P_\nu''(0)=-2(\sqrt{\pi})(\nu+1)/\Gamma(\tfrac{1}{2}(1-\nu))\Gamma(\tfrac{1}{2}\nu).$$

where ϕ is the incoming meson. For S-wave decays, ϕ is a pseudoscalar meson and we should use the identification of 1 and $i\gamma_5$ as in (4.1). For p-wave decays, ϕ is a scalar meson and we should use

$$1 \quad \text{for} \quad \tfrac{3}{2}^-, \tfrac{7}{2}^-, \ldots \quad \text{or} \quad \tfrac{1}{2}^+, \tfrac{5}{2}^+, \ldots \text{ exchanges}$$

and

$$i\gamma_5 \quad \text{for} \quad \tfrac{3}{2}^+, \tfrac{7}{2}^+, \ldots \quad \text{or} \quad \tfrac{1}{2}^-, \tfrac{5}{2}^-, \ldots \text{ exchanges.}$$

Finally the propagator[8] for the fermion of mass M_l and spin $J=l+\frac{1}{2}$ is taken to be

$$\frac{-i(M_l-i\gamma\cdot K)}{M_l^2+K^2}\left(\frac{l+1}{2l+3}\right)\gamma_{\mu 1}\gamma_{\nu 1}G_{\mu 1\cdots\mu_{l+1};\nu_1\cdots\nu_{l+1}}(K)\,, \quad (4.3)$$

where $K=p-q'$ and $G_{\mu 1\cdots\mu_{l+1};\nu_1\cdots\nu_{l+1}}(K)$ is given in (2.2).

From (4.1)–(4.3), we find the amplitude for the spin-J exchange process to be

$$T_J^{\text{scatt}}=\frac{Fg_lh_l}{m_l^2+K^2}\left(\frac{l+1}{2l+3}\right)\sum_r(-1)^r\frac{(2l-2r+2)!}{r!(l+1-2r)!(l+1-r)!(2l+2)!}(l!)^2$$
$$\times[4r^2(\gamma\cdot\theta\cdot b)(\gamma\cdot\theta\cdot b')(b\cdot\theta\cdot b')^{l-2r+1}(b\cdot\theta\cdot b)^{r-1}(b'\cdot\theta\cdot b')^{r-1}$$
$$+(l+1-2r)(\gamma\cdot\theta\cdot\gamma)(b\cdot\theta\cdot b)^r(b'\cdot\theta\cdot b')^r(b\cdot\theta\cdot b')^{l-2r}$$
$$+2r(l+1-2r)(\gamma\cdot\theta\cdot b)(\gamma\cdot\theta\cdot b)(b\cdot\theta\cdot b)^{r-1}(b'\cdot\theta\cdot b')^r(b\cdot\theta\cdot b')^{l-2r}$$
$$+2r(l+1-2r)(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b')(b'\cdot\theta\cdot b')^{r-1}(b\cdot\theta\cdot b)^r(b\cdot\theta\cdot b')^{l-2r}$$
$$+(l+1-2r)(l-2r)(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b)(b\cdot\theta\cdot b)^r(b'\cdot\theta\cdot b')^r(b\cdot\theta\cdot b')^{l-2r-1}]\,, \quad (4.4)$$

where $b=p+q'$, $b'=p'+q$, and F is given as follows for the different types of decays and exchanges listed in Table I.

Type	F
1	$-(M_l-i\gamma\cdot K)$
2	$-i\gamma_5(M_l-i\gamma\cdot K)$
3	$-i\gamma_5(M_l+i\gamma\cdot K)$
4	$(M_l+i\gamma\cdot K)$

(4.5)

As a preliminary to "Reggeization," we next express (4.4) in terms of Legendre polynomials:

$$T_J^{\text{scatt}}=g_lh_lF\left[\frac{l+1}{2l+3}\frac{(l!)^2}{(2l+2)!}2^{l+1}\right]\frac{1}{K^2+M_l^2}$$
$$\times\{E_1[(l+1)^2P_{l+1}-(2l+1)ZP_{l+1}'+Z^2P_{l+1}'']$$
$$+E_2P_{l+1}'+E_3(lP_{l+1}'-ZP_{l+1}'')+E_4P_{l+1}''\}\,, \quad (4.6)$$

with

$$E_1=(\gamma\cdot\theta\cdot b)(\gamma\cdot\theta\cdot b')(b\cdot\theta\cdot b)^{(l-1)/2}(b'\cdot\theta\cdot b')^{(l-1)/2}\,,$$
$$E_2=(\gamma\cdot\theta\cdot\gamma)(b\cdot\theta\cdot b)^{l/2}(b'\cdot\theta\cdot b')^{l/2}\,,$$
$$E_3=(b\cdot\theta\cdot b)^{l/2}(b'\cdot\theta\cdot b')^{l/2}\times\left[\frac{(\gamma\cdot\theta\cdot b)(\gamma\cdot\theta\cdot b)}{b\cdot\theta\cdot b}+\frac{(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b')}{b'\cdot\theta\cdot b'}\right],$$
$$E_4=(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b)(b\cdot\theta\cdot b)^{(l-1)/2}(b'\cdot\theta\cdot b')^{(l-1)/2}\,,$$

and

$$Z=b\cdot\theta\cdot b'/[(b\cdot\theta\cdot b)(b'\cdot\theta\cdot b')]^{1/2}.$$

We note, as an aside, that for ordinary π-nucleon scattering the sum of types 1 and 4 would correspond to both members of a parity-doublet trajectory being exchanged. Reference to (4.5) shows that the M_l factor will then cancel so that the fixed cuts of Carlitz and Kislinger[6] will not appear on "Reggeization."

We are interested in summing (4.6) over all members of a Regge family as before for the kinematics of the decay situation. Some of the kinematical factors become in this case

$$(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b')\to -M'^2[1-(M'/M_l)^2]^2,$$
$$(\gamma\cdot\theta\cdot b')(\gamma\cdot\theta\cdot b)\to -M'\left[1-\left(\frac{M'}{M_l}\right)^2\right]\times\left[\pm 2M-M'+\frac{M'}{M_l^2}(\mu^2-M^2)\right],$$
$$(\gamma\cdot\theta\cdot b)(\gamma\cdot\theta\cdot b')\to\left[1-\left(\frac{M'}{M_l}\right)^2\right]\left[(\mu^2-M^2)\times\left(2-\frac{M'^2}{M_l^2}\right)-M'(M'\mp 2M)\right], \quad (4.7)$$
$$(b'\cdot\theta\cdot b')\to -M'^2[1-(M'/M_l)^2],$$
$$(b\cdot\theta\cdot b')\to(\mu^2-M^2)[1-(M'/M_l)^2],$$
$$K^2\to -M'^2.$$

In (4.7) the upper sign holds for type-1 and -4 amplitudes while the lower sign holds for type-2 and -3. The kinematical factors we have not written do not go to zero as $M'\to M_l$. Finally the values of the quantity

[8] R. E. Behrends and C. Fronsdal, Phys. Rev. 106, 345 (1957); C. Fronsdal, Nuovo Cimento 9, 416 (1957).

TABLE I. Amplitude types.

Amplitude type	Strong vertex	Weak vertex	Decay	Exchange particle
1	1	1	s wave	$\frac{3}{2}^+, \frac{7}{2}^+, \ldots$ or $\frac{1}{2}^-, \frac{5}{2}^-, \ldots$
2	1	$i\gamma_5$	p wave	$\frac{3}{2}^+, \frac{7}{2}^+, \ldots$ or $\frac{1}{2}^-, \frac{5}{2}^-, \ldots$
3	$i\gamma_5$	1	p wave	$\frac{3}{2}^-, \frac{7}{2}^-, \ldots$ or $\frac{1}{2}^+, \frac{5}{2}^+, \ldots$
4	$i\gamma_5$	$i\gamma_5$	s wave	$\frac{3}{2}^-, \frac{7}{2}^-, \ldots$ or $\frac{1}{2}^+, \frac{5}{2}^+, \ldots$

F in the decay situation are

$$\begin{array}{cc} \text{Type} & F \\ \hline 1 & -(M_l+M') \\ 2 & -i\gamma_5(M_l-M') \\ 3 & -i\gamma_5(M_l+M') \\ 4 & +(M_l-M') . \end{array} \tag{4.8}$$

Now the Reggeization according to the Van Hove model proceeds as in the t-channel case. Again we shall only consider the Regge-pole singularity. In the zero-width approximation the position of the pole in the complex l plane is given by the solution of

$$M_l^2-M'^2=0. \tag{4.9}$$

We denote this solution by $\alpha=l+\frac{1}{2}$. Thus, because the kinematical factors of (4.1) vanish when (4.9) holds, we will have an analogous situation to the t-channel case. Specifically (4.6) will behave like

$$F[1-(M'/M_l)^2]^{\alpha/2-1/4}, \tag{4.10}$$

where F is given by (4.8). Thus type-1 and -3 amplitudes will behave like

$$(M_l-M')^{\alpha/2-1/4} \tag{4.11}$$

near $l=\alpha-\frac{1}{2}$. We see that these vanish for $\alpha>\frac{1}{2}$ and diverge for $\alpha<\frac{1}{2}$. Again we interpret the divergence as meaning that the given trajectory *can* contribute. On the other hand, type-2 and -4 amplitudes will behave like

$$(M_l-M')^{\alpha/2+3/4} \tag{4.12}$$

near $l=\alpha-\frac{1}{2}$. Trajectories of this type will not contribute for $\alpha>-\frac{3}{2}$. Thus we have given the justification of the rule (1.2).

V. s-CHANNEL POLES

The relevant diagram here is Fig. 1(a) with a meson of 4-momentum q coming in to the weak vertex. The calculation for this case is almost identical to the u-channel case. We can still use (4.6) to describe the associated scattering process when we redefine

$$K=p'+q', \quad b=p-q, \quad b'=p'-q'.$$

The position of the Regge pole is now given by the solution of

$$M_l^2-M^2=0.$$

Let the solution be $l=\alpha-\frac{1}{2}$. Then as in the u-channel case, the amplitude will behave like $(M_l-M)^{\alpha/2-1/4}$ for type-1 and -4 amplitudes and as $(M_l-M)^{\alpha/2+3/4}$ for type-2 and -3 amplitudes. Thus rules (1.2) also hold for the s-channel case.

VI. DISCUSSION

We have seen that the structure of the Van Hove model is such as to forbid a number of trajectories from contributing to the Regge pole for various nonleptonic decay amplitudes. As mentioned previously, we have no guarantee that Regge-pole terms are important here. However, our result seems to be almost "kinematical" in nature so it may be worthwhile to see what its consequences are.

First consider the t channel. The first thing that comes to mind is the K^* trajectory. In this case $\alpha(\mu^2)>0$, so by (1.1) it should not contribute. On the other hand, the K or K_A trajectories could contribute, on the assumption that they are parallel to the K^* trajectory. Thus the t-channel contribution to the s waves (which would come from K^*) should vanish and we might expect that nonpole terms (which adequately explain s waves in the current-algebra approach) could be important. Alternatively, baryon poles could contribute to the s waves. In the case of the p waves, the poles must contribute even in the current-algebra picture. Baryon poles furthermore must be important because, as is well known, meson poles (K or K_A trajectory) by themselves predict Σ_+^+ (p wave)$=0$.

The situation is much more complicated for the baryon poles. Table II summarizes the results of using the criterion (1.2) on the usual trajectories.[9] For simplicity, the assumption is made that all trajectories are parallel to the nucleon trajectory.[10] It is clear that there are a sufficient number of contributions for us to be able to make a number of fits to the data. We may

TABLE II. Baryon trajectory contributions. (x means no contribution. OK means that a possible contribution exists.)

Trajectory	s wave s channel	s wave u channel	p wave s channel	p wave u channel
$\frac{1}{2}^+$ octet	x	x	x	OK
$\frac{3}{2}^+$ decuplet	x	OK	x	x
$\frac{1}{2}^-$ singlet	x	OK	x	x
$\frac{3}{2}^-$ singlet	x	x	x	OK
$\frac{1}{2}^-$ octet	OK	OK	x	OK
$\frac{3}{2}^-$ octet	x	x	OK	OK
$\frac{5}{2}^-$ octet	x	OK	x	x

[9] The trajectories listed are those corresponding to the particles listed on p. 197 of Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

[10] We have considered the $\frac{1}{2}^-$ octet and $\frac{5}{2}^-$ octet given in Ref. 9 to belong to different Regge families. If the members of the $\frac{5}{2}^-$ octet are assumed to be Regge recurrences of the corresponding members of the $\frac{1}{2}^-$ octet, they will contribute to all the cases in Table II.

ask if a very simple model can be constructed to fit the p waves (assuming that the nonpole current-algebra result fits the s waves). The simplest model might contain the $\frac{1}{2}^+$ octet trajectory by itself. In this case, only the u channel contributes. If only $SU(3)$ Clebsch-Gordan coefficients are used, all the p-wave amplitudes stand correctly in relation to one another when reasonable d/f ratios are used for the weak and strong vertices. However, if the magnitude *and* sign of $(d/f)_{\rm weak}$ is taken from the current-algebra results, the relative sign of the s waves and the p waves is incorrect. Thus it may be that a number of trajectories are required to fit the data. Our criterion (1.2) may be useful in limiting this number.

We would like to remark that several authors have proposed "duality" models[11] for nonleptonic hyperon decays where the s-wave amplitude is given by the K^*-trajectory exchange in the t channel. Essentially these models claim to pick up the K^* Regge-pole term so that our result would have very serious consequences for them if it is assumed that any result which is true in the Van Hove model is true in any Regge model.

Finally, we note that a more detailed study of the l-plane analyticity of the decay amplitudes could be useful (though rather complicated). If ideas like the one presented here turn out to be useful, they may, of course, be applied to other decay processes.

[11] S. Nussinov and J. L. Rosner, Phys. Rev. Letters **23**, 1264 (1969); K. Kawarabayashi and S. Kitakado, *ibid.* **23**, 440 (1969); Mahiko Suzuki, *ibid.* **22**, 1217 (1969); **22**, 1413(E) (1969).

PHYSICAL REVIEW D VOLUME 3, NUMBER 9 1 MAY 1971

Study of the Decay $\omega \to \pi\pi\gamma$*†

NAOMI LEVY‡ AND PAUL SINGER
Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel
(Received 21 October 1970)

The effect of π-π interaction in isospin-zero states on the characteristics of the $\omega \to \pi\pi\gamma$ decay is investigated by means of partial-wave dispersion relations. With the ρ-exchange contribution as the driving term and elastic unitarity for π-π scattering on the right-hand cut, we arrive at integral equations whose solutions explicitly depend on the π-π phase shift. Various scattering-length and resonance models are used to represent the π-π phase shift. The high-energy contribution to the integral equation is treated in three ways: by subtraction, adding an effective pole on the left-hand cut, and normalizing the amplitude at the f_0 resonance to a Veneziano model for $\omega \to \pi\pi\gamma$. Numerical results for rate and spectrum are discussed in the light of the possibility of discriminating by experimental results among the various models for π-π interaction.

I. INTRODUCTION

In this paper we present a detailed study of the first-order electromagnetic process $\omega \to 2\pi+\gamma$. Apart from the natural challenge of a theoretical understanding of the radiative transitions among vector and pseudoscalar mesons, there is an additional feature making this process an interesting object for experimental and theoretical study, since it provides the opportunity for investigating the dynamics of π-π interaction in even angular momentum states because of the particular final-state configuration of the pion pair.

It is well known that the analysis of π-π interaction[1] is quite difficult, mainly because of the complications arising from the presence of additional hadrons in the final states of the processes investigated. As a result, our knowledge of the details of the π-π interaction, especially in the S-wave angular momentum state, is still in a quite unsettled position, which occasions the understandable interest in any process which could contribute to advancing the study of π-π interaction.

In the process $\omega \to \pi\pi\gamma$, only even angular momentum states are allowed for the two pions, as a result of the charge-conjugation invariance of the electromagnetic interactions. Moreover, the pions are restricted to the isospin-zero state when the process is considered to first order in the fine-structure constant. This determines the relation between the two possible final charge states, namely, $\Gamma(\omega \to \pi^+\pi^-\gamma) = 2\Gamma(\omega \to \pi^0\pi^0\gamma)$.

The first theoretical estimate of this process,[2] describing it as a transition $\omega \to (\rho)+\pi \to \gamma+\pi+\pi$, predicts a relative decay rate[3] of $\Gamma(\omega \to \pi^+\pi^-\gamma+\pi^0\pi^0\gamma)/\Gamma(\omega \to \text{all}) = 1.8\times10^{-4}$. This number is obtained by

* Based in part on a thesis submitted by N. L. to the Senate of the Technion in partial fulfilment of the requirements for the D.Sc. degree.

† Research supported in part by Stiftung Volkswagenwerk.

‡ Present address: Theoretical Group, DESY, 2 Hamburg 52, Germany.

[1] G. L. Kane, in Proceedings of the 1970 Conference on Meson Spectroscopy, Philadelphia, Pa. (unpublished). See also the Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, edited by F. Loeffler and E. Malamud, Argonne National Laboratory, 1969 (unpublished).

[2] P. Singer, Phys. Rev. **128**, 2789 (1962).

[3] The values quoted for ω decay in the compilation tables [Particle Data Group, Phys. Letters **33B**, 1 (1970)] are $\Gamma(\omega \to \text{all}) = 11.9\pm1.3$ MeV, out of which $(90\pm4)\%$ is made of the 3π decay mode, while the dominant electromagnetic decay to $\pi^0\gamma$ accounts for $(9.4\pm1.2)\%$.