Decay Correlations in Resonance Production Using Quarks of Spin 3/2

JERROLD FRANKLIN
*Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel**
and
Temple University, Philadelphia, Pennsylvania 19122

(Received 31 August 1970)

The decay correlation predictions of Białas and Zalewski for the process $P+B\to V+B^*$ are derived using spin-$\frac{3}{2}$ quarks, obeying normal Fermi statistics. An additional assumption is required of an approximate left-right symmetry in the transversity frame for quark-quark scattering.

I. INTRODUCTION

In previous papers,[1] we have shown that many results of the quark model do not depend on the magnitude of the quark spin. We drew the conclusion from that work that there was little indication from experiment that spin-$\frac{1}{2}$ quarks were to be preferred over quarks of any other spin. There are also quark-model predictions for decay correlations in resonance production[2] that are in good agreement with experiment.[3] These predictions have thus far been made using spin-$\frac{1}{2}$ quarks and their success could be used to infer that there is evidence to prefer spin $\frac{1}{2}$ for the quark.

In this paper, we treat the process

$$P+B\to V+B^*,\tag{1}$$

where P represents a pseudoscalar meson, B represents a spin-$\frac{1}{2}$ baryon, V represents a vector meson, and B^* represents a spin-$\frac{3}{2}$ baryon, using spin-$\frac{3}{2}$ quarks (satisfying normal Fermi statistics), and obtain all the predictions previously found for spin-$\frac{1}{2}$ quarks.[2] To achieve this, we have to make one additional assumption about quark-quark scattering (that it is approximately left-right symmetric in the transversity frame—i.e., in the frame with the quantization axis normal to the production plane), but it does not seem to us to be an unreasonable assumption, nor need the symmetry be an exact one. We follow the calculation of Białas and Zalewski[2] (BZ) rather closely, except that we use spin-$\frac{3}{2}$ quarks where they have used spin $\frac{1}{2}$. We treat process (1) in detail, but a similar derivation could be made for other processes they consider.

II. DERIVATION OF BZ CLASS-(A) RELATIONS

Białas and Zalewski[2] use transversity amplitudes for the quarks and for the hadrons. For the $PB\to VB^*$ process, the most general transversity-amplitude description is (assuming parity conservation)

$$\begin{array}{ccccccc} & B^*\to & \tfrac{3}{2} & \tfrac{1}{2} & -\tfrac{1}{2} & -\tfrac{3}{2} & B\downarrow \end{array}$$

$$N_1=\begin{pmatrix} A' & 0 & B & 0\\ 0 & A & 0 & B'\end{pmatrix}\begin{matrix}\tfrac{1}{2}\\ -\tfrac{1}{2}\end{matrix},$$

$$N_0=\begin{pmatrix} 0 & E & 0 & -\bar{F}\\ F & 0 & -\bar{E} & 0\end{pmatrix},\tag{2}$$

$$N_{-1}=\begin{pmatrix} \bar{B}' & 0 & \bar{A} & 0\\ 0 & \bar{B} & 0 & \bar{A}'\end{pmatrix}.$$

Here N can be considered a three-dimensional matrix with the element $N_{i,jk}$ representing the amplitude for $PB_j\to V_iB_k^*$ (e.g., B represents $PB_{1/2}\to V_1B_{-1/2}^*$), where the subscripts (ijk) refer to the respective transversities.

The full content of the spin-$\frac{1}{2}$-quark-model class-(A) predictions (as defined in BZ) are that

$$\begin{aligned} &A'=\sqrt{3}A\,,\quad B'=\sqrt{3}B\,,\quad \bar{A}'=\sqrt{3}\bar{A}\,,\\ &\bar{B}'=\sqrt{3}\bar{B}\,,\quad F=0=\bar{F}\,,\quad \bar{E}=E.\end{aligned}\tag{3a}$$

Obviously any model that reproduces these equalities will be equivalent, in all its detailed predictions, to the usual quark model. The additional class-(B) predictions of BZ have

$$A=\bar{A}\,,\quad A'=\bar{A}'\,,\tag{3b}$$

and their class-(C) predictions are

$$B=\bar{B}\,,\quad B'=\bar{B}'.\tag{3c}$$

For spin-$\frac{1}{2}$ quarks, these added predictions follow from established symmetry principles, but put a strain on the quark additivity assumption and have frame-dependent difficulties.[4]

For spin-$\frac{3}{2}$ quarks, the $PB\to VB^*$ amplitudes in terms of quark-quark amplitudes can be calculated in a straightforward but lengthy manner using hadron wave functions derived previously.[1] The resulting ampli-

* Address until June, 1971.

[1] J. Franklin, Phys. Rev. **180**, 1583 (1969); **181**, 1984 (1969).

[2] A. Białas and K. Zalewski, Nucl. Phys. **B6**, 465 (1968). We denote this paper by BZ. Earlier references can be found therein.

[3] A. Białas and K. Zalewski, Phys. Letters **26B**, 170 (1968), find agreement with five earlier experiments for their class-(A) predictions. K. Böckmann *et al.*, *ibid.* **28B**, 72 (1968), find agreement for the BZ class-(A) and class-(B) relations, but disagreement for the class-(C) predictions in the case $\pi^+p\to N^*\omega$. The cases of best agreement correspond to equalities between statistical tensors or density-matrix combinations satisfied to within 10–20% with about twice that much experimental uncertainty, although in many cases the deviation and the experimental uncertainties are considerably greater.

[4] H. Lipkin, Phys. Rev. **183**, 1221 (1969).

tudes are

$$\begin{aligned}
A={}&-[3(3\bar{1},1\bar{3})+3(33,11)+2\sqrt{3}(31,1\bar{1})+\sqrt{3}(1\bar{1},\bar{1}\bar{3})\\
&+\sqrt{3}(13,\bar{1}1)+2(11,\bar{1}\bar{1})]/(1200)^{1/2},\\
A'={}&-\{[\sqrt{3}(3\bar{1},1\bar{3})+2\sqrt{3}(\bar{1}3,\bar{3}1)]\\
&+[\sqrt{3}(33,11)+2\sqrt{3}(\bar{1}\bar{1},\bar{3}\bar{3})]\\
&+[2(31,1\bar{1})+4(\bar{1}1,\bar{3}\bar{1})]+3(1\bar{1},\bar{1}\bar{3})\\
&+3(13,\bar{1}1)+2\sqrt{3}(11,\bar{1}\bar{1})\}/(1200)^{1/2},\\
B={}&[3(\bar{3}\bar{1},\bar{1}\bar{3})+3(\bar{3}3,\bar{1}1)+2\sqrt{3}(\bar{3}1,\bar{1}\bar{1})\\
&+\sqrt{3}(\bar{1}\bar{1},1\bar{3})+\sqrt{3}(\bar{1}3,11)+2(\bar{1}1,1\bar{1})]/(1200)^{1/2},\\
B'={}&\{[\sqrt{3}(\bar{3}\bar{1},\bar{1}\bar{3})+2\sqrt{3}(13,31)]+[\sqrt{3}(\bar{3}3,\bar{1}1)\\
&+2\sqrt{3}(1\bar{1},3\bar{3})]+[2(\bar{3}1,\bar{1}\bar{1})+4(11,3\bar{1})]+3(\bar{1}\bar{1},1\bar{3})\\
&+3(\bar{1}3,11)+2\sqrt{3}(\bar{1}1,1\bar{1})\}/(1200)^{1/2},\\
E={}&[6(\bar{3}3,\bar{3}3)-6(\bar{3}\bar{3},\bar{3}\bar{3})+2(\bar{3}1,\bar{3}1)-2(\bar{3}\bar{1},\bar{3}\bar{1})\\
&-3(13,13)+3(1\bar{3},1\bar{3})-(11,11)\\
&+(1\bar{1},1\bar{1})]/(2400)^{1/2},\\
F={}&[3(33,\bar{1}3)-3(1\bar{3},\bar{3}\bar{3})+3(13,\bar{3}3)-3(3\bar{3},\bar{1}\bar{3})\\
&+(31,\bar{1}1)-(1\bar{1},\bar{3}\bar{1})+(11,\bar{3}1)\\
&-(3\bar{1},\bar{1}\bar{1})]/(2400)^{1/2}.
\end{aligned}\tag{4}$$

Our notation is that (mn,rs) is a quark-quark amplitude with m, n, r, and s representing twice the quark transversities. A bar over an index is used to denote negative quark transversity, so that, e.g., $(3\bar{1},1\bar{3})$ is the amplitude for initial quark transversities $\frac{1}{2}$, $-\frac{3}{2}$ scattering to final quark transversities $\frac{3}{2}$, $-\frac{1}{2}$. The barred hadron amplitudes $\bar{A}$, $\bar{B}$, $\bar{A}'$, $\bar{B}'$, $\bar{E}$, and $\bar{F}$, respectively, are given by Eqs. (4) with all quark transversities reversed.

Observation of Eqs. (4) indicates that Eqs. (3a), and hence all the class-(A) BZ relations, follow if the additional assumption is made for quark-quark amplitudes that

$$(mn,rs)=(\bar{r}\bar{s},\bar{m}\bar{n}). \tag{5}$$

Assumption (5) does not follow from any established hadron symmetry, but is an additional assumption about quark-quark scattering. It corresponds to a left-right symmetry (plus time reversal invariance) for quark-quark scattering in the transversity description. The symmetry need not be exact but only approximately true to the extent that the BZ class-(A) predictions agree with experiment.[3] It can be noted further from Eqs. (4) that a large part of Eqs. (3a) follow without (5) so that any deviation from the symmetry in quark-quark scattering will have a smaller effect on the BZ class-(A) predictions. The symmetry (5) can be put in terms of quark-quark helicity amplitudes, where it corresponds to the condition

$$H_{cd,ab}=0 \text{ if } b+c-a-d=\text{odd integer}, \tag{6}$$

where cd, ab are the quark helicities. It does not seem to have a simple physical meaning in terms of quark helicities.

The additional class-(B) and class-(C) BZ predictions would follow here, just as in BZ, if further equalities are assumed between quark-quark amplitudes. These further equalities follow from established invariance principles, as in BZ, and have the same interpretation difficulties and ambiguities in comparison with experiment.[4]

III. SUMMARY

We have shown that the quark-model decay correlation predictions of BZ for $PB\to VB^*$ follow also for spin-$\frac{3}{2}$ quarks. This required an additional assumption of a left-right symmetry in the transversity frame for quark-quark scattering, but even if this symmetry is not exact, there would still be a strong tendency for the BZ relations to follow. Our conclusion is that the remarkable success of the BZ class-(A) predictions need not be taken as evidence that quarks have spin $\frac{1}{2}$. If quarks do indeed underly the success of these predictions, we have shown that spin-$\frac{3}{2}$ quarks could just as reasonably give similar results.

We have not treated other processes here, but the process $PB\to VB^*$ probably contains enough spin complexity to be typical of the BZ predictions. Application of spin-$\frac{3}{2}$ quarks to other hadron reactions would be straightforward but tedious, and we feel that the BZ results would still follow as they do here. We could also extend our efforts to higher-spin quarks with a correspondingly lengthy calculation. We do not feel this is worth the effort at this stage, but we expect that the results of this work are general enough that, with the additional assumption (5), use of any spin quark would lead to the BZ relations for any hadron process.

Although it is our conclusion, from this and previous investigations,[1] that there is not as yet experimental evidence for any particular spin of the quark, we remark here that the quark would be tested by a measurement of the Ξ^- or Σ^- magnetic moments,[1] both of which are in principle measurable.[5,6]

Note added in proof. The approximate left-right symmetry assumed for the quark transversity amplitudes can be achieved by relating q-q amplitudes to the corresponding $\bar{q}$-q amplitudes. This is because the meson-baryon amplitude always equals the sum of q-q and $\bar{q}$-q amplitudes. For example, for the process $\pi^+p\to\omega N^{*++}$, the left-right symmetry can be by having

$$(mn,rs)_{\mathcal{P}\mathcal{N}\to\mathcal{N}\mathcal{P}}=(\bar{m}\bar{n},\bar{r}\bar{s})_{\bar{\mathcal{N}}\mathcal{N}\to\bar{\mathcal{P}}\mathcal{P}}.$$

Here $\mathcal{P}$ and $\mathcal{N}$ refer to the quark types and $\bar{\mathcal{P}}$ and $\bar{\mathcal{N}}$ to the respective antiquarks. This is seen to be a left-right symmetry between a q-q amplitude and the corresponding "crossed" $\bar{q}$-q amplitude. This, in fact, is the same type of symmetry as the observed "mirror symmetry" in $\pi^\pm-p$ polarization (experimentally, the

[5] C. E. Wiegand, Phys. Rev. Letters **22**, 1235 (1969), suggests using the fine structure in Σ^- atoms (for which spectral lines have been observed) to measure the Σ^- magnetic moment.

[6] A Brookhaven group has Ξ^- data that, when analyzed, could give a value for the Ξ^- moment [T. Kycia (private communication)].

π^--p polarization is approximately the negative of the π^+-p polarization[7]). Since there does not seem to be mirror symmetry in $K^\pm$-p polarization,[7] we would not expect the BZ predictions to be as well satisfied for K reactions as for π reactions. This observation is not in conflict with the data,[3] but greater accuracy is required in the K experiments to test it.

ACKNOWLEDGMENTS

I would like to thank Professor Paul Singer and Simcha Rosendorff for their hospitality at the Technion, where this work was completed. I would also like to thank Harry Lipkin for a useful communication and Arnon Dar for calling may attention to the approximate mirror symmetry in $\pi^\pm$-p polarization.

[7] M. Borghini *et al.*, Phys. Letters **21**, 1141 (1966); **24B**, 77 (1967); **31B**, 405 (1970); R. J. Esterling *et al.*, Phys. Rev. Letters **21**, 1410 (1968).

PHYSICAL REVIEW D VOLUME 3, NUMBER 1 1 JANUARY 1971

Symmetry Breaking and Spin-Zero Mass Spectrum*

J. Schechter and Y. Ueda
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 3 September 1970)

We discuss the mass spectrum in a very general chiral $SU(3)\times SU(3)$ model of pseudoscalar and scalar mesons. The case of an exactly symmetric Lagrangian and spontaneous breakdown is covered, as well as the more realistic case when there is also some intrinsic symmetry breaking in the Lagrangian. Our treatment is facilitated by the derivation of a mass formula which holds for all situations. In the realistic case the structure of the theory forces the mass of the ninth pseudoscalar meson to come out right and leaves us with the freedom to choose the masses of all scalar mesons (except the κ) as high as we like. The structure of the intrinsic symmetry breaking term is predicted. We also include some discussion of the baryons in this model and estimate the corrections to the Cabibbo theory of semileptonic hyperon decay.

I. INTRODUCTION

At the present time it seems likely that the structure of strong-interaction physics is related to a broken $SU(3)\times SU(3)$ chiral symmetry group.[1] The discussion of symmetry breaking inevitably involves one with a fairly complicated nonlinear system, so it is advisable to study a relatively simple model in detail. This is done in the present paper for the $SU(3)$ "σ" model of mesons, from the "spontaneous-breakdown" point of view.[2]

Although there have been a number of interesting previous papers, we believe that the subject is far from exhausted. Generally, the previous work has been concerned either with special forms of interactions,[3] with discussions of nonlinear realizations of the chiral group,[4] or with generalizations[5] to gauge field theories. Here we develop a simple method for calculating the mass spectrum for all cases of interest. This lets us easily obtain the preceding results as well as interesting new ones, both for the situation when the Lagrangian is exactly chiral symmetric and when there is some symmetry breaking added. The former case represents a complete spontaneous breakdown of symmetry and gives some zero-mass bosons,[6] a thorough discussion of the resulting spectrum is presented. The latter case gives a more realistic mass spectrum, and we find that the mass of the ninth pseudoscalar meson is well predicted.

Taking the symmetric part of the interaction to be the most general nonderivative one possible, we use fields transforming as linear realizations of the chiral group. It will be noted that one advantage which has been claimed for nonlinear realizations[4]—that of suppressing information about hard-to-observe particles—actually occurs also for the mass spectrum when linear realizations are used. We do not include gauge fields, in order to avoid more complication of an already complicated system; this addition may be desirable. (It should be noted, though, that certain predictions of chiral theories, such as the meson-nucleon scattering lengths, seem to be independent[7] of the presence of gauge fields in the theory.)

In Sec. II the formalism for computing the meson mass spectrum is discussed. Isotopic spin invariance is not assumed at first, and in the following paper the

* Work supported by the U. S. Atomic Energy Commission.
[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[2] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).
[3] M. Lévy, Nuovo Cimento **52A**, 23 (1967); P. de Mottoni and E. Fabri, *ibid.* **54A**, 42 (1968); G. Cicogna, F. Strocchi, and R. V. Caffarelli, Phys. Rev. D **1**, 1197 (1970); G. Cicogna, *ibid.* **1**, 1786 (1970).
[4] W. A. Bardeen and B. W. Lee, Phys. Rev. **177**, 2389 (1969).
[5] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969). This review article contains a large bibliography.
[6] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962); S. A. Bludman and A. Klein, *ibid.* **131**, 2364 (1963).
[7] J. Schechter and Y. Ueda, Phys. Rev. **188**, 2184 (1969).