Expectations for Particle-Nucleus Cross Sections at Very High Energies and the Hadron Mass Spectrum

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Striking decreases in particle-nucleus total cross sections at high energies are predicted, due to the presence of coherent intermediate resonance contributions. A method of utilizing this effect to probe the high-mass hadron spectrum is outlined.

EXTENSIVE theoretical[1] and experimental[2] studies have been made of elastic and particle production processes on nuclei at energies now available to accelerators. In the near future, the range of available energies will increase, and one can begin to consider the question of whether or not effects might appear at high energies which have not yet been seen. We will examine the problem of elastic proton-nucleus scattering (the extension to other elastic processes is trivial) and show that dramatic reductions in the total p-A cross section for light nuclei are to be expected because of the presence of coherent intermediate states which, although suppressed at energies $\sim$30 GeV/c, become increasingly important at energies $\gtrsim$100 GeV/c. In addition to the interest inherent in the particle-nucleus cross sections themselves, we show that certain gross properties of the higher-mass hadronic states can be investigated by measuring the energy dependence of this reduction.

For the reaction

$$p+A \to p+A \tag{1}$$

the usual Glauber[3]-theory approach considers only those multiple-scattering diagrams like Fig. 1(a), where the proton undergoes a number of collisions without changing its identity. It has been pointed out, however,[4] that diagrams such as that in Fig. 1(b), in which N^* are created coherently in the nucleus and then are coherently converted back to protons farther downstream, should be present and should interfere with the "standard" diagrams in Fig. 1(a). Such effects have not yet been observed because (1) the amplitude to produce N^* from a proton is small[5] (if we characterize the N^* production amplitude by G and the elastic scattering by σ, then experimentally[6] $G^2/\sigma^2 \sim 10^{-3}$), and (2) there is a momentum transfer $\Delta_m=(m_{N^*}{}^2-m^2)/2P_{\rm lab}$ associated with the change of mass at the production vertex, so that at low energies the nuclear form factor suppresses the coherent production process.

At high energies the momentum transfer will become smaller for a given N^* mass, so that the form-factor suppression will become less important. In addition, higher-mass N^* will then be able to participate in the reaction, so that the number of intermediate N^* in Fig. 1(b) will increase, thus counteracting, at least partially, the smallness of G^2/σ^2. Thus, a study of the energy dependence of the proton-nucleus total cross section should shed some light on the occurrence of resonances at very high masses without actually calling for a detailed study of N^* production in the high-mass region.

We shall proceed by calculating the particle-nucleus elastic scattering amplitude, and then use the optical theorem to get the total particle-nucleus cross section. For simplicity, we shall write the ground-state density for all nuclei as

$$|\Psi|^2=\left(\frac{1}{\sqrt{\pi}R}\right)^{3A}\prod_{i=1}^{A}e^{-r_i{}^2/R^2}, \tag{2}$$

where R^2 is related to the rms radius of the nucleus by $R^2=\frac{2}{3}r_{\rm rms}{}^2$. The validity of the approximation for small-momentum-transfer reactions has been discussed elsewhere.[7] The Glauber theory for the case of standard elastic scattering [i.e., the sum of all graphs such as that in Fig. 1(a)] has been worked out,[8] and we simply note that the result for the amplitude F_0 for this process is

$$F_0=\tfrac{1}{2}iP(R^2+2a)\exp\left(\frac{R^2\Delta^2}{4A}\right)\sum_{K=1}\binom{A}{K}(-1)^{K+1}\times\left[\frac{\sigma(1-i\alpha)}{2\pi(R^2+2a)}\right]^K\exp\left[\frac{-(R^2+2a)\Delta^2}{4K}\right], \tag{3}$$

where we have written the elementary proton-nucleon

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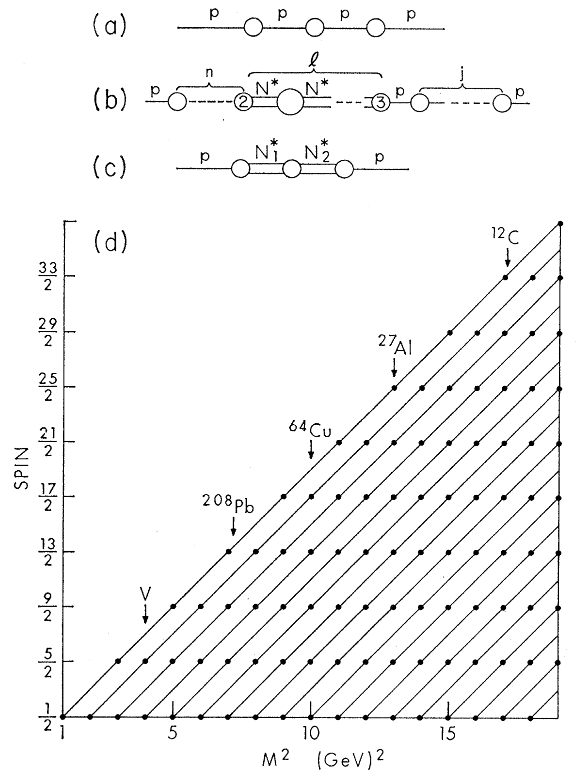

FIG. 1. (a)–(c) Some multiple-scattering diagrams; (d) the occurrence of Regge recurrences and secondaries for the protons. The arrows mark $m_{\max}^*$ for the nuclei in case (i) when $p=100$ GeV/c, and V marks the limit beyond which the residue in Eq. (11) suppresses the N^* contribution in the Veneziano model.

amplitude for momentum transfer δ as

$$f(\delta)=(P/4\pi)(i+\alpha)\sigma e^{-a\delta^2/2} \tag{4}$$

and Δ (δ) is the total transverse momentum transferred to the nucleus (nucleon).

Let us call the amplitude which sums all two-step graphs of the type in Fig. 1(b) F_{2i}, where the i refers to the fact that we consider the ith type of N^* in the intermediate state (we shall discuss this point more fully below). Let us also write the amplitude for the reaction $pN\to N^*N$ on the nth nucleon as

$$g(\delta)=(G_iP/4\pi)e^{-a\delta^2/2}e^{i\Delta_m z_n}.$$

Then the two-step analog to Eq. (3) is just

$$F_{2i}=\tfrac{1}{2}iPG_i^2e^{R^2\Delta^2/4A}(R^2+2a)\times\sum_{K=2}^{A}\sum_{j=0}^{K-2}\sum_{l=0}^{K-j-2}\sum_{n=0}^{K-j-l-2}(-1)^{K+1}\times\binom{A}{K}\frac{1}{K}\left[\frac{\sigma(1-i\alpha)}{2\pi(R^2+2a)}\right]^{n+j}\left[\frac{\sigma^*(1-i\alpha^*)}{2\pi(R^2+2a)}\right]^{l}\times e^{-\Delta^2(R^2+2a)/4K}I_z\,, \tag{5}$$

where σ^* and α^* denote the total cross section and phase for N^*-N scattering just as σ and α do for proton-N scattering. To make the notation a little clearer, we are summing over those processes in which the incident proton scatters n times, converts to the N^* on the nucleon labeled "2," the N^* then scatters l times, reconverts to a proton at "3", and the proton scatters j times before leaving the nucleus. The quantity I_z is the result of the integration over the z position of the nucleons, and is given by

$$I_z=\left(\frac{1}{(\sqrt{\pi})R}\right)^{A}\int_{-\infty}^{\infty}dz_3\int_{-\infty}^{\infty}dz_2\,\theta(z_3-z_2)e^{i(z_3+z_2)\Delta_m(1-2/A)}\times e^{-(z_3^2+z_2^2)/R^2}\left[\int_{-\infty}^{z_2}e^{2i\Delta_m z/A}e^{-z^2/R^2}dz\right]^{n}\times\left[\int_{z_2}^{z_3}e^{2i\Delta_m z/A}e^{-z^2/R^2}dz\right]^{l}\left[\int_{z_3}^{\infty}e^{2i\Delta_m z/A}e^{-z^2/R^2}dz\right]^{j}\times[(\sqrt{\pi})R]^{A-K}e^{-\Delta_m^2R^2(A-K)/A^2}. \tag{6}$$

The θ functions and truncated integrals arise, of course, from the ordering implicit in the fact that the proton must be converted to an N^* to the left of the point where the N^* is reconverted.

The general evaluation of I_z is rather cumbersome, but there is a reasonable approximation scheme which will allow a simple evaluation of the integrals to be made. We know that the integrals over the impact-parameter plane will ensure that in processes such as those considered here, the most important contribution to the cross section comes from a region of the nucleus in a ring at the outer edge.[3] We therefore replace $\int_{-\infty}^{z_2}$ and $\int_{z_3}^{\infty}$ in Eq. (6) by $\int_{-\infty}^{0}$ and $\int_0^{\infty}$, respectively. The error involved in this replacement is small, since these integrals contribute small terms of order $\Delta_m R/A$ to third- and higher-order scattering, and do not contribute in the leading term at all. If, in addition, we drop terms of order z^2/R^2 in the $\int_{z_2}^{z_3}$ and terms of order Δ^2R^2 in $\int dz_3dz_2$, a relatively simple expression for I_z is found:

$$I_z=\frac{1}{2}\left(\frac{1}{\sqrt{2\pi}}\right)^{l}\frac{\Gamma(l+1)}{\Gamma(\frac{1}{2}l+1)}e^{-\frac{1}{2}\Delta_m^2R^2(1-2/A)}\times\left[\frac{1}{2}\left(1-i\frac{\Delta_m R}{(\sqrt{\pi})A}\right)\right]^{n+j}, \tag{7}$$

which, substituted back into Eq. (5), gives us the result for F_{2i} which we sought. The complete elastic scattering amplitude will then be given by

$$F_{2i}=F_0+\textstyle\sum_i F_{2i}. \tag{8}$$

We note that the exponential factor in I_z explicitly suppresses the large-mass contributions in the sum over i. This is the nuclear form-factor effect we mentioned earlier. We also note that this result is similar to those derived earlier for photon-nucleus[9] and neutrino-nucleus[10] total cross sections.

The leading term in the amplitude F_{2i} is a double-scattering term, and hence will *increase* the Glauber shadowing effect and *decrease* the cross section. In addition, we see that F_{2i} contains a factor $G_i^2(m_i) \times\exp[-(m_i^2-m^2)^2R^2/4P^2]$, where m_i is the mass of the ith resonance contributing. This means that σ_{pA} will depend on the masses of the higher N^* resonances (or, more exactly, on the masses of those N^* which can be made in a p-p collision by Pomeranchon exchange), on the number of such resonances, and on their coupling strengths (or Regge residues). While a detailed investigation aimed at untangling the various N^* is being carried out, it is possible to make some general statements about the high-mass spectrum simply by observing the energy dependence of σ_{pA} for various nuclei.

For the sake of definiteness, let us assume that the N^* which will contribute to F_{2i} are located at masses given by the Regge picture, in which a resonance will occur when a given trajectory or one of its secondaries crosses a physical value of the angular momentum in a Chew-Frautschi plot. The situation for the proton is shown in Fig. 1(d). (Note that only the first few resonances on the leading trajectory and the first secondary have actually been experimentally seen.[6]) The resonances occur at

$$m^*=m(2m+n+1)^{1/2} \tag{9}$$

for a particle with angular momentum $J=(4m+1)/2$ on the nth secondary.

We see that the number of resonances which could conceivably contribute to the sum over i in Eq. (8) is, in principle, infinite. In practice, of course, for a given projectile momentum, there will be a maximum above which resonances will not make significant contributions to the total cross section. By investigating the manner in which the sum over i is cut off, we can gain some knowledge of the properties of the higher-mass resonances.

In order to see how this could occur, let us consider two extreme models of the high-mass hadron spectrum.

Case (i). Assume that all residues G_i are equal up to a mass greater than $m_{\max}^*$ defined below, so that the cutoff in the sum over i arises entirely because of the factor $\exp(-\Delta_m^2R^2/2)$ in F_{2i}. In this case we have a model in which high-mass resonances continue to couple strongly to other particles, and the nuclear form factor suppresses the highest-mass contributions to the cross section. In this case, the maximum mass which can contribute significantly to F_{2i} will be given by

$$m_{\max}^*\sim 2P/R. \tag{10}$$

Case (ii). Assume that for the higher masses, $G_i(m_i)$ decreases. For example, in the Veneziano model,[11] it has been shown that the residues decrease as a polynomial in mass times an exponential. Let us therefore consider the case where

$$G_i(m_i)=G_0e^{-m_i/m}, \tag{11}$$

where G_0 is picked to match the $N^*(1480)$ data. We assume that whatever the decrease in the coupling strength is, the scale will be set by m, the proton mass. The very-high-mass resonances in this case couple very weakly, so whether they are "there" or not becomes largely a matter of semantics.

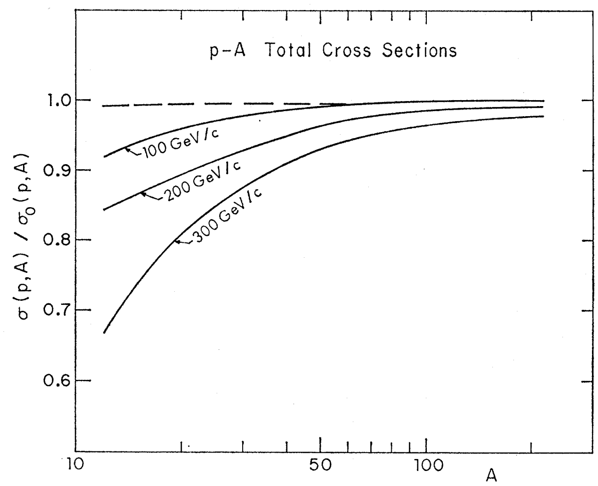

FIG. 2. Ratio of σ_{pA} to p-A cross section calculated neglecting F_{2i} at various energies. The heavy lines correspond to case (i), the dashed line (which is the same for all energies) to case (ii). We have taken $\sigma=\sigma^*=40$ mb. Changes in σ^* do not affect the results very much. For reference at 30 GeV/c, the reduction of σ/σ_0 for ^{12}C is less than 1%.

The important difference between these two models is the fact that in case (i) the cutoff depends on the incident-particle momentum, while in case (ii) it is independent of the momentum, and depends only on the mass of the resonance. Thus the energy dependence of the total cross section should allow us to distinguish between the two cases. In Fig. 2 we show some predicted values of σ_{pA}. We see that, as expected, if the coupling of high-mass resonances stays appreciable, a sharp energy-dependent reduction in σ_{pA} should be observed for light nuclei, while if the coupling decreases on a scale set by the nucleon mass, σ_{pA} should be essentially independent of energy. [Note that in an intermediate case, where the residue cutoff is slower than in Eq. (9), one would observe a drop in σ_{pA} until the momentum reached a value where $m_{\max}^*$ exceeded the residue cutoff, and then σ_{pA} would become energy independent.]

The reason that the coherent intermediate states have their greatest effect for large P and small A is easily understood if we note that in Eq. (10) $m_{\max}^*$ will be largest for large P and small R, so that the

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number of intermediate N^* which contribute to the sum over i increases with P and is largest for small A. For reference, we show $m_{\max}^*$ for $P=100$ GeV/c for various nuclei in Fig. 1(d). We note that for sizeable reductions in the cross section, large numbers of coherent intermediate states are necessary.

It should be noted that the reductions which we have calculated depend very critically on the assumption which has been implicit in our development that the phases of the inelastic amplitudes g_i are all roughly the same, and that all g_i are nearly pure imaginary. If the phases were not roughly equal, then the effects of the intermediate N^* would tend to cancel out among themselves, and little or no reduction would occur. On the basis of the quark model, however,[6,12] one expects that the phases of all of the g_i should approach zero at high energies (more exactly, that they should be proportional to the phase of the p-p amplitude, which approaches zero), so that this assumption is probably not going to lead us into difficulties.

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One can imagine many uses for this technique. One could look at the high-mass spectrum of the pion and the kaon, or one could compare p and $\bar{p}$ total cross sections to test particle-antiparticle universality. In addition, by measuring the photon-nucleus total cross section, one could look for evidence of heavy vector mesons.

In closing, one word of caution is in order. We have explicitly looked only at lowest-order terms in G^2. If the effect of these terms is appreciable, it will be necessary to consider G^3 diagrams, like the one in Fig. 1(c), which tend to lessen the decrease in σ_{pA} which we have predicted. However, since in most cases the contributions of order G^2 seem to be $\lesssim 20\%$, the inclusion of G^3 terms probably is not necessary.

Thus we see that measurement of the total particle-nucleus cross sections as a function of energy allows one to determine the general behavior of coupling strengths as a function of resonance mass, and also to make some statements about the occurrence of resonances at very high mass without actually having to produce the resonances separately.

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Beta Decay of Hyperons*

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The formulas for spin and angular correlations in hyperon β decay are brought into forms which can give specific information about the character of possible deviations from the universal $SU(3)$ scheme. The recent experimental results for Λ-particle β decay are discussed qualitatively in terms of the proposed combinations of integrated correlation coefficients.

I. INTRODUCTION

A CONCISE description of semileptonic weak interactions has been given on the basis of the following assumptions[1,2]: (1) The weak vector current and the electromagnetic current consist of the appropriate components of a common octet current of $SU(3)$; (2) the axial-vector current is the same component of another octet current; and (3) there is a universal suppression factor for strangeness-changing transitions. In particular, the measured rates and the lepton-neutrino angular correlations of $\Delta S=1$ transitions appear to fit rather well into such a framework. However, more recently, preliminary data on spin correlations for $\Lambda \to p+e^-+\bar{\nu}$ decays have become available[3,4]; these data can give a new and sensitive test of the assumptions (1)–(3). It is therefore of interest to have some means of analytically exploring the qualitative features of specific proposals for corrections to the $SU(3)$ scheme and for possible CP-violating terms.

In this paper, we suggest particular combinations of *presently observed*, integrated correlation coefficients. These combinations are constructed in order to exhibit a dominant dependence upon specific form factors. The functions Σ and Ω are sensitive to the induced tensor (f_2) and/or pseudotensor (g_2) form factors; they are independent of vector–axial-vector interference terms ($\mathrm{Re} f_1 g_1^*$), and they vanish in the allowed approxima-

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