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Cabibbo's Theory and Second-Order $SU(3)$-Symmetry-Breaking Corrections

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We discuss second-order corrections to the vector and axial-vector baryon currents, arising from partial conservation of axial-vector current and partial conservation of vector current in a field-algebra model. It is shown that the amount of symmetry breaking is severely limited if an acceptable fit to leptonic-baryon-decay data is required. This does not seem to be consistent with information obtained from meson data and K^+-nucleon scattering.

I. INTRODUCTION

Leptonic baryon decays (lbd) have been the subject of rather intense experimental study.[1] These data are usually analyzed in the light of Cabibbo's $SU(3)$-symmetric theory,[2] which has fared extremely well in these tests, in which a three- to four-parameter fit is made to a set of 11 data points.[3] Since $SU(3)$ is necessarily a broken symmetry, the excellent agreement between theory and experiment is an intriguing fact. It is thus to be expected that lbd will set rather strict limitations on the type and amount of $SU(3)$-symmetry breaking.

In the present paper we analyze lbd in an $SU(3)\times SU(3)$-symmetric field-algebra model, broken by imposing the requirement of partial conservation of vector current (PCVC) and of axial-vector current (PCAC). Symmetry-breaking effects have up to now only been considered in connection with the axial-vector current and only in a rather crude and phenomenological manner[4] by allowing the axial-vector Cabibbo angle θ_A to be different from the vector angle θ_V. Symmetry-breaking effects on the vector current were supposed to be negligible by appealing to the Ademollo-Gatto theorem.[5] We explicitly calculate these effects and show them to be prohibitively large if as much breaking is included as suggested by an analysis of meson data[6,7] and K^+-nucleon scattering.[8]

In Sec. II we summarize some generalities. For more details the reader is referred to Ref. 6, the notation of which we use whenever pertinent. To get an idea of what is to be expected from a theoretical calculation of lbd, we present a fit to the most recent data in the symmetry limit. This is done in Sec. III. Section IV contains the discussion of the realistic case, when symmetry-breaking effects are included.

[1] See, e.g., H. Filthuth, in *Proceedings of the Topical Conference on Weak Interactions*, edited by J. S. Bell (CERN, Geneva, 1969).

[2] N. Cabibbo, Phys. Rev. Letters **10**, 521 (1963).

[3] F. Eisele *et al.*, Z. Physik **225**, 383 (1969).

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[7] L. E. Wood *et al.*, Washington University, St. Louis, Mo., report, 1970 (unpublished); H. T. Nieh and H. S. Tsao, Phys. Rev. D **1**, 2663 (1970).

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II. GENERALITIES

As usual we assume the existence of 18 spin-zero mesons contained in the meson matrix M, which transform according to the $(3,3^*)$ representation of chiral $SU(3)\times SU(3)$. Out of the nine scalar mesons, we actually only need the two strange doublets κ, since the divergence of the strangeness-changing vector current will be proportional to the κ field. The vector and axial-vector gauge fields $V_\mu{}^\alpha$, $A_\mu{}^\alpha$, as well as the left-handed baryons $L=\frac{1}{2}(1+\gamma_5)B$ and the right-handed baryons $R=\frac{1}{2}(1-\gamma_5)B$, transform as $[(8,1),(1,8)]$.

A "super-Lagrangian" is now constructed according to well-known rules employing covariant derivatives.[9] Since the transverse parts of our currents are vector- or axial-vector-meson dominated, we only need the free-baryon term and the vector-meson–baryon term of the total Lagrangian in order to calculate the relevant matrix elements. The longitudinal parts are fixed by appealing to PCAC and PCVC, which are implemented by adding a symmetry-breaking term of the form[10]

$$\mathcal{L}_{\mathrm{SB}}=\mathrm{Tr}\{F(M+M^\dagger)\}\,, \tag{1}$$

where

$$F=\begin{pmatrix} f_1 & & \\ & f_1 & \\ & & f_2 \end{pmatrix}$$

and $M+M^\dagger=2\sigma$ contains the nonet of scalar mesons. Only σ_0 and σ_8 acquire nonvanishing vacuum expectation values and we may parametrize $\langle 0|\sigma|0\rangle$ as follows:

$$\langle 0|\sigma|0\rangle=\begin{pmatrix} a & & \\ & a & \\ & & b \end{pmatrix}. \tag{2}$$

$a\neq 0$ breaks chiral symmetry and $b\neq a$ breaks $SU(3)$ symmetry.

In the tree approximation for the case at hand, it is sufficient to consider the following[11] baryon Lagrangian

[9] See, e.g., Ref. 8 for more details in setting up the Lagrangian.

[10] Since we are not interested in the details of the meson spectrum, we do not include the π-η $U(3)\times U(3)$-violating mass-splitting term in $\mathcal{L}_{SB}$.

[11] We have omitted terms of the form $\mathrm{Tr}(\bar{L}MM^\dagger)\,\mathrm{Tr}(M^\dagger MR)$. Since L and R are traceless, one would only shift the Λ mass by amounts proportional to $(b^2-a^2)^2$.

$\mathfrak{L}_B$:

$$\begin{aligned}\mathfrak{L}_B=-\tfrac{1}{2}\operatorname{Tr}\{&\sum_{n=0}^{1}c_n[\bar{L}(MM^\dagger)^n\gamma_\mu D_\mu L+\bar{R}(M^\dagger M)^n\gamma_\mu D_\mu R]+\sum_{m=0}^{1}d_m[\bar{L}\gamma_\mu D_\mu L(MM^\dagger)^m+\bar{R}\gamma_\mu D_\mu R(MM^\dagger)^m]\\&+e_2[\bar{L}MM^\dagger\gamma_\mu D_\mu LMM^\dagger+\bar{R}M^\dagger M\gamma_\mu D_\mu RM^\dagger M]+\sum_{n=0}^{1}\alpha_n[\bar{L}M(M^\dagger M)^nRM^\dagger+\bar{R}M^\dagger(MM^\dagger)^nLM]\\&+\sum_{m=0}^{1}\beta_m[\bar{L}MRM^\dagger(MM^\dagger)^m+\bar{R}M^\dagger LM(M^\dagger M)^m]+\gamma_2[\bar{L}MM^\dagger MRM^\dagger MM^\dagger+\bar{R}M^\dagger MM^\dagger LMM^\dagger M]\}+\text{H.c.}\end{aligned}\tag{3}$$

In order to allow for renormalization of the baryon currents, we have to add

$$\begin{aligned}\mathfrak{L}_R=h_1&\operatorname{Tr}\{\bar{L}\gamma_\mu D_\mu MM^\dagger L+\bar{R}\gamma_\mu D_\mu M^\dagger MR\}\\&+h_2\operatorname{Tr}\{\bar{L}\gamma_\mu LD_\mu MM^\dagger+\bar{R}\gamma_\mu RD_\mu M^\dagger M\}+\text{H.c.},\end{aligned}\tag{4}$$

where the covariant derivatives are

$$\begin{aligned}D_\mu M&=\partial_\mu M-i\sqrt{2}g(V_\mu+A_\mu)M+i\sqrt{2}gM(V_\mu-A_\mu),\\D_\mu L&=\partial_\mu L-i(g/2)[V_\mu+A_\mu,L],\\D_\mu R&=\partial_\mu R-i(g/2)[V_\mu-A_\mu,R],\end{aligned}\tag{5}$$

and g is the vector-meson coupling constant.

c_n, d_n, e_2, α_n, β_n, and γ_2 are constants to be determined from the baryon mass spectrum and contain the model dependence of our approach. In terms of baryon renormalization constants and masses, we have

$$\begin{aligned}Z_N^{-1}&=c_0+d_0+c_1a^2+d_1b^2+e_2a^2b^2,\\Z_\Xi^{-1}&=c_0+d_0+c_1b^2+d_1a^2+e_2a^2b^2,\\Z_\Sigma^{-1}&=c_0+d_0+(c_1+d_1)a^2+e_2a^4,\\Z_\Lambda^{-1}&=c_0+d_0+\tfrac{1}{3}(c_1+d_1)(a^2+2b^2)+\tfrac{1}{3}e_2(a^4+2b^4),\end{aligned}\tag{6}$$

$$\begin{aligned}m_N&=Z_Nab[\alpha_0+\beta_0+\alpha_1a^2+\beta_1b^2+\gamma_2a^2b^2],\\m_\Xi&=Z_\Xi ab[\alpha_0+\beta_0+\alpha_1b^2+\beta_1a^2+\gamma_2a^2b^2],\\m_\Sigma&=Z_\Sigma a^2[\alpha_0+\beta_0+\alpha_1a^2+\beta_1a^2+\gamma_2a^4],\\m_\Lambda&=Z_\Lambda[\tfrac{1}{3}(\alpha_0+\beta_0)(a^2+2b^2)+\tfrac{1}{3}(\alpha_1+\beta_1)(a^4+2b^4)\\&\qquad+\tfrac{1}{3}\gamma_2(a^6+2b^6)].\end{aligned}\tag{7}$$

Finally we state the partial conservation laws:

$$\begin{aligned}\partial_\mu\left(\frac{m_0^2}{g}V_\mu{}^a\right)&=-\frac{1}{\sqrt{2}}i(f_1-f_2)\sigma^a,\quad &a=4,5,6,7\\\partial_\mu\left(\frac{m_0^2}{g}A_\mu{}^a\right)&=\sqrt{2}f_1\varphi^a,\quad &a=1,2,3\\\partial_\mu\left(\frac{m_0^2}{g}A_\mu{}^a\right)&=\frac{1}{\sqrt{2}}(f_1+f_2)\varphi^a,\quad &a=4,5,6,7\end{aligned}\tag{8}$$

where φ^a is the pseudoscalar nonet and m_0 may be identified with the ρ-meson mass.

III. $SU(3)$-SYMMETRIC FIT TO HYPERON DECAYS

We consider reactions of the type

$$B\to B'+l+\nu_l,\tag{9}$$

where B and B' are the initial and final baryons, l is either an electron or a muon, and ν_l the corresponding neutrino. The four-momenta and masses of the baryons are P, M and P', M'.

In terms of the usual parametrization, the matrix element of the hadronic current is

$$\begin{aligned}\langle B'|\mathcal{J}_\mu(0)|B\rangle=\bar{u}_{B'}(P')\bigg[&f_1(Q^2)\gamma_\mu+\frac{f_2(Q^2)}{M}\sigma_{\mu\nu}Q_\nu\\&+i\frac{f_3(Q^2)}{M}Q_\mu+g_1(Q^2)\gamma_\mu\gamma_5+i\frac{g_3(Q^2)}{M}Q_\mu\gamma_5\bigg]u_B(P),\end{aligned}\tag{10}$$

with

$$Q=P-P',\quad\sigma_{\mu\nu}=(1/2i)[\gamma_\mu,\gamma_\nu],\quad\gamma_\mu{}^\dagger=\gamma_\mu$$

and the metric is chosen such that $P^2=-M^2$.

In the tree approximation, reaction (9) proceeds via spin-one and spin-zero meson intermediate states. With the aid of $\mathfrak{L}_B$ and $\mathfrak{L}_R$, we obtain the following matrix

TABLE I. Coupling constants used in the matrix elements of the vector and axial-vector currents.

Decay	f_V	f_A
$n\to p$	1	$(Z_N^{-1}-4a^2h_1)Z_N/Z_\pi$
$\Sigma^\pm\to\Lambda$	0	$-(\sqrt{\tfrac{2}{3}})2a^2(h_1+h_2)(Z_\Sigma Z_\Lambda)^{1/2}/Z_\pi$
$\Lambda\to p$	$-(\sqrt{\tfrac{3}{2}})[\tfrac{1}{2}(Z_N^{-1}+Z_\Lambda^{-1})-\tfrac{1}{3}(b-a)^2(2h_1-h_2)](Z_NZ_\Lambda)^{1/2}/Z_\kappa$	$-(\sqrt{\tfrac{3}{2}})[\tfrac{1}{2}(Z_N^{-1}+Z_\Lambda^{-1})-\tfrac{1}{3}(b+a)^2(2h_1-h_2)](Z_NZ_\Lambda)^{1/2}/Z_K$
$\Sigma^-\to n$	$-[\tfrac{1}{2}(Z_N^{-1}+Z_\Sigma^{-1})+(b-a)^2h_2](Z_NZ_\Sigma)^{1/2}/Z_\kappa$	$-[\tfrac{1}{2}(Z_N^{-1}+Z_\Sigma^{-1})+(b+a)^2h_2](Z_NZ_\Sigma)^{1/2}/Z_K$
$\Xi^-\to\Lambda$	$(\sqrt{\tfrac{3}{2}})[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Lambda^{-1})-\tfrac{1}{3}(b-a)^2(h_1-2h_2)](Z_\Xi Z_\Lambda)^{1/2}/Z_\kappa$	$(\sqrt{\tfrac{3}{2}})[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Lambda^{-1})-\tfrac{1}{3}(b+a)^2(h_1-2h_2)](Z_\Xi Z_\Lambda)^{1/2}/Z_K$
$\Xi^-\to\Sigma^0$	$(1/\sqrt{2})[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Sigma^{-1})-(b-a)^2h_1](Z_\Xi Z_\Sigma)^{1/2}/Z_\kappa$	$(1/\sqrt{2})[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Sigma^{-1})-(b+a)^2h_1](Z_\Xi Z_\Sigma)^{1/2}/Z_K$
$\Xi^0\to\Sigma^+$	$[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Sigma^{-1})-(b-a)^2h_1](Z_\Xi Z_\Sigma)^{1/2}/Z_\kappa$	$[\tfrac{1}{2}(Z_\Xi^{-1}+Z_\Sigma^{-1})-(b+a)^2h_1](Z_\Xi Z_\Sigma)^{1/2}/Z_K$

TABLE II. Experimental data and predicted values in a one-angle fit for $b=a$.

Decay	Experimental data Width (sec^{-1})	g_1/f_1	Predicted values Width (sec^{-1})	g_1/f_1	f_1	f_2	g_1
$\Sigma^- \to \Lambda e\bar{\nu}$	$(4.02\pm0.40)\times10^5$	-0.35 ± 0.18[a]	4.04×10^5	...	0	1.449	0.604
$\Sigma^+ \to \Lambda e\nu$	$(2.51\pm0.58)\times10^5$	...	2.44×10^5	...	0	1.449	0.604
$\Lambda \to pe\bar{\nu}$	$(3.38\pm0.40)\times10^6$	0.77 ± 0.06	3.62×10^6	0.72	-0.300	-0.320	-0.217
$\Lambda \to p\mu\bar{\nu}$	$(5.37\pm2.30)\times10^5$	...	5.92×10^5	0.72	-0.300	-0.320	-0.217
$\Sigma^- \to ne\bar{\nu}$	$(7.11\pm0.30)\times10^6$	-0.25 ± 0.11	6.84×10^6	-0.29	-0.245	0.318	0.073
$\Sigma^- \to n\mu\bar{\nu}$	$(3.20\pm0.20)\times10^6$	...	3.22×10^6	-0.29	-0.245	0.318	0.073
$\Xi^- \to \Lambda e^-$	$(4.03\pm1.00)\times10^6$	...	3.36×10^6	0.21	0.300	-0.025	0.063

[a] This v lue corresponds to f_1/g_1.

elements:

$\Delta S=0$,

$$\langle B'|j_\mu|B\rangle=f_V(BB'\rho)\bar{u}_{B'}\left[\gamma_\mu+\frac{iQ_\mu(M-M')}{m_\rho^2}\right]u_B\frac{1}{Q^2+m_\rho^2},$$

$$\langle B'|j_{\mu5}|B\rangle=f_A(BB'A_1)\bar{u}_{B'}\left[\gamma_\mu\gamma_5\frac{m_{A_1}^2}{Q^2+m_{A_1}^2}-iQ_\mu\gamma_5(M+M')\left(\frac{1}{Q^2+m_{A_1}^2}-\frac{1}{Q^2+\mu_\pi^2}\right)\right]u_B, \tag{11}$$

$\Delta S\neq0$,

$$\langle B'|j_\mu|B\rangle=f_V(BB'K_V)\bar{u}_{B'}\left[\gamma_\mu\frac{m_{K_V}^2}{Q^2+m_{K_V}^2}+iQ_\mu(M-M')\left(\frac{1}{Q^2+m_{K_V}^2}-\frac{1}{Q^2+\mu_\kappa^2}\right)\right]u_B,$$

$$\langle B'|j_{\mu5}|B\rangle=f_A(BB'K_A)\bar{u}_{B'}\left[\gamma_\mu\gamma_5\frac{m_{K_A}^2}{Q^2+m_{K_A}^2}-iQ_\mu\gamma_5(M+M')\left(\frac{1}{Q^2+m_{K_A}^2}-\frac{1}{Q^2+\mu_K^2}\right)\right]u_B. \tag{12}$$

The coupling constants are given in Table I, where

$$Z_\pi=m_{A_1}^2/m_\rho^2,\quad Z_K=1+\tfrac{1}{4}(Z_\pi-1)(1+b/a)^2,\quad Z_\kappa=1+\tfrac{1}{4}(Z_\pi-1)(1-b/a)^2 \tag{13}$$

are the spin-zero meson-renormalization constants.

In the $SU(3)$-symmetry limit $b\to a$, we obtain the usual Cabibbo matrix elements, once the $\Delta S=0$ matrix elements are multiplied by $\cos\theta$ and the $\Delta S\neq0$ ones by $\sin\theta$. Since the couplings depend only on ratios of renormalization constants and b/a, we may, as far as Table I is concerned, put $Z_N=1$ and $a=\frac{1}{2}$. In this case we have the identification (for $b=a$)

$$h_1=1-(F+D)Z_\pi,\quad h_2=-1+(F-D)Z_\pi. \tag{14}$$

In order to get some feeling for how well the theory may be expected to agree with experiment, a fit to the available data was made in the case $b=a$. Since the experimental error of (g_1/f_1) for $n\to p$ is so much smaller than the other experimental errors, we may impose from the beginning[12]

$$F+D=1.2310, \tag{15}$$

neglecting the error of 0.0098.

The numbers needed to calculate the decay rates are taken from Ref. 13. We have also included $SU(3)$-symmetric weak magnetism terms, obtained by putting

$$\left(\frac{D}{F+D}\right)_{\text{e.m.}}=\frac{3}{2}\frac{1}{1-\mu_p/\mu_n}. \tag{16}$$

Table II displays the experimental data[14] and the predicted values in a one-angle fit. The following masses were used:

$$m_{A_1}=1070\text{ MeV},\quad m_{K_V}=980\text{ MeV},\quad m_{K_A}=1243\text{ MeV},\quad \mu_\kappa=1100\text{ MeV}.$$

We obtain a $\chi^2=7.3$ for eight degrees of freedom ($n_D=8$), giving a confidence level C.L.$=50\%$. The fitted parameters are

$$\theta=0.248\pm0.01,\quad \alpha=D/(F+D)=0.62\pm0.02\quad(h_2=-1.58\pm0.08).$$

For a two-angle fit, we obtain very similar results.

$$\chi^2=6.3,\quad n_D=7,\quad \text{C.L.}=53\%,\quad \theta_V=0.262\pm0.01,\quad \theta_A=0.217\pm0.03,\quad \alpha=0.60\pm0.02\quad(h_2=-1.50\pm0.09).$$

The agreement is excellent. It may be added that we are not interested in testing the hypothesis of conserved

[12] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).
[13] M. M. Nieto, Rev. Mod. Phys. **40**, 140 (1968).
[14] All data are taken from Ref. 12, except $g_1(0)/f_1(0)$ for $\Lambda\to p$, which is taken from M. Bagett *et al.*, Inst. für Hochenergiephysik, University of Heidelberg, report (unpublished).

TABLE III. Results of a two-angle fit for model ($\alpha_0+\beta_0$, α_1, β_1, d_1).

b/a	1.4	1.5	1.6
χ^2	11.1	7.7	8.9
C.L. (%)	12	38	28
θ_V	0.243	0.298	0.318
θ_A	0.248	0.178	0.153
$\alpha\|_{\Lambda\to p}$	0.55	0.51	0.48

TABLE IV. Results of a one-angle fit for $b/a=1.2$.

Model	χ^2	C.L. (%)	θ	h_2
($\alpha_0+\beta_0$, α_1, β_1, d_1)	7.8	44	0.245±0.01	−1.31±0.07
($\alpha_0+\beta_0$, α_1, β_1, e_2)	7.7	45	0.240±0.01	−1.23±0.07
($\alpha_0+\beta_0$, α_1, β_1, γ_2)	8.9	34	0.244±0.01	−1.44±0.08
(c_0+d_0, c_1, d_1, α_1)	12.3	10	0.246±0.01	−1.11±0.06

vector current (CVC), which predicts $f_1=0$ for $\Sigma\to\Lambda$ decays. This piece of information contributes the value 3.7 to χ^2. Omitting it increases the C.L. to 82 and 90% for the one- and two-angle fits, respectively.

We see that symmetry-breaking corrections must be very small, in order not to spoil the agreement obtained with pure $SU(3)$.

IV. $SU(3)$-BREAKING CORRECTIONS

In a number of recent publications, meson properties have been analyzed within the context of our approach.[6,7] These analyses differ essentially only in their assumption about the κ-meson parameters; namely, the mass μ_κ and decay constant f_κ. Typically the range of b/a is $1.2<b/a<2$. Another process sensitive to symmetry-breaking effects is K^+-nucleon scattering, the analysis of which also suggests $b/a\sim1.8$.[8]

As we shall see, values of b/a larger than ~1.3 are excluded by lbd in an essentially model-independent way, respecting of course the assumptions of Sec. II.

Given b/a, the matrix elements of our currents depend on the following eight parameters:

$$c_0+d_0,\ c_1,\ d_1,\ e_2,\ \alpha_0+\beta_0,\ \alpha_1,\ \beta_1,\ \text{and}\ \gamma_2.$$

The most obvious solution suggesting itself would be to assume pure mass mixing, i.e., putting $c_1=d_1=e_2=\gamma_2=0$, $c_0+d_0=1$, obtaining a mass formula of the Gell-Mann–Okubo type,[8]

$$3m_\Lambda+m_\Sigma(2b^2/a^2-1)-2(b/a)(m_N+m_\Xi)=0. \quad (17)$$

Inserting the baryon masses into Eq. (17) gives $b/a=0.95$ and a fit to lbd essentially identical to the one for $b=a$ is obtained. Yet this value of b/a is inconsistent with the results of Refs. 6 and 7. We thus look for a model which gives a good fit to lbd with b/a at least ≥1.2.

A large variety of models may be constructed, ranging from pure mass mixing to pure current mixing ($\alpha_0+\beta_0=1$, $\alpha_1=\beta_1=\gamma_2=e_2=0$), all of them collapsing to the model of Sec. III in the limit $b\to a$.

We start from $b/a\sim2$ and lower this value until an acceptable χ^2 is obtained. For such large values, all models are very bad. $\Sigma\to\Lambda$ decays are the most sensitive ones to b/a, since the $SU(3)$-symmetric anchor is absent, i.e., $f_1=0$ and g_1 depends on h_2, the function of which is to cancel large values of b/a.

For example, mass mixing with $\alpha_0+\beta_0$, α_1, β_1, and d_1[15] yields the following χ^2 for $\theta_A=\theta_V$:

$b/a=$	1.4	1.5	1.6
χ^2	11.2	14.1	19.3
C.L. (%)	17	8	1

For other models the χ^2 are even bigger. For two-angle fits, an acceptable χ^2 can only be found at the

TABLE V. Predicted values for ($\alpha_0+\beta_0$, α_1, β_1, d_1). Numbers in brackets correspond to the two-angle fit. $\delta\chi^2$ refers to the value contributed by the relevant item to the total χ^2.

	Width (sec^{-1})	g_1/f_1	f_1	f_2	g_1	$\delta\chi^2$ width (sec^{-1})	g_1/f_1
$\Sigma^-\to\Lambda e\bar{\nu}$	4.13×10^5	···	0.0	1.450	0.611	0.07	3.7
	[3.93×10^5]	[···]	[0.0]	[1.441]	[0.596]	[0.04]	[3.7]
$\Sigma^+\to\Lambda e\nu$	2.49×10^5	···	0.0	1.440	0.611	0.0006	···
	[2.37×10^5]	[···]	[0.0]	[1.432]	[0.596]	[0.05]	[···]
$\Lambda\to pe\bar{\nu}$	3.69×10^6	0.75	−0.296	−0.316	−0.223	0.63	0.07
	[3.36×10^6]	[0.77]	[−0.322]	[−0.345]	[−0.192]	[0.0007]	[0.0001]
$\Lambda\to p\mu^-$	6.05×10^5	0.75	−0.296	−0.316	−0.223	0.08	···
	[5.50×10^5]	[0.77]	[−0.322]	[−0.345]	[−0.192]	[0.003]	[···]
$\Sigma^-\to ne\bar{\nu}$	6.83×10^6	−0.34	−0.237	0.314	0.081	0.82	0.68
	[6.87×10^6]	[−0.28]	[−0.259]	[0.343]	[0.057]	[0.61]	[0.10]
$\Sigma^-\to n\mu^-$	3.21×10^6	−0.34	−0.237	0.314	0.081	0.98	···
	[3.25×10^6]	[−0.28]	[−0.259]	[0.343]	[0.057]	[1.34]	[···]
$\Xi^-\to\Lambda e^-$	3.20×10^6	0.21	0.293	−0.025	0.062	0.67	···
	[3.73×10^6]	[0.24]	[0.319]	[−0.027]	[0.061]	[0.09]	[···]

[15] We label the different models by the parameters used to adjust the baryon spectrum, e.g., ($\alpha_0+\beta_0$, α_1, β_1, d_1).

TABLE VI. Results of a two-angle fit for $b/a=1.2$.

Model	χ^2	C.L. (%)	θ_V	θ_A	h_2
$(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$	6.0	57	0.267 ± 0.01	0.205 ± 0.03	-1.22 ± 0.09
$(\alpha_0+\beta_0, \alpha_1, \beta_1, c_2)$	7.3	45	0.255 ± 0.01	0.218 ± 0.03	-1.19 ± 0.09
$(\alpha_0+\beta_0, \alpha_1, \beta_1, \gamma_2)$	8.3	34	0.258 ± 0.01	0.219 ± 0.03	-1.39 ± 0.09
$(c_0+d_0, c_1, d_1, \alpha_1)$	7.2	43	0.272 ± 0.007	0.190 ± 0.02	-0.97 ± 0.08

expense of having very unreasonable values for θ_A and θ_V. For $(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$, we get the results displayed in Table III.

For each decay, an effective $\alpha=D/(D+F)$ may be defined. An independent determination of this number can be obtained for $\Lambda\to p$ and $\Sigma\to\Lambda$ decays,[16] giving as some sort of average for these two decays an α in the range

$$0.58\leq\alpha\leq0.60. \tag{18}$$

Our two-angle fits for $b/a\geq1.4$ are excluded if we impose the requirement (18). Acceptable fits are only obtained for $b/a\leq1.2$. From now on we fix b/a to be

$$b/a=1.2$$

as the maximum value compatible with lbd.

We now list some results of a two-parameter fit. Table IV contains the χ^2 and parameters for the models giving the lowest χ^2; details are collected in Table V. For a two-angle fit we obtain the results of Table VI.

The case $(\alpha_0+\beta_0, \alpha_1, \beta_1, d_1)$ actually has a lower χ^2 than the symmetric two-angle fit, again at the expense of a greater value for $\theta_V-\theta_A$. Since $Z_K\neq Z_\pi\neq Z_\kappa$, one may hope to get good fits for θ_A closer to θ_V, since our symmetry breaking distinguishes between A and V. Unfortunately, this is not the case.

We conclude by stressing that any model of $SU(3)$-symmetry breaking should be tested with lbd. For the type of symmetry breaking used in this paper, the breaking parameter b/a cannot be larger than about 1.2. One may of course also take for granted the Gell-Mann–Okubo formula and pure mass mixing with $b/a\sim1$ and try to fit meson data and K^+-nucleon scattering into this picture.

[16] See, e.g., R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969), p. 441.

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Inclusive Reactions and Dual-Resonance Models*

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The theoretical and phenomenological advantages of looking at inclusive-type reactions in dual-resonance models are discussed. The case of single-particle distributions is considered in some detail, and part of the contributions explicitly evaluated. We find that such contributions exhibit "scaling" behavior of the type observed long ago by Amati, Fubini, and Stanghellini in the multiperipheral model and which was reproposed in more precise terms recently by Feynman and by Benecke, Chou, Yang, and Yen. The explicit form of the limiting distribution is given. A similar behavior is found in the case of the multiparticle distribution. The need of manageable techniques for computing high-energy limits of dual loop discontinuities is stressed. We finally comment on the possible relevance of our results on deriving bootstrap-type conditions from duality and in providing a way to understand the connection between the dual-resonance model and other models of current interest.

I. INTRODUCTION

ALMOST ten years ago, prior to any substantial high-energy data, Amati, Stanghellini, and Fubini[1] noted a remarkable *scaling* property of their multiperipheral model,[1,2] namely, that in the laboratory system, fixed low-mass clusters of secondary particles from high-energy collisions have a transverse momentum distribution which is independent of the incident energy and of the energy of the cluster.

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[1] D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento **26**, 896 (1962). See also K. Wilson, Acta Phys. Austriaca **17**, 37 (1963).

[2] L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento **25**, 626 (1962).