Unitarity Corrections to the Hard-Meson Analysis of K_{l3} Decay*

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We calculate the unitarity correction to the approximation of κ dominance of the scalar, strangeness-changing channel in the hard-meson analysis of K_{l3} decay by Chang and Leung. New ranges are obtained for the possible values of F_K/F_π and $f_+(0)$. In particular, the latter satisfies $f_+(0)<1$.

In the analysis of chiral symmetry breaking in K_{l3} decays by Chang and Leung,[1] the following assumptions were made within the context of the hard-meson approach to current-algebra calculations[2,3]: (i) The term in the strong-interaction Hamiltonian which breaks the chiral symmetry transforms under chiral $SU(3)$ according to the representation $(3,\bar{3})+(\bar{3},3)$[4,5]; (ii) the κ-π-K vertex is no more than quadratic in momenta, thus restricting the form of the primitive three-point vertex; and (iii) all three channels, including the scalar strangeness-changing one, are dominated by single-particle states. In the analysis of other three- and four-point functions, there has been some interest recently, following the work of Amatya, Pagnamenta, and Renner,[6] in retaining assumption (iii) for the pseudoscalar channels only [i.e., the hypothesis of partial conservation of the axial-vector current (PCAC)], and employing the constraints of elastic unitarity in the other channels.[7]

The simple κ-dominance approximation of Ref. 1 was used to obtain limits between which the physical values of F_K/F_π and $f_+(0)$ must lie. In this paper we calculate the unitarity correction to this approximation and find that the sign is such as to decrease the likely values of these quantities, with the magnitude estimated from elastic unitarity. In particular, we find that $f_+(0)$ satisfies the inequality $f_+(0)<1$.[8] Before turning to the calculation, we should mention that a possible feature of our solution is a broad resonance of mass ~ 1 BeV in the S-wave, $I=\frac{1}{2}$ πK channel. However, its exact mass and width depend rather sensitively on the value of F_K/F_π and the magnitude of the contributions to the unitarity sum coming from inelastic channels. For this reason, as well as the paucity of experimental data, we content ourselves in this paper with deriving the inequalities mentioned above, which are not so sensitive to these details.

We define the relevant three-point function by

$$\langle A_\mu{}^{\pi^+}(q_1)A_\nu{}^{K^0}(q_2)V_\lambda{}^{K^-}(q_3)\rangle = \int\int d^4x_1 d^4x_2 e^{-iq_1\cdot x_1}e^{-q_2\cdot x_2} \times\langle T(A_\mu{}^{\pi^+}(x_1)A_\nu{}^{K^0}(x_2)V_\lambda{}^{K^-}(0))\rangle_0\,, \quad (1)$$

with $q_1+q_2+q_3=0$. The normalization of the currents is such that the vector currents, for example, obey the equal-time commutation relations

$$[V_0{}^i(x),V_\mu{}^j(y)]_{x_0=y_0} = i\epsilon^{ijk}V_\mu{}^k(x)\delta^3(\mathbf{x}-\mathbf{y})+\text{Schwinger term}\,,$$

and $A_\mu{}^{\pi^+}=(1/\sqrt{2})(A_\mu{}^1+iA_\mu{}^2)$; our metric is such that on the mass shell, $q_1{}^2=-m_\pi{}^2$ and $q_2{}^2=-m_K{}^2$.

The hypothesis of PCAC implies that the diagonalization of the π and K channels can be effected by defining, in momentum space,[2]

$$\bar{A}_\mu{}^{\pi^+}(q_1)=A_\mu{}^{\pi^+}(q_1)-i(q_{1\mu}/m_\pi{}^2)\partial^\lambda A_\lambda{}^{\pi^+}(q_1)$$

and

$$\bar{A}_\nu{}^{K^0}(q_2)=A_\nu{}^{K^0}(q_2)-i(q_{2\nu}/m_K{}^2)\partial^\lambda A_\lambda{}^{K^0}(q_2)\,. \quad (2)$$

The way to proceed in the remaining channel, without assuming single-particle dominance, has been indicated by Gerstein, Schnitzer, and Weinberg.[3] It is necessary to define two spin-0 propagators in this channel, namely,

$$\langle\partial^\mu V_\mu{}^{K^+}(q_3)\partial^\nu V_\nu{}^{K^-}(-q_3)\rangle=-i\Delta(q_3)\,, \quad (3)$$

where $\Delta(q)$ has the spectral representation

$$\Delta(q)=\int du\,\frac{\rho(u)}{(q^2+u)} \quad (4)$$

and

$$\langle\partial^\mu V_\mu{}^{K^+}(q_3)V_\lambda{}^{K^-}(-q_3)\rangle=-q_{3\lambda}D(q_3)\,, \quad (5)$$

where $D(q)$ has the spectral representation

$$D(q)=\int du\,\frac{\rho(u)}{u(q^2+u)}. \quad (6)$$

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They are connected by the relations

$$q^2 D(q)+\Delta(q)=\Delta(0). \tag{7}$$

Thus, if we define

$$\bar{V}_\lambda{}^{K^-}(q_3)=V_\lambda{}^{K^-}(q_3)-iq_{3\lambda}\frac{D(q_3)}{\Delta(q_3)}\partial^\lambda V_\lambda{}^{K^-}(q_3), \tag{8}$$

we obtain the diagonalization requirement

$$\langle \partial^\mu V_\mu{}^{K^+}(q_3)\bar{V}_\lambda{}^{K^-}(-q_3)\rangle=0,$$

and the "primitive" three-point vertex is defined by[3]

$$\langle \bar{A}_\mu{}^{\pi^+}(q_1)\bar{A}_\nu{}^{K^0}(q_2)\bar{V}_\lambda{}^{K^-}(q_3)\rangle$$
$$=-\Delta_{\mu\mu'}{}^{A,\pi}(q_1)\Delta_{\nu\nu'}{}^{A,K}(q_2)\Delta_{\lambda\lambda'}{}^{V,K}(q_3)\times\Gamma^{\mu'\nu'\lambda'}(q_1,q_2,q_3), \tag{9}$$

where, on the right-hand side, we have the three spin-1 propagators corresponding to each channel. The form of the primitive three-point vertex $\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$, of course, lies outside the framework of current algebra. However, since we will be considering only the three-point function which corresponds to zero spin in each channel, we are interested only in the quantity $q_{1\mu}q_{2\nu}q_{3\lambda}\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$. Assuming that the primitive vertex is as smooth a function of the momenta as possible, we find the linear expansion[1,3]

$$\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$$
$$=A_1(g^{\mu\nu}q_1{}^\lambda+g^{\mu\lambda}q_1{}^\nu)+A_2(g^{\mu\nu}q_2{}^\lambda+g^{\nu\lambda}q_2{}^\mu)+A_3(g^{\mu\lambda}q_3{}^\nu+g^{\nu\lambda}q_3{}^\mu)+B_1(g^{\mu\nu}q_1{}^\lambda-g^{\mu\lambda}q_1{}^\nu)+B_2(g^{\mu\nu}q_2{}^\lambda-g^{\nu\lambda}q_2{}^\mu)+B_3(g^{\mu\lambda}q_3{}^\nu-g^{\nu\lambda}q_3{}^\mu). \tag{10}$$

Thus

$$q_{1\mu}q_{2\nu}q_{3\lambda}\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$$
$$=\tfrac{1}{2}A_1[q_1{}^4-(q_2{}^2-q_3{}^2)^2]+\tfrac{1}{2}A_2[q_2{}^4-(q_3{}^2-q_1{}^2)^2]+\tfrac{1}{2}A_3[q_3{}^4-(q_1{}^2-q_2{}^2)^2]. \tag{11}$$

Let us define the off-shell spin-0 matrix element by

$$F(q_1,q_2,q_3)=-i\frac{\Delta_\pi{}^{-1}(q_1)}{F_\pi m_\pi{}^2}\frac{\Delta_K{}^{-1}(q_2)}{F_K m_K{}^2}\times\langle \partial A^{\pi^+}(q_1)\partial A^{K^0}(q_2)\partial V^{K^-}(q_3)\rangle, \tag{12}$$

such that, on-shell, it can be written in terms of the K_{l3} form factors $f_\pm(q_3)$ as

$$F(q_1,q_2,q_3)\,|_{q_1{}^2=-m_\pi{}^2,\,q_2{}^2=-m_K{}^2}$$
$$\equiv F(q_3)=(m_K{}^2-m_\pi{}^2)f_+(q_3)-q_3{}^2f_-(q_3). \tag{13}$$

In relating $F(q_1,q_2,q_3)$ to $q_{1\mu}q_{2\nu}q_{2\lambda}\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$, we encounter several equal-time commutators between the time component of a current and a current divergence. We are therefore forced, at this stage, to make a particular choice of symmetry-breaking model. We choose the model of Gell-Mann, Oakes, and Renner,[4] namely,

$$\mathcal{H}'=-u_0-cu_8, \tag{14}$$

with the scalar nonet u_i, and their chiral partners v_i, forming a basis of the representation $(3,\bar{3})+(\bar{3},3)$ of chiral $SU(3)$. The constant c can be expressed in terms of the masses, decay constants, and renormalization constants of π and K, namely,

$$c=-\sqrt{2}\frac{F_K m_K{}^2 Z_K{}^{-1/2}-F_\pi m_\pi{}^2 Z_\pi{}^{-1/2}}{F_K m_K{}^2 Z_K{}^{-1/2}+\frac{1}{2}F_\pi m_\pi{}^2 Z_\pi{}^{-1/2}}. \tag{15}$$

In this model, we find

$$\Delta(0)=(F_K Z_K{}^{1/2}-F_\pi Z_\pi{}^{1/2})\times(F_K m_K{}^2 Z_K{}^{-1/2}-F_\pi m_\pi{}^2 Z_\pi{}^{-1/2}). \tag{16}$$

Using the definitions of the various diagonalized currents and the equal-time commutation relations of both current algebra and the symmetry-breaking model, we find

$$F(q_1,q_2,q_3)=\frac{C^{A,\pi}}{F_\pi}\frac{C^{A,K}}{F_K}C^{V,K}\frac{\Delta(q_3)}{\Delta(0)}q_{1\mu}q_{2\mu}q_{3\lambda}\Gamma^{\mu\nu\lambda}(q_1,q_2,q_3)$$
$$+\frac{\Delta(q_3)}{\Delta(0)}\left[\frac{Z_\pi{}^{1/2}}{Z_K{}^{1/2}}\Delta_K{}^{-1}(q_2)-\frac{Z_K{}^{1/2}}{Z_\pi{}^{1/2}}\Delta_\pi{}^{-1}(q_1)\right]$$
$$+\frac{q_1{}^2-q_2{}^2}{2F_K F_\pi}\left[(F_K{}^2+F_\pi{}^2)\frac{\Delta(q_3)}{\Delta(0)}-D(q_3)\right]$$
$$-\frac{q_3{}^2}{2F_K F_\pi}\left[(F_K{}^2-F_\pi{}^2)\frac{\Delta(q_3)}{\Delta(0)}-\frac{F_K Z_K{}^{1/2}+F_\pi Z_\pi{}^{1/2}}{F_K Z_K{}^{1/2}-F_\pi Z_\pi{}^{1/2}}D(q_3)\right], \tag{17}$$

where $q_{1\mu}\Delta_{\mu\mu'}{}^{A,\pi}(q_1)=C^{A,\pi}q_{1\mu'}$, with analogous definitions in the other channels.

Note that, apart from the first term, the coefficients of $\Delta(q_3)$ and $D(q_3)$ on the right-hand side of Eq. (17) are, at most, quadratic in momenta. In the κ-dominance approximation, these coefficients sum up to give the off-shell κ-K-π vertex, which is Eq. (5) of Chang and Leung.[1] Their assumption, that there are no coefficients of $\Delta(q_3)$ and $D(q_3)$ quartic in momenta in the total expression for $F(q_1,q_2,q_3)$, leads to the vanishing of the first term on the right-hand side of Eq. (17). In dispersion-theoretic language this assumption, which we follow, amounts to requiring that

$$\langle \partial A^{\pi^+}(q_1)\partial A^{K^0}(q_2)\partial V^{K^-}(q_3)\rangle$$

satisfy at least once-subtracted dispersion relations in $q_i{}^2$, $i=1$, 2, and 3, with the other squared momenta fixed. Equations (13) and (17) then yield

$$2F_K F_\pi F(q_3)=(m_K{}^2-m_\pi{}^2)\left[(F_K{}^2+F_\pi{}^2)\frac{\Delta(q_3)}{\Delta(0)}-D(q_3)\right]-q_3{}^2\left[(F_K{}^2-F_\pi{}^2)\frac{\Delta(q_3)}{\Delta(0)}-\frac{F_K Z_K{}^{1/2}+F_\pi Z_\pi{}^{1/2}}{F_K Z_K{}^{1/2}-F_\pi Z_\pi{}^{1/2}}D(q_3)\right]. \tag{18}$$

Following Ref. 1, we will assume further that $F(q_3)$ satisfies an unsubtracted dispersion relation. Thus, from Eq. (18), we have

$$\int du\,\rho(u)=\frac{F_K Z_K^{1/2}+F_\pi Z_\pi^{1/2}}{F_K Z_K^{1/2}-F_\pi Z_\pi^{1/2}}\frac{[\Delta(0)]^2}{F_K^2-F_\pi^2}. \quad (19)$$

Equations (16) and (19) are expressions with strictly positive left-hand sides. Now Chang and Leung[1] and Glashow and Weinberg[5] make the hypothesis that $(Z_K/Z_\pi)^{1/2}>0$, and the former also take $F_K/F_\pi>0$. Making the same requirements and neglecting the second term of the second bracket on the right-hand side of (16), we find the inequality

$$F_K/F_\pi>(Z_\pi/Z_K)^{1/2}. \quad (20)$$

From Eq. (19), this implies

$$F_K/F_\pi>1. \quad (21)$$

It is convenient at this stage to introduce two positive constants m_+^2 and m_-^2, which are defined by

$$m_+^2-m_-^2=\frac{(F_K Z_K^{-1/2}-F_\pi Z_\pi^{-1/2})(F_K m_\pi^2 Z_K^{1/2}+F_\pi m_K^2 Z_\pi^{1/2})}{F_K^2-F_\pi^2} \quad (22)$$

and

$$m_+^2 m_-^2=\frac{(m_K^2-m_\pi^2)\Delta(0)}{F_K^2-F_\pi^2}. \quad (23)$$

From (21) and (23), m_+^2 and m_-^2 have the same sign, defined to be positive. Then the sign of the right-hand side of (22) determines which root is m_+^2 and which is m_-^2. It is also clear from the definitions that in general one root is zeroth order in $SU(3)$-symmetry violations, while the other is second order. In fact, we have

(i) if $F_K/F_\pi>(Z_K/Z_\pi)^{1/2}$, then m_+^2 (zeroth order) $>(m_K^2-m_\pi^2)>m_-^2$ (second order),

whereas

(ii) if $F_K/F_\pi<(Z_K/Z_\pi)^{1/2}$, then m_-^2 (zeroth order) $>(m_K^2-m_\pi^2)>m_+^2$ (second order).

Then we may rewrite Eq. (18) as

$$2F_K F_\pi F(q_3)=(m_K^2-m_\pi^2)\left(\frac{q_3^2}{m_+^2}+1\right)\left(\frac{q_3^2}{m_-^2}-1\right)D(q_3)+(m_K^2-m_\pi^2)(F_K^2+F_\pi^2)-(F_K^2-F_\pi^2)q_3^2. \quad (24)$$

We might note that if $F_K/F_\pi=(Z_K/Z_\pi)^{1/2}$, which corresponds to the value of c used by Gell-Mann, Oakes, and Renner,[4] then $m_+^2=m_-^2=(m_K^2-m_\pi^2)$.

Finally, using condition (19), Eq. (24) may itself be rewritten as

$$F(q_3)=\frac{(m_K^2-m_\pi^2)}{2F_K F_\pi}\int du\,\frac{(u/m_+^2-1)(u/m_-^2+1)}{u(q_3^2+u)}\rho(u). \quad (25)$$

Now, from definitions (3), (4), and (13), we obtain the unitarity condition

$$\rho(u)=\phi(u)R(u)|F(u)|^2,\quad u\geqslant(m_K+m_\pi)^2 \quad (26)$$

with

$$\phi(u)=\frac{3}{32\pi^2}\frac{[u-(m_K+m_\pi)^2]^{1/2}[u-(m_K-m_\pi)^2]^{1/2}}{u}$$

and $R(u)$ the inelasticity factor such that

$$R(u)=1\quad\text{for}\quad (m_K+m_\pi)^2\leq u\leq(m_K+2m_\pi)^2$$

and

$$R(u)>1\quad\text{for}\quad u>(m_K+2m_\pi)^2.$$

Also, from Eq. (25), we have

$$\mathrm{Im}F(u)=\pi\frac{m_K^2-m_\pi^2}{2F_K F_\pi}\frac{(u/m_+^2-1)(u/m_-^2+1)}{u}\rho(u), \quad (27)$$

which, together with the unitarity condition (26), leads to

$$\mathrm{Im}F^{-1}(u)=-\pi\frac{m_K^2-m_\pi^2}{2F_K F_\pi}\frac{(u/m_+^2-1)(u/m_-^2+1)}{u}\times\phi(u)R(u). \quad (28)$$

We solve this equation by writing a twice-subtracted dispersion relation for $F^{-1}(q_3)$, subtracted at $q_3^2=-m_+^2$ and $q_3^2=m_-^2$, to yield

$$F^{-1}(q_3)=\frac{F_K Z_K^{1/2}+F_\pi Z_\pi^{1/2}}{(m_K^2-m_\pi^2)(F_K Z_\pi^{1/2}+F_\pi Z_K^{1/2})}+\frac{q_3^2(F_K^2-F_\pi^2)}{(m_K^2-m_\pi^2)(F_K Z_\pi^{1/2}+F_\pi Z_K^{1/2})(F_K m_K^2 Z_K^{-1/2}-F_\pi m_\pi^2 Z_\pi^{-1/2})}-\frac{m_K^2-m_\pi^2}{2F_K F_\pi}\left(\frac{q_3^2}{m_+^2}+1\right)\left(\frac{q_3^2}{m_-^2}-1\right)\int_{(m_K+m_\pi)^2}^{\Lambda}du\,\frac{\phi(u)R(u)}{u(q_3^2+u)}. \quad (29)$$

The first point to note about this solution is that the first two terms constitute the result of Ref. 1. The last term is therefore the unitarity correction to the κ-dominance approximation. Secondly, note that the integral has been given a cutoff Λ. Thus, if $R(u)$ should increase too rapidly with u, the cutoff ensures convergence of the integral. However, even when inelastic effects are neglected, the cutoff is required to prevent

(a) the appearance of Castillejo-Dalitz-Dyson poles and (b) the superconvergence of $F(q_3)$. That $F(q_3)$ cannot be superconvergent follows from the positivity of $\mathrm{Im}F(q_3)$. This in turn follows from the fact that $m_+^2<(m_K^2+m_\pi^2)$, which can be seen by rewriting definition (22) as

$$m_+^2-m_-^2=(m_K^2+m_\pi^2)-\frac{F_K m_K^2 Z_K^{-1/2}+F_\pi m_\pi^2 Z_\pi^{-1/2}}{F_K m_K^2 Z_K^{-1/2}-F_\pi m_\pi^2 Z_\pi^{-1/2}}\frac{\Delta(0)}{F_K^2-F_\pi^2}, \tag{30}$$

from which it follows that

$$[(m_K^2+m_\pi^2)-m_+^2][(m_K^2+m_\pi^2)+m_-^2]>0.$$

From the solution (29), evaluated at $q_3^2=0$, we find

$$\frac{F_K}{F_\pi f_+(0)}=\frac{F_K(F_K Z_K^{1/2}+F_\pi Z_\pi^{1/2})}{F_\pi(F_K Z_\pi^{1/2}+F_\pi Z_K^{1/2})}+\frac{(m_K^2-m_\pi^2)^2}{2F_\pi^2}\int_{(m_K+m_\pi)^2}^{\Lambda} du\,\frac{\phi(u)R(u)}{u^2}. \tag{31}$$

The left-hand side of this relation has the experimental value 1.23 ± 0.05. We will assume that a reliable estimate of the integral on the right-hand side may be obtained by neglecting inelastic effects, i.e., taking $R(u)=1$, and using a large value of the cutoff Λ. This is not unreasonable owing to the u^{-2} convergence factor in the integrand. Then the last term on the right-hand side makes the small but significant contribution ~0.05. If we define

$$\alpha=\frac{F_K(F_K Z_K^{1/2}+F_\pi Z_\pi^{1/2})}{F_\pi(F_K Z_\pi^{1/2}+F_\pi Z_K^{1/2})},\quad \frac{Z_K^{1/2}}{Z_\pi^{1/2}}=\frac{(\alpha-1)F_K F_\pi}{F_K^2-\alpha F_\pi^2}, \tag{32}$$

then $\alpha=1.18\pm0.05$.

From (20) and (32), F_K/F_π satisfies

$$\left(\frac{\alpha}{2-\alpha}\right)^{1/2}>\frac{F_K}{F_\pi}>\alpha^{1/2}. \tag{33}$$

Numerically, the upper limit is 1.20 ± 0.06 and the lower limit is 1.08 ± 0.02. The corresponding range for $f_+(0)$ is

$$0.97\pm0.01>f_+(0)>0.88\mp0.02,$$

which implies that the second-order $SU(3)$ symmetry-breaking corrections to $f_+(0)$ are negative,[8] and no larger than 15% in magnitude. The value of c used by Gell-Mann, Oakes, and Renner[4] corresponds to $F_K/F_\pi=(2\alpha-1)^{1/2}=1.16\pm0.04$ and $f_+(0)=0.95$.

In conclusion, by comparing these results with those of the κ-dominance approximation corresponding to the input $F_K/F_\pi f_+(0)=1.23\pm0.05$, we find that the unitarity correction decreases the likely values of both F_K/F_π and $f_+(0)$.

After the details of this paper had been worked out, we received a paper by de Alwis,[9] in which possible second-order symmetry-breaking corrections to the K_{l3} theorem of Dashen and Weinstein[10] were calculated, using ideas similar to those presented in this paper. Although the results agree to the extent that the quantity $\frac{1}{2}(F_K/F_\pi-F_\pi/F_K)$ could be as small as ~0.08, the approaches markedly differ concerning the hard-meson assumptions. Whereas we have assumed a particularly smooth primitive three-point vertex,[1,5] de Alwis assumes instead the validity of perturbation expansions around the chiral $SU(3)$ symmetry limit, applied to nonprimitive vertices.

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