product of the amplitude to produce $q\bar{q}$ states of spin J and the amplitude for a state of spin J to decay into a given number of (relatively stable) particles.

ACKNOWLEDGMENTS

One of us (J.S.B.) would like to thank Professor G. F. Chew for his hospitality at the Lawrence Radiation Laboratory, Berkeley, where most of the numerical calculations were performed and, further, to express his gratitude to the Associated Western Universities, Inc., and the Atomic Energy Commission, whose financial support made possible this faculty orientation and training program at the Lawrence Radiation Laboratory.

PHYSICAL REVIEW D VOLUME 3, NUMBER 7 1 APRIL 1971

Implications of Local Duality in a Set of Coupled Reactions*

M. J. King and Kameshwar C. Wali
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 25 September 1970)

This is an extension of the study in an earlier paper on meson-decuplet scattering to two sets of coupled reactions: (1) $PB_{10}\to PB_{10}$, $PB_{10}\to PB_8$, $PB_8\to PB_8$ and (2) $PV\to PV$, $PV\to PP$, $PP\to PP$. P and V refer to the pseudoscalar-meson octet and vector-meson nonet, respectively, while B_8 and B_{10} correspond to the octet of $J^P=\frac{1}{2}^+$ baryons and decuplet $J^P=\frac{3}{2}^+$ baryons, respectively. Some consequences of local duality are examined and compared with experiment in limited energy regions for the above reactions. The assumption that the imaginary parts of the direct-channel helicity amplitudes vanish in the forward (backward) scattering if the crossed t channel (u channel) is exotic leads to systems of equations relating resonance contributions in the direct channel. The solutions predict certain patterns of particles degenerate in mass but with different $SU(3)$, spin, and parity assignments. In particular, the inclusion of spin considerations yields the result that the particles on leading trajectories must be accompanied by daughters with prescribed ratios of coupling constants between the parent and daughter states. We discuss the link between our results and those which follow from a complementary description in terms of Regge residues.

I. INTRODUCTION

Combining crossing, $SU(3)$ symmetry, and duality, several authors have predicted certain exchange-degeneracy patterns for hadronic trajectories.[1,2] These considerations have presented a new approach to the classification of hadrons. In this approach it is assumed that the imaginary part of the resonant scattering amplitude is expressible, at high energies, in terms of Regge trajectories in the crossed channels. Hence if a particular scattering amplitude is characterized by internal quantum numbers for which no resonances exist, the Regge trajectories in the crossed channels must exhibit exchange degeneracy so that the corresponding imaginary parts cancel.

A complementary description of the resonant part of the scattering amplitude exists in terms of direct-channel resonances. One can assume, therefore, that there is a region of s and small t on the one hand, and a region of s and small u on the other, within which the imaginary part can be calculated in two alternative ways: in terms of direct-channel resonances *or* in terms of Regge trajectories of the crossed channels.[3] If we select the s-channel reactions so that their t or u channels are characterized by exotic quantum numbers, the imaginary parts due to s-channel resonances must add up to zero. We have investigated the consequences of this assumption in a local energy region (local duality) in the case of the following set of coupled reactions:

(1) $PB_{10}\to PB_{10},\ PB_{10}\to PB_8,\ PB_8\to PB_8,$

(2) $PV\to PV,\quad PV\to PP,\quad PP\to PP,$

where P represents the pseudoscalar meson octet, B_{10} represents the decuplet of $J^P=\frac{3}{2}^+$ baryons, B_8 represents the octet of $J^P=\frac{1}{2}^+$ baryons, and V represents the nonet of vector mesons.

In each of these reactions, we consider a set of direct-channel resonances degenerate in mass but with different spins and parities, and examine the constraints on their coupling constants. The choice of such a system of resonances degenerate in mass is, in part, motivated by the experimental evidence in certain energy regions for a number of πN (and $\pi\pi$) resonances approximately equal in mass although differing in spin, isospin, and parity. Further, there is evidence that they are coupled to $\pi\Delta$ (and $\pi\rho$) systems. $SU(3)$ symmetry would then require the consideration of reactions (1) and (2). The assumption, in part, is also motivated by the properties of Veneziano-type models[4] which attempt to incor-

* Work supported by the U. S. Atomic Energy Commission.

[1] R. H. Capps, Phys. Rev. Letters 22, 215 (1969); J. Mandula *et al.*, *ibid.* 22, 1147 (1969); V. Barger and C. Michael, Phys. Rev. 186, 1592 (1969).

[2] L. K. Chavda and R. H. Capps, Phys. Rev. D 1, 1845 (1970).

[3] J. Mandula, J. Weyers, and G. Zweig, Phys. Rev. Letters 23, 627 (1969).

[4] G. Veneziano, Nuovo Cimento 57A, 190 (1968).

porate crossing symmetry and Regge asymptotic behavior. The occurrence of parity partners and lower-spin daughter states degenerate in mass with the parent states is a fundamental property of such models.

Spin is an essential complication in the problem. By including the consideration of spin, we show that the particles on leading trajectories must be accompanied by daughters with prescribed ratios of coupling constants between the parent and the daughter states. We discuss the general pattern of nontrivial solutions and compare the results with experiments in the case of nonstrange baryons ($\approx$1700-MeV region) and mesons ($\approx$1300-MeV region). Considerations along these lines were suggested and partial results reported in Ref. 3.

In Sec. II, we present the basic equations and some kinematic details. Section III contains the solutions for the different reactions and the discussion of certain restrictive assumptions regarding some coupling constants. These assumptions in addition to local duality play a crucial role in our results. In Sec. IV, we attempt to find a link between our results and those that follow from a complementary description in terms of Regge residues.[1,2] Section V is devoted to a summary and discussion of the results.

II. BASIC EQUATIONS OF KINEMATICS

We are concerned with a four-particle scattering amplitude for $A+B\to C+D$. A and C denote the members of the octet of pseudoscalar mesons, while B_8 and D_{10} denote hadrons of spin S and S', respectively. The imaginary parts of the s-channel helicity amplitudes can be written as

$$\begin{aligned}\mathrm{Im}\bar{G}_{0\mu;0\lambda}{}^{R}(s,\theta_s)&\\ =K(s)\sum_{JLL'R'}&X^{RR'}(2L'+1)^{1/2}(2L+1)^{1/2}\\ &\times C(L'S'J;0\mu)C(LSJ;0\lambda)\\ &\times\chi_{R'}{}^{J}(L')\omega_{R'}{}^{J}(L)\hat{d}_{\lambda\mu}{}^{J}(\theta_s)\,,\end{aligned}\tag{2.1}$$

where R characterizes the irreducible $SU(3)$ representation of the crossed t or u channel and $X_{R'}{}^{R}$ are the elements of the corresponding crossing matrix. L, λ (L',μ) denote the orbital angular momentum and helicity of the initial (final) state and $K(s)$ is a kinematic factor. The χ^J's are related to the partial widths through

$$\chi_{R'}{}^{J}(L)=(\Gamma_{CD}{}^{J}/\Gamma^{J})^{1/2}\times\{[(W^*-W_{\mathrm{c.m.}})/\tfrac{1}{2}\Gamma^J]^2+1\}^{-1/2},\tag{2.2}$$

where Γ^J is the total width and $\Gamma_{CD}{}^{J}$ the partial width of a resonance of angular momentum J and mass W^*. A similar definition holds for $\omega_{R'}{}^{J}$. $W_{\mathrm{c.m.}}$ is the c.m. energy. Finally

$$\hat{d}_{\lambda\mu}{}^{J}(\theta_s)=(\cos\tfrac{1}{2}\theta_s)^{-|\lambda+\mu|}(\sin\tfrac{1}{2}\theta_s)^{-|\lambda-\mu|}d_{\lambda\mu}{}^{J}(\theta_s)\,.\tag{2.3}$$

Also note that $\bar{G}_{0\mu;0\lambda}{}^{R}(s,\theta_s)$ are amplitudes for which the physical boundary singularities have been removed by dividing out the appropriate powers of the usual half-angle factors $\cos\frac{1}{2}\theta_s$ and $\sin\frac{1}{2}\theta_s$. Therefore $\bar{G}_{0\mu;0\lambda}{}^{R}(s,\theta_s)$ will not vanish at $\theta_s=0$ or π just because of kinematic reasons. Further, the helicity labels of independent amplitudes can be chosen so that $\lambda-\mu$ and $\lambda+\mu$ are non-negative integers. Then,

$$\hat{d}_{\lambda\mu}{}^{J}(0)=\frac{1}{(\lambda-\mu)!}\left[\frac{(J+\lambda)!}{(J-\lambda)!}\frac{(J-\mu)!}{(J+\mu)!}\right]^{1/2}\tag{2.4}$$

and

$$\begin{aligned}&\hat{d}_{\lambda\mu}{}^{J}(\pi)\\ &=(-1)^{-|\lambda-\mu|+|\lambda+\mu|}(\lambda-\mu)!(-1)^{J+\lambda}\hat{d}_{\lambda,-\mu}{}^{J}(0)\,.\end{aligned}\tag{2.5}$$

Using (2.5) and

$$C(L'S'J;0-\mu)=(-1)^{L'+S'-J}C(L'S'J;0\mu)\,,\tag{2.6}$$

we can write

$$\begin{aligned}\mathrm{Im}\bar{G}_{0\mu;0\lambda}{}^{R}(s^*,0)&\\ =K(s^*)\sum_{JLL'R'}&X_{ts}{}^{RR'}(2L'+1)^{1/2}(2L+1)^{1/2}\\ &\times C(L'S'J;0\mu)C(LSJ;0\lambda)\\ &\times\chi_{R'}{}^{J}(L')\omega_{R'}{}^{J}(L)\hat{d}_{\lambda\mu}{}^{J}(0)\end{aligned}\tag{2.7}$$

in the forward direction and

$$\begin{aligned}\mathrm{Im}\bar{G}_{0-\mu;0\lambda}{}^{R}(s^*,\pi)&=(-1)^{-|\lambda+\mu|+|\lambda-\mu|}(\lambda-\mu)!\\ &\times(-1)^{S'+\lambda}K(s^*)\sum_{JLL'R'}X_{us}{}^{RR'}(-1)^{L'}\\ &\times(2L'+1)^{1/2}(2L+1)^{1/2}C(L'S'J;0\mu)C(LSJ;0\lambda)\\ &\times\chi_{R'}{}^{J}(L')\omega_{R'}{}^{J}(L)\hat{d}_{\lambda\mu}{}^{J}(0)\end{aligned}\tag{2.8}$$

in the backward direction.

Our basic set of equations is obtained from (2.7) and (2.8) by setting the corresponding imaginary parts to zero when R denotes an "exotic" representation. One of our subsequent assumptions will be that for a given J, the coupling corresponding to the lowest allowed L-value is the dominant coupling. Parity conservation demands that the allowed L-values differ by units of two. For J-values considered in this paper, there are at most two L-values. If L_0 (L_0') is the smaller of the two allowed values for the initial (final) state, denote

$$A_\lambda{}^{R,J\pm}(L_0)=\left[1+\frac{(2L_0+5)^{1/2}}{(2L_0+1)^{1/2}}\frac{C(L_0+2,S,J;0\lambda)}{C(L_0,S,J;0\lambda)}\times\frac{\chi_R{}^{J\pm}(L_0+2)}{\chi_R{}^{J\pm}(L_0)}\right],\tag{2.9}$$

$$B_\lambda{}^{R,J\pm}(L_0)=\left[1+\frac{(2L_0+5)^{1/2}}{(2L_0+1)^{1/2}}\frac{C(L_0+2,S,J;0\lambda)}{C(L_0SJ;0\lambda)}\times\frac{\omega_R{}^{J\pm}(L_0+2)}{\omega_R{}^{J\pm}(L_0)}\right],\tag{2.10}$$

$$Y_{\mu,\lambda}{}^{R,J^\pm}=\sum_{R'} X_{ts}{}^{RR'}[\chi_{R'}{}^{J^\pm}(L_0')\omega_{R'}{}^{J^\pm}(L_0)] \times A_\mu{}^{R',J^\pm}(L_0')B_\lambda{}^{R',J^\pm}(L_0)\,, \quad (2.11)$$

$$Z_{\mu,\lambda}{}^{R,J^\pm}=\sum_{R'} X_{us}{}^{RR'}[\chi_{R'}{}^{J^\pm}(L_0')\omega_{R'}{}^{J^\pm}(L_0)] \times A_\mu{}^{R',J^\pm}(L_0')B_\lambda{}^{R',J^\pm}(L_0)\,. \quad (2.12)$$

Then our basic equations are

$$\sum_{J^\pm}(2L_0'+1)^{1/2}(2L_0+1)^{1/2}C(L_0'S'J;0\mu) \times C(L_0SJ;0\lambda)Y_{\mu,\lambda}{}^{R,J^\pm}\hat{d}_{\lambda\mu}{}^J(0)=0\,, \quad (2.13)$$

$$\sum_{J^\pm}(2L_0'+1)^{1/2}(2L_0+1)^{1/2}(-1)^{L_0'}C(L_0'S'J;0\mu) \times C(L_0SJ;0\lambda)Z_{\mu,\lambda}{}^{R,J^\pm}\hat{d}_{\lambda\mu}{}^J(0)=0\,, \quad (2.14)$$

where R corresponds to an exotic representation. From (2.9) it can be verified that

$$A_{-\lambda}{}^{J^\pm}(L_0)=A_\lambda{}^{J^\pm}(L_0)\,,\quad B_{-\lambda}{}^{J^\pm}(L_0)=B_\lambda{}^{J^\pm}(L_0)\,, \quad (2.15)$$

and consequently

$$\begin{aligned} Y_{\mu,\lambda}{}^{J^\pm}&=Y_{\mu,-\lambda}{}^{J^\pm}=Y_{-\mu,\lambda}{}^{J^\pm}=Y_{-\mu,-\lambda}{}^{J^\pm}\,,\\ Z_{\mu,\lambda}{}^{J^\pm}&=Z_{\mu,-\lambda}{}^{J^\pm}=Z_{-\mu,\lambda}{}^{J^\pm}=Z_{-\mu,-\lambda}{}^{J^\pm}\,. \end{aligned} \quad (2.16)$$

The above symmetry relations are useful in the choice of independent helicity amplitudes. We also give here explicit expressions for $Y_{\mu,\lambda}{}^{R,J^\pm}$ and $Z_{\mu,\lambda}{}^{R,J^\pm}$ in the different reactions under consideration.[5] For this purpose we henceforth use $\chi_R{}^{J^\pm}$ and $\omega_R{}^{J^\pm}$ to denote the couplings or partial widths of a $J^\pm$ (J half-integer) to PB_{10} and PB_8, respectively. Similarly, $\chi_R{}^{J^\pm}$, $\omega_R{}^{J^\pm}$ denote the couplings of a $J^\pm$ (J integral) resonance to PV and PP, respectively.

$PB_{10}\rightarrow PB_{10}$:

$$\begin{aligned} Y_{\mu,\lambda}{}^{J^\pm}&=4A_{\mu,\lambda}{}^{8,J^\pm}[\chi_8{}^{J^\pm}(L_0)]^2 -5A_{\mu,\lambda}{}^{10,J^\pm}[\chi_{10}{}^{J^\pm}(L_0)]^2 \quad (t\,27)\,,\\ Z_{\mu,\lambda}{}^{J^\pm}&=A_{\mu,\lambda}{}^{8,J^\pm}[\chi_8{}^{J^\pm}(L_0)]^2 +\tfrac{5}{8}A_{\mu,\lambda}{}^{10,J^\pm}[\chi_{10}{}^{J^\pm}(L_0)]^2 \quad (u\,27,35)\,. \end{aligned} \quad (2.17)$$

$PB_8\rightarrow PB_{10}$:

$$\begin{aligned} Y_\mu{}^{J^\pm}&=(1/18)^{1/2}A_\mu{}^{10,J^\pm}\chi_{10}{}^{J^\pm}(L_0)\omega_{10}{}^{J^\pm} -A_\mu{}^{8,J^\pm}\chi_8{}^{J^\pm}(L_0) \times[\tfrac{2}{5}\omega_S{}^{J^\pm}+(2/3\sqrt{5})\omega_A{}^{J^\pm}] \quad (t\,27)\,,\\ Y_\mu{}^{J^\pm}&=-(\tfrac{1}{2})^{1/2}A_\mu{}^{10,J^\pm}\chi_{10}{}^{J^\pm}(L_0)\omega_{10}{}^{J^\pm} +A_\mu{}^{8,J^\pm}\chi_8{}^{J^\pm}(L_0) \times[\tfrac{2}{5}\omega_S{}^{J^\pm}-(2/\sqrt{5})\omega_A{}^{J^\pm}] \quad (t\,10)\,,\\ Z_\mu{}^{J^\pm}&=-(1/18)^{1/2}A_\mu{}^{10,J^\pm}\chi_{10}{}^{J^\pm}(L_0)\omega_{10}{}^{J^\pm} -A_\mu{}^{8,J^\pm}\chi_8{}^{J^\pm}(L_0) \times[\tfrac{2}{5}\omega_S{}^{J^\pm}-(2/3\sqrt{5})\omega_A{}^{J^\pm}] \quad (u\,27)\,. \end{aligned} \quad (2.18)$$

[5] The superscript R on Y and Z will usually be omitted as a matter of convenience.

$PB_8\rightarrow PB_8$:

$$\begin{aligned} Y^{J^\pm}&=-\tfrac{1}{12}(\omega_{10}{}^{J^\pm})^2+\tfrac{1}{5}(\omega_S{}^{J^\pm})^2 -\tfrac{1}{3}(\omega_A{}^{J^\pm})^2+\tfrac{1}{8}(\omega_1{}^{J^\pm})^2 \quad (t\,27)\,,\\ Y^{J^\pm}&=\tfrac{1}{4}(\omega_{10}{}^{J^\pm})^2-\tfrac{2}{5}(\omega_S{}^{J^\pm})^2 +\tfrac{1}{8}(\omega_1{}^{J^\pm})^2 \quad (t\,10,10^*)\,,\\ Z^{J^\pm}&=\tfrac{1}{12}(\omega_{10}{}^{J^\pm})^2+\tfrac{1}{5}(\omega_S{}^{J^\pm})^2 +\tfrac{1}{3}(\omega_A{}^{J^\pm})^2+\tfrac{1}{8}(\omega_1{}^{J^\pm})^2 \quad (u\,27)\,,\\ Z^{J^\pm}&=\tfrac{1}{4}(\omega_{10}{}^{J^\pm})^2+\tfrac{2}{5}(\omega_S{}^{J^\pm})^2 -(2/\sqrt{5})\omega_S{}^{J^\pm}\omega_A{}^{J^\pm} -\tfrac{1}{8}(\omega_1{}^{J^\pm})^2 \quad (u\,10^*)\,. \end{aligned} \quad (2.19)$$

In (2.17), $A_{\mu,\lambda}{}^{R,J^\pm}=A_\mu{}^{R,J^\pm}A_\lambda{}^{R,J^\pm}$. Since total angular momentum and parity determines a unique value for L, quantities corresponding to A_λ's are equal to unity in the case of PB_8 system, thus giving rise to the different helicity dependences of the Y's and Z's in Eqs. (2.17)–(2.19). From the octet-octet $SU(3)$ crossing matrix,[6] one can easily derive analogous expressions in the case of $PV\rightarrow PV$, $PV\rightarrow PP$, and $PP\rightarrow PP$. For later discussion, we list below the A_λ's which are different from unity[7] (these occur in the couplings to PB_{10} and PV systems only):

PB_{10}:

$$\begin{aligned} A_{\frac{3}{2}}{}^{\frac{5}{2}+}(1)&=1+(\sqrt{\tfrac{3}{2}})\frac{\chi^{\frac{5}{2}+}(3)}{\chi^{\frac{5}{2}+}(1)}\,, & A_{\frac{3}{2}}{}^{\frac{5}{2}-}(2)&=1+(\sqrt{\tfrac{1}{6}})\frac{\chi^{\frac{5}{2}-}(4)}{\chi^{\frac{5}{2}-}(2)}\,,\\ A_{\frac{1}{2}}{}^{\frac{5}{2}+}(1)&=1-(\sqrt{\tfrac{2}{3}})\frac{\chi^{\frac{5}{2}+}(3)}{\chi^{\frac{5}{2}+}(1)}\,, & A_{\frac{1}{2}}{}^{\frac{5}{2}-}(2)&=1-(\sqrt{6})\frac{\chi^{\frac{5}{2}-}(4)}{\chi^{\frac{5}{2}-}(2)}\,,\\ A_{\frac{3}{2}}{}^{\frac{3}{2}+}(1)&=1+\frac{1}{3}\frac{\chi^{\frac{3}{2}+}(3)}{\chi^{\frac{3}{2}+}(1)}\,, & A_{\frac{3}{2}}{}^{\frac{3}{2}-}(0)&=1+\frac{\chi^{\frac{3}{2}-}(2)}{\chi^{\frac{3}{2}-}(0)}\,,\\ A_{\frac{1}{2}}{}^{\frac{3}{2}+}(1)&=1-3\frac{\chi^{\frac{3}{2}+}(3)}{\chi^{\frac{3}{2}+}(1)}\,, & A_{\frac{1}{2}}{}^{\frac{3}{2}-}(0)&=1-\frac{\chi^{\frac{3}{2}-}(2)}{\chi^{\frac{3}{2}-}(0)}\,. \end{aligned} \quad (2.20)$$

PV:

$$\begin{aligned} A_1{}^{2^-}&=1+(\sqrt{\tfrac{2}{3}})\frac{\chi^{2^-}(3)}{\chi^{2^-}(1)}\,, & A_1{}^{1^+}&=1+(\sqrt{\tfrac{1}{2}})\frac{\chi^{1^+}(2)}{\chi^{1^+}(0)}\,,\\ A_0{}^{2^-}&=1-(\sqrt{\tfrac{3}{2}})\frac{\chi^{2^-}(3)}{\chi^{2^-}(1)}\,, & A_0{}^{1^+}&=1-\sqrt{2}\frac{\chi^{1^+}(2)}{\chi^{1^+}(0)}\,. \end{aligned} \quad (2.21)$$

Finally, we remark that we choose the independent helicity amplitudes in such a way that $\lambda-\mu$ and $\lambda+\mu$ are non-negative integers. Thus, for example, in $PB_{10}\rightarrow PB_{10}$ the six independent amplitudes correspond to $\lambda=\frac{3}{2}$,

[6] J. J. DeSwart, Nuovo Cimento **31**, 420 (1964).

[7] We assume that the leading baryonic trajectory is $\Delta(1236)$, so that in the energy region under consideration, only resonances of spin $\frac{5}{2}$ or below will contribute. Similarly, the leading mesonic ρ-A_2 trajectory limits the contributions in the meson-meson systems to spin 2 and below.

$\mu=\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, and $\lambda=\frac{1}{2}, \mu=\frac{1}{2}, -\frac{1}{2}$. Furthermore, the equations fall naturally into pairs due to the symmetry relation (2.16); that is, for instance, the equations corresponding to $\mathrm{Im}\bar{G}_{0\frac{3}{2};0\frac{3}{2}}{}^{R}(s^*,0)$ $[\mathrm{Im}\bar{G}_{0\frac{3}{2};0\frac{3}{2}}{}^{R}(s^*,\pi)]$ and $\mathrm{Im}\bar{G}_{0-\frac{3}{2};0\frac{3}{2}}{}^{R}(s^*,0)=0$ $[\mathrm{Im}\bar{G}_{0-\frac{3}{2};0\frac{3}{2}}{}^{R}(s^*,\pi)=0]$ both can be expressed in terms of the quantities $Y_{\frac{3}{2}\frac{3}{2}}{}^{R,J^\pm}$ $[Z_{\frac{3}{2}\frac{3}{2}}{}^{R,J^\pm}]$.

III. DISCUSSION OF SOLUTIONS

In a previous paper,[8] a set of particles degenerate in mass and $J^P=\frac{5}{2}^\pm, \frac{3}{2}^\pm, \frac{1}{2}^\pm$ were considered in the case of $PB_{10}\to PB_{10}$. It was shown that the solutions to Eqs. (2.13) and (2.14) can be expressed on terms of two constant parameters p and p' and certain combinations of $A_{\mu,\lambda}{}^{J\pm}$. A physically interesting feature of these solutions was the existence of a set of daughter states corresponding to particles on the leading or parent trajectories. In the present paper we begin this section by a more detailed examination of the solutions with special attention to the positivity conditions on the residues. For the sake of completeness, we write here our additional assumptions concerning $A_\mu{}^{R,J\pm}$:

(i) $A_\mu{}^{R,J\pm}$ is independent of R, i.e.,

$$A_{\mu,\lambda}{}^{8,J\pm}=A_{\mu,\lambda}{}^{10,J\pm}=A_{\mu,\lambda}{}^{J\pm}. \tag{3.1}$$

(ii) The ratios $\chi_R{}^{J\pm}(L_0+2)/\chi_R{}^{J\pm}(L_0)$ are small. Theoretical arguments based on centrifugal-barrier effects suggest

$$|\chi_R{}^{J\pm}(L_0+2)/\chi_R{}^{J\pm}(L_0)|\sim(q^*/W^*)^4, \tag{3.2}$$

where q^* is the c.m. momentum corresponding to the mass of the resonance W^*. In the limited energy region of interest for the present considerations, this is indeed small. The smallness of this ratio implies [see (2.20) and (2.21)] that the $A_\mu{}^{J\pm}$'s are positive and of order of magnitude unity. A given resonance, therefore, decays predominantly in the lowest allowed angular momentum state and in general is coupled to all helicity states. This is indeed a restrictive assumption in addition to local duality and is not likely to be true at higher energies.

To write the solutions of (2.13) and (2.14) in a simple form, we denote

$$\alpha=A_{\frac{1}{2}}{}^{\frac{5}{2}+}/A_{\frac{3}{2}}{}^{\frac{5}{2}+},\quad \beta=A_{\frac{1}{2}}{}^{\frac{3}{2}+}/A_{\frac{3}{2}}{}^{\frac{3}{2}+},$$
$$\gamma=A_{\frac{1}{2}}{}^{\frac{5}{2}-}/A_{\frac{3}{2}}{}^{\frac{5}{2}-},\quad \delta=A_{\frac{1}{2}}{}^{\frac{3}{2}-}/A_{\frac{3}{2}}{}^{\frac{3}{2}-}. \tag{3.3}$$

Then,

$$Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-}=-\frac{7}{25}\frac{\alpha-\beta}{\beta-\gamma}Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+},\quad Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-}=\frac{7}{25}\frac{\alpha-\beta}{\beta-\gamma}Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+}, \tag{3.4}$$

$$Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{3}{2}-}=\frac{3}{7}\frac{9\alpha-\gamma}{\alpha-\delta}Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-},\qquad Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{3}{2}-}=\frac{3}{7}\frac{9\alpha-\gamma}{\alpha-\delta}Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-}, \tag{3.5}$$

$$\begin{aligned} Y^{1/2-}&=\frac{3}{14}[\alpha(6\gamma-5\beta-9\delta)+\gamma(5\beta-4\gamma+\delta)]Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-},\\ Z^{1/2-}&=\frac{3}{14}[\alpha(6\gamma-5\beta-9\delta)+\gamma(5\beta-4\gamma+\delta)]Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}-}, \end{aligned} \tag{3.6}$$

$$Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{3}{2}+}=\frac{\alpha-\gamma}{\beta-\gamma}Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+},\quad Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{3}{2}+}=\frac{\alpha-\gamma}{\beta-\gamma}Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+}, \tag{3.7}$$

$$\begin{aligned} Y^{1/2+}&=\tfrac{1}{10}[\alpha(-36\alpha+\beta+45\delta)+\gamma(6\alpha-\beta-5\delta)]Y_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+},\\ Z^{1/2+}&=\tfrac{1}{10}[\alpha(-36\alpha+\beta+45\delta)+\gamma(6\alpha-\beta-5\delta)]Z_{\frac{3}{2}\frac{3}{2}}{}^{\frac{5}{2}+}, \end{aligned} \tag{3.8}$$

where α, β, γ, and δ satisfy the condition

$$(\beta-\alpha)(9\delta-\gamma)=25(\beta-\gamma)(\delta-\alpha). \tag{3.9}$$

From (3.3)–(3.9), using (2.17), we can easily derive the results in Table I of Ref. 8. Because of assumption (ii) α, β, γ, and δ are constrained to be positive. A careful examination of the requirement that $(\chi_R{}^{J\pm})^2$ be nonnegative leads to the possibilities

$$\alpha<\beta<\gamma \quad\text{and}\quad \alpha<\delta<\tfrac{1}{9}\gamma, \tag{3.10a}$$

$$\gamma>\beta>\alpha>\delta>\tfrac{1}{9}\gamma. \tag{3.10b}$$

If we concentrate our attention on the energy region $\sim$1700 MeV, (3.10a) appears to be ruled out. Although experimental evidence is not definitive, the condition $\alpha<\frac{1}{9}\gamma$ appears to be ruled out by $F_{15}(1688)$ and $D_{15}(1670)$ data. Consequently, we consider only case (3.10b) and look for a range of values for the parameters such that all the residues are non-negative. It turns out that it is impossible to obtain nonvanishing positive residues for both $(\chi_R{}^{1/2+})^2$ and $(\chi_R{}^{1/2-})^2$. The only solution consistent with (3.10b) is

$$\alpha=\tfrac{1}{3}\gamma,\quad \beta=\tfrac{3}{4}\gamma,\quad \delta=\tfrac{1}{4}\gamma, \tag{3.11}$$

for which

$$Z^{1/2+}=Y^{1/2+}=Z^{1/2-}=Y^{1/2-}=0. \tag{3.12}$$

Hence $J^P=\frac{1}{2}^\pm$ states are *decoupled* from the PB_{10} system. The solutions for the couplings of other particles can be expressed in terms of γ and any two other $(\chi_R{}^{J\pm})^2$. We choose these to be $(\chi_8{}^{5/2+})^2$ and $(\chi_8{}^{5/2-})^2$. Then,

$$(\chi_{10}{}^{5/2-})^2=\left[\frac{2}{7}\left(\frac{\gamma+6}{\gamma+2}\right)^2(\chi_8{}^{5/2+})^2-\tfrac{2}{5}(\chi_8{}^{5/2-})^2\right],$$

$$(\chi_{10}{}^{5/2+})^2=\frac{2}{5}\left[\frac{63}{5}\left(\frac{\gamma+2}{\gamma+6}\right)^2(\chi_8{}^{5/2-})^2-(\chi_8{}^{5/2+})^2\right], \tag{3.13}$$

$$(\chi_8{}^{3/2-})^2=\frac{63}{8}\left(\frac{\gamma+4}{\gamma+6}\right)^2(\chi_8{}^{5/2-})^2,$$

$$(\chi_8{}^{3/2+})^2=\frac{3}{8}\left(\frac{\gamma+12}{\gamma+2}\right)^2(\chi_8{}^{5/2+})^2, \tag{3.14}$$

[8] M. J. King and K. C. Wali, Phys. Rev. Letters **24**, 1460 (1970).

$$(\chi_{10}{}^{3/2-})^2=\frac{63}{8}\left(\frac{\gamma+4}{\gamma+6}\right)^2(\chi_{10}{}^{5/2-})^2\,,\qquad (\chi_{10}{}^{3/2+})^2=\frac{3}{8}\left(\frac{\gamma+12}{\gamma+2}\right)^2(\chi_{10}{}^{5/2+})^2\,, \tag{3.15}$$

$$(\chi_8{}^{1/2-})=0\,,\qquad (\chi_8{}^{1/2+})=0\,, \tag{3.16}$$

$$(\chi_{10}{}^{1/2-})=0\,,\qquad (\chi_{10}{}^{1/2+})=0\,. \tag{3.17}$$

Substituting these solutions in $PB_8\to PB_{10}$ equations (2.18), we can derive relations for $\omega_R{}^{3/2\pm}$ in terms of $\omega_S{}^{5/2\pm}$, $\omega_A{}^{5/2\pm}$, $\omega_{10}{}^{5/2\pm}$, $\chi_8{}^{5/2\pm}$, and γ. These are

$$\pm\omega_S{}^{3/2+}=\tfrac{1}{4}\omega_S{}^{5/2+}+\left(\frac{5\sqrt{14}}{4\sqrt{3}}\right)\left(\frac{\gamma+2}{\gamma+6}\right)\left[\left(\frac{\sqrt{5}}{4}\right)\left(\frac{\chi_{10}{}^{5/2-}}{\chi_8{}^{5/2+}}\right)\omega_{10}{}^{5/2-}-\frac{1}{\sqrt{2}}\left(\frac{\chi_8{}^{5/2-}}{\chi_8{}^{5/2+}}\right)\omega_A{}^{5/2-}\right], \tag{3.18}$$

$$\pm\omega_A{}^{3/2+}=\tfrac{1}{4}\omega_A{}^{5/2+}-\frac{\sqrt{14}}{4\sqrt{3}}\left(\frac{\gamma+2}{\gamma+6}\right)\left[\frac{5}{4}\left(\frac{\chi_{10}{}^{5/2-}}{\chi_8{}^{5/2+}}\right)\omega_{10}{}^{5/2-}-\frac{3}{\sqrt{2}}\left(\frac{\chi_8{}^{5/2-}}{\chi_8{}^{5/2+}}\right)(\omega_S{}^{5/2-}+\tfrac{2}{3}(\sqrt{5})\omega_A{}^{5/2-})\right], \tag{3.19}$$

$$\pm\omega_{10}{}^{3/2+}=\tfrac{1}{4}\omega_{10}{}^{5/2+}-\frac{\sqrt{35}}{2\sqrt{6}}\left(\frac{\gamma+2}{\gamma+6}\right)\left[\frac{1}{\sqrt{2}}\left(\frac{\chi_{10}{}^{5/2-}}{\chi_{10}{}^{5/2+}}\right)\omega_{10}{}^{5/2-}-\frac{12}{5}\left(\frac{\chi_8{}^{5/2-}}{\chi_{10}{}^{5/2+}}\right)(\omega_S{}^{5/2-}-\tfrac{1}{3}(\sqrt{5})\omega_A{}^{5/2-})\right], \tag{3.20}$$

$$\pm\omega_S{}^{3/2-}=\tfrac{1}{4}\omega_S{}^{5/2-}+\frac{25\sqrt{8}}{24\sqrt{21}}\left(\frac{\gamma+6}{\gamma+2}\right)\left[\frac{\sqrt{5}}{4}\left(\frac{\chi_{10}{}^{5/2+}}{\chi_8{}^{5/2-}}\right)\omega_{10}{}^{5/2+}-\frac{1}{\sqrt{2}}\left(\frac{\chi_8{}^{5/2+}}{\chi_8{}^{5/2-}}\right)\omega_A{}^{5/2+}\right], \tag{3.21}$$

$$\pm\omega_A{}^{3/2-}=\tfrac{1}{4}\omega_A{}^{5/2-}-\frac{5\sqrt{8}}{24\sqrt{21}}\left(\frac{\gamma+6}{\gamma+2}\right)\left[\frac{5}{4}\left(\frac{\chi_{10}{}^{5/2+}}{\chi_8{}^{5/2-}}\right)\omega_{10}{}^{5/2+}-\frac{3}{\sqrt{2}}\left(\frac{\chi_8{}^{5/2+}}{\chi_8{}^{5/2-}}\right)(\omega_s{}^{5/2+}+\tfrac{2}{3}(\sqrt{5})\omega_A{}^{5/2+})\right], \tag{3.22}$$

$$\pm\omega_{10}{}^{3/2-}=\tfrac{1}{4}\omega_{10}{}^{5/2-}-\frac{5\sqrt{40}}{24\sqrt{21}}\left(\frac{\gamma+6}{\gamma+2}\right)\left[\frac{1}{\sqrt{2}}\left(\frac{\chi_{10}{}^{5/2+}}{\chi_{10}{}^{5/2-}}\right)\omega_{10}{}^{5/2+}-\frac{12}{5}\left(\frac{\chi_8{}^{5/2+}}{\chi_{10}{}^{5/2-}}\right)(\omega_S{}^{5/2+}-\tfrac{1}{3}(\sqrt{5})\omega_A{}^{5/2+})\right]. \tag{3.23}$$

In the equations above, the sign of $\omega_R{}^{3/2\pm}$ depends on the relative sign of $\chi_R{}^{3/2\pm}$ and $\chi_R{}^{5/2\pm}$. The ratios involving χ's are known from Eqs. (3.13)–(3.17) provided γ and $\chi_8{}^{5/2\pm}$ are given. Thus the couplings of octet and decuplet $J^P=\frac{3}{2}\pm$ particles can be given in terms of the couplings of particles with $J^P=\frac{5}{2}\pm$, i.e., particles on the leading trajectories. Since $\chi_8{}^{1/2\pm}$, $\chi_{10}{}^{1/2\pm}$ vanish, the equations involving $\chi_8{}^{1/2\pm}$, $\omega_S{}^{1/2\pm}$, etc., vanish identically. Thus the couplings of $J^P=\frac{1}{2}\pm$ particles are undetermined from $PB_{10}\to PB_{10}$ and $PB_8\to PB_{10}$ equations.

The solutions to these equations must also be consistent with the system of $PB_8\to PB_8$ equations, which involve not only the $\omega_{S,A,10}{}^{5/2\pm}$ and $\omega_{S,A,10}{}^{3/2\pm}$ occurring above, but also the spin-$\frac{1}{2}$ couplings $\omega_{S,A,10}{}^{1/2\pm}$ and the singlet couplings $\omega_1{}^{J\pm}$. There are eight equations to be satisfied, corresponding to Eqs. (2.13), (2.14), and (2.19), and therefore in view of the large number of new couplings introduced the system is not very restrictive. Rather than enumerate all the equations here, we defer discussion of the $PB_8\to PB_8$ system till Sec. IV, where some particular solutions are considered.

We have applied similar considerations to the reactions $PV\to PV$, $PV\to PP$, and $PP\to PP$. In this case we limit our discussion to the 1300-MeV region and to a set of particles degenerate in mass but with $J^P=2^\pm$, $1^\pm$, $0^\pm$ and belonging to unitary singlets and octets. Charge-conjugation invariance requires the octet couplings to be *either* F- or D-type in the case of mesons. Thus, the usual tensor particles have F-type coupling to the PV system and D-type to PP. The pure singlet component of the tensor nonet is not coupled to the PV system. We assume that from the known decay widths of the tensor particles we can calculate $\chi_A{}^{2+}$, $\omega_S{}^{2+}$, and $\omega_1{}^{2+}$. The corresponding quantities for other particles can then be expressed in terms of $\chi_A{}^{2+}$, $\omega_S{}^{2+}$, and $\omega_1{}^{2+}$. We have listed the $A_\mu{}^{J\pm}$'s that can be different from unity in (2.21). However, the requirement that the residues be non-negative leads to restriction on them in a manner analogous to Eq. (3.11). We have

$$A_0{}^{1^+}/A_1{}^{1^+}=\tfrac{2}{3}A_0{}^{2^-}/A_1{}^{2^-}. \tag{3.24}$$

Using (3.24), we have given the solution to the system of equations under consideration in Table I. From the existence of a tensor nonet ($J^{PC}=2^{++}$ where C refers to the charge-conjugation quantum number of the neutral particles in the nonet) and our restrictive assumption concerning $A_\mu{}^{2^-}$ and $A_\mu{}^{1^+}$ we predict the existence of a set of $J^{PC}=2^{--}$, 1^{--}, 1^{++}, and 0^{++} particles. The 2^{--} particles form a nonet whose couplings depend on $A_1{}^{2^-}$ in addition to $\chi_A{}^{2+}$. The existence of a vector nonet coupled strongly to both PV and PP systems is an important feature of the results. For the even-parity case, an octet of 1^{++} and a singlet of 0^{++} particles are necessary. The charge-conjugation

TABLE I. Solution to the system of equations $PV \to PV$, $PP \to PV$, $PP \to PP$.

Singlet	D-type	F-type
$(\chi_1^{2-})^2 = (80/27)(\chi_A^{2+})^2/(A_1^{2-})^2$	$(\chi^{2-})^2 = (25/27)(\chi_A^{2+})^2/(A_1^{2-})^2$	$(\chi_A^{1+})^2 = (\chi_A^{2+})^2/(A_1^{1+})^2$
$(\chi_1^{1-})^2 = (80/9)(\chi_A^{2+})^2$	$(\chi_s^{1-})^2 = (25/9)(\chi_A^{2+})^2$	$(\omega_A^{1-})^2 = 3(\omega_s^{2+})^2$
$(\omega_1^{0+})^2 = 16(\omega_s^{2+})^2 - 5(\omega_1^{2+})^2$		

quantum numbers of the predicted particles have been derived from the nature of their $SU(3)$ couplings to PV and PP systems. We can easily show that for D-type and F-type couplings of three octets, the product of the corresponding charge-conjugation quantum numbers is $+1$ and -1, respectively. The singlet coupling to two octets obeys the same rule as the D-type. The above assignments of C to the predicted mesons follow from these rules and the known charge-conjugation properties of the pseudoscalar and vector multiplets.

IV. DISCUSSION OF SOME SPECIAL SOLUTIONS AND COMPARISON WITH EXPERIMENTS

As remarked earlier in Sec. I, several authors[1,2] have derived exchange-degeneracy patterns and relations between effective Regge residues by combining $SU(3)$ symmetry, crossing, and the absence of exotic resonances (global duality). Of special relevance to the present investigation is the work of Chavda and Capps,[2] who discuss the residues of baryonic trajectories that contribute to the reactions $PB_{10} \to PB_{10}$, $PB_{10} \to PB_8$, and $PB_8 \to PB_8$. The Regge residues when extrapolated are related to the residues of resonances on *the leading trajectories*. It is, therefore, natural to interpret the different solutions of Chavda and Capps as sets of relations between the residues of $J^P = \frac{5}{2}^{\pm}$ resonances in our considerations of local duality.

From this point of view, it is interesting to note that Eqs. (3.11) and (3.13) for $PB_{10} \to PB_{10}$ imply

$$Z_{\frac{3}{2}\frac{3}{2}}^{\frac{5}{2}+} = \frac{15}{7} Z_{\frac{3}{2}\frac{3}{2}}^{\frac{5}{2}-}, \quad Y_{\frac{3}{2}\frac{3}{2}}^{\frac{5}{2}+} = -\frac{15}{7} Y_{\frac{3}{2}\frac{3}{2}}^{\frac{5}{2}-}, \tag{4.1}$$

$$Z_{\frac{1}{2}\frac{1}{2}}^{\frac{5}{2}+} = \frac{5}{21} Z_{\frac{1}{2}\frac{1}{2}}^{\frac{5}{2}-}, \quad Y_{\frac{1}{2}\frac{1}{2}}^{\frac{5}{2}+} = -\frac{5}{21} Y_{\frac{1}{2}\frac{1}{2}}^{\frac{5}{2}-} \tag{4.2}$$

along with similar relations between the corresponding quantities for $J^P = \frac{3}{2}^{\pm}$ particles. These relations in turn show that Eqs. (2.7) and (2.8) are satisfied in a somewhat special manner; viz., in a subset of equations

$$\mathrm{Im}\bar{G}_{0\lambda,0\lambda}{}^R(s^*,0) = 0, \quad \mathrm{Im}\bar{G}_{0-\lambda;0\lambda}{}^R(s^*,\pi) = 0 \tag{4.3}$$

the contributions of $J^P = \frac{5}{2}^{\pm}$ states on the parent trajectory cancel among themselves. This enforces cancellations between the $J^P = \frac{3}{2}^{\pm}$ states on the daughter trajectory. The remaining equations in which $|\lambda| \neq |\mu|$ provide relationships between the residues of the parent and daughter states. Thus the special solutions (3.13)–(3.17) are consistent with global duality in the following sense: The contributions from particles on the leading trajectory cancel among themselves in (4.3) as one would expect from global duality, since the corresponding helicity amplitudes do not vanish either in the forward or backward direction for kinematic reasons.[9] We now assume that this feature continues to hold for the corresponding equations in the case of $PB_8 \to PB_{10}$ and $PB_8 \to PB_8$ and look for solutions to the complete system of equations. The solutions, as in the case of Ref. 2, depend on the assumed number of multiplets. We examine some of the simpler cases.

(a) The leading trajectory $SU(3)$ multiplets are 8^+, 10^+, 8^-, 1^-. We assume that $\chi_8^{5/2\pm}$, $\chi_{10}^{5/2+}$, $\omega_{S,A}^{5/2\pm}$, $\omega_{10}^{5/2+}$, and $\omega_1^{5/2-}$ are nonzero[10] and that the remaining $\frac{5}{2}$ couplings vanish. If we insist the contributions due to the corresponding states satisfy (4.3) in $PB_{10} \to PB_{10}$, $PB_{10} \to PB_8$, and $PB_8 \to PB_8$, we obtain

$$\begin{aligned}
\frac{\omega_S^{5/2-}}{\omega_A^{5/2-}} &= \left(\sqrt{\frac{5}{9}}\right)\left[(\sqrt{5})\frac{\omega_A^{5/2+}}{\omega_S^{5/2+}} - 2\right], \\
\omega_A^{5/2-} &= (\sqrt{\tfrac{3}{5}})\omega_S^{5/2+}, \\
\chi_8^{5/2-} &= -\left(\sqrt{\frac{5}{7}}\right)\left(\frac{6+\gamma}{2+\gamma}\right)\chi_8^{5/2+}, \\
\omega_{1}^{5/2+} &= (\sqrt{\tfrac{2}{5}})[3\omega_S^{5/2+} - (\sqrt{5})\omega_A^{5/2+}], \\
\chi_{10}^{5/2+} &= \frac{4}{\sqrt{5}}\chi_8^{5/2+}, \\
\omega_1^{5/2-} &= -\left(\sqrt{\frac{4}{15}}\right)[\omega_S^{5/2+} + (\sqrt{5})\omega_A^{5/2+}].
\end{aligned} \tag{4.4}$$

These solutions correspond exactly to the solutions

[9] In the global duality considerations of Refs. 1 and 2, the helicity dependences are not specified. We assume that the considerations apply to helicity amplitudes corresponding to (4.3) for the reasons mentioned in the text. Therefore, if we had started with a model satisfying global duality, we would have expected relations (4.1) and (4.2) in a local energy region. From these relations, it is easy to show that solutions (3.13)–(3.17) follow.

[10] This does not mean that we exclude such cases in which some ω "accidentally" vanishes. For example, if $\omega_s^{5/2+} = (\frac{1}{3}\sqrt{5})\omega_A^{5/2+}$, Eq. (4.4) implies that $\omega_{10}^{5/2+} = 0$.

[Eq. (1)] of Ref. 2, *the identifications being given by*[11]

$$\begin{aligned} f_R&=(2L_0+1)^{1/2}C(L_0\tfrac{3}{2}\,\tfrac{5}{2};0\,\tfrac{3}{2})A_{\frac{3}{2}}^{\frac{5}{2}+}\chi_R^{5/2^+},\\ gc_+&=(2L_0+1)^{1/2}C(L_0\tfrac{1}{2}\,\tfrac{5}{2};0\,\tfrac{1}{2})\omega_S^{5/2^+},\\ gs_+&=(2L_0+1)^{1/2}C(L_0\tfrac{1}{2}\,\tfrac{5}{2};0\,\tfrac{1}{2})\omega_A^{5/2^+},\\ g_{10,1}&=(2L_0+1)^{1/2}C(L_0\tfrac{1}{2}\,\tfrac{5}{2};0\,\tfrac{1}{2})\omega_{10,1}^{5/2^+}, \end{aligned}\tag{4.5}$$

with similar relations for quantities pertaining to $J^P=\frac{5}{2}^-$ particles. Solutions (4.4) are expressed in terms of three arbitrary parameters $\omega_S^{5/2^+}$, $\omega_A^{5/2^+}$, and $\chi_8^{5/2^+}$ (g, θ_+, and f in Ref. 2). In addition we have the relations between the daughter and parent states. The relations between $\chi^{3/2^\pm}$ and $\chi^{5/2^\pm}$ are given in Eqs. (3.14) and (3.15). Using these, we can derive from $PB_8\to PB_{10}$ equations

$$\begin{aligned} &\pm\omega_S^{3/2^\pm}=\tfrac{3}{2}\omega_S^{5/2^\pm},\quad \pm\omega_A^{3/2^\pm}=\tfrac{3}{2}\omega_A^{5/2^\pm},\\ &\pm\omega_{10}^{3/2^+}=(9/\sqrt{10})[\omega_S^{5/2^+}-(\tfrac{1}{3}\sqrt{5})\omega_A^{5/2^+}]. \end{aligned}\tag{4.6}$$

The above equations imply that the F/D ratio for $\frac{3}{2}^\pm$ states is the same as that for $\frac{5}{2}^\pm$. The solutions so far are all still expressible in terms of the same three parameters $\omega_S^{5/2^+}$, $\omega_A^{5/2^+}$, and $\chi_8^{5/2^+}$. Since $\chi_R^{1/2^\pm}=0$, we do not obtain any information concerning $\omega_R^{1/2^+}$ from $PB_8\to PB_{10}$. Similarly, because of the assumption $\chi_{10}^{5/2^-}=\omega_{10}^{5/2^-}=0$ nothing is learned about $\omega_{10}^{3/2^-}$.

The $PB_8\to PB_8$ set of equations places constraints on the couplings $\omega_R^{1/2^\pm}$, $\omega_{10}^{3/2^-}$, along with the singlet couplings $\omega_1^{J^\pm}$. After some simplification of this set of equations and using (4.4) and (4.6), we are able to obtain the relations

$$\begin{aligned} &\omega_S^{1/2^-}[\omega_S^{1/2^-}-(\tfrac{1}{3}\sqrt{5})\omega_A^{1/2^-}]=0,\\ &(\omega_{10}^{1/2^-})^2-(3/\sqrt{5})\omega_A^{1/2^-}[\omega_S^{1/2^-}-(\tfrac{1}{3}\sqrt{5})\omega_A^{1/2^-}]\\ &\qquad=(\omega_{10}^{3/2^-})^2,\\ &\omega_A^{1/2^-}[\omega_S^{1/2^-}+(\sqrt{5})\omega_A^{1/2^-}]=(45/4)^{1/2}(\omega_1^{3/2^+})^2,\\ &(\omega_1^{1/2^-})^2+\tfrac{2}{5}\omega_S^{1/2+}-[\omega_S^{1/2^-}+(\sqrt{5})\omega_A^{1/2^-}]\\ &\qquad=(\omega_1^{3/2^-})^2-(9/4)(\omega_1^{5/2^-})^2,\\ &\omega_S^{1/2^+}[\omega_S^{1/2^+}-(\tfrac{1}{3}\sqrt{5})\omega_A^{1/2^+}]=(5/3)(\omega_{10}^{3/2^-})^2,\\ &(\omega_{10}^{1/2^+})^2-(3/\sqrt{5})\omega_A^{1/2^+}[\omega_S^{1/2^+}-(\tfrac{1}{3}\sqrt{5})\omega_A^{1/2^+}]=0,\\ &\omega_A^{1/2^+}[\omega_S^{1/2^+}+(\sqrt{5})\omega_A^{1/2^+}]\\ &\qquad=(45/4)^{1/2}[(\omega_1^{3/2^-})^2-(9/4)(\omega_1^{5/2^-})^2],\\ &(\omega_1^{1/2^+})^2+\tfrac{2}{5}\omega_S^{1/2^+}[\omega_S^{1/2^+}+(\sqrt{5})\omega_A^{1/2^+}]=(\omega_1^{3/2^+})^2. \end{aligned}\tag{4.7}$$

These eight equations are not enough to determine the eleven couplings $\omega_{S,A,10}^{1/2^\pm}$, $\omega_1^{1/2^\pm}$, $\omega_1^{3/2^\pm}$, and $\omega_{10}^{3/2^-}$ in terms of the parent coupling $\omega_1^{5/2^-}$ (and thus in terms of the parameters $\omega_A^{5/2^+}$ and $\omega_S^{5/2^+}$). The simplest additional assumption would be to generalize the absence of 10^- and 1^+ to include the daughter trajectories, i.e.,

$$\omega_{10}^{3/2^-}=0\quad\text{and}\quad\omega_1^{3/2^+}=0.\tag{4.8}$$

[11] Under conditions (3.11), the ratios $A_\lambda^{J^\pm}$ can be expressed in terms of γ. We have made use of such expressions in the derivation of solutions (3.13)–(3.23) and consequently in the particular solution (4.4).

In this case the remaining couplings become

$$\pm\omega_1^{3/2^-}=\tfrac{3}{2}\omega_1^{5/2^-},\quad \omega_{S,A,10,1}^{1/2^\pm}=0.\tag{4.9}$$

Here all spin-$\frac{1}{2}$ couplings to PB_8 and PB_{10} are zero, and the contributions of the spin-$\frac{3}{2}$ states are enough to insure satisfaction of local and global duality in the sense described in the beginning of this section. However, this may not be the most realistic solution in view of experimental evidence for πN resonances of spin-$\frac{1}{2}$ as well as a decimet particle $J^P=\frac{3}{2}^-$ in the 1700-MeV region. We come back to this point at the end of this section.

(b) The leading trajectory $SU(3)$ multiplets are 8^+, 10^+, 8^-, 10^-, 1^+, 1^-. Equations (3.13)–(3.17) give $PB_{10}\to PB_{10}$ solutions in terms of the parameters $\chi_8^{5/2^+}$ and $\chi_8^{5/2^-}$. Our assumptions concerning global duality in $PB_8\to PB_{10}$ in regard to Eqs. (4.3) imply the relations

$$\begin{aligned} &\omega_S^{5/2^-}\chi_8^{5/2^-}\\ &=\frac{5}{2}\left(\sqrt{\frac{5}{21}}\right)\left(\frac{6+\gamma}{2+\gamma}\right)\left[\frac{1}{3\sqrt{2}}\omega_{10}^{5/2^+}\chi_{10}^{5/2^+}\right.\\ &\qquad\left.-\frac{2}{3\sqrt{5}}\omega_A^{5/2^+}\chi_8^{5/2^+}\right],\\ &\omega_A^{5/2^-}\chi_8^{5/2^-}\\ &=-\frac{5}{2\sqrt{21}}\left(\frac{6+\gamma}{2+\gamma}\right)\left[\frac{1}{3\sqrt{2}}\omega_{10}^{5/2^+}\chi_{10}^{5/2^+}\right.\\ &\qquad\left.+\frac{4}{3\sqrt{5}}\chi_8^{5/2^+}\left(\omega_A^{5/2^+}+\frac{3}{2\sqrt{5}}\omega_S^{5/2^+}\right)\right],\\ &\omega_{10}^{5/2^-}\chi_{10}^{5/2^-}\\ &=-\left(\sqrt{\frac{10}{21}}\right)\left(\frac{6+\gamma}{2+\gamma}\right)\left[\frac{1}{3\sqrt{2}}\omega_{10}^{5/2^+}\chi_{10}^{5/2^+}\right.\\ &\qquad\left.+\frac{4}{3\sqrt{5}}\chi_8^{5/2^+}\left(\omega_A^{5/2^+}-\frac{3}{\sqrt{5}}\omega_S^{5/2^+}\right)\right]. \end{aligned}\tag{4.10}$$

Therefore, in addition to $\chi_8^{5/2^+}$ and $\chi_8^{5/2^-}$, three more parameters are necessary to specify a solution. The remaining $PB_8\to PB_{10}$ equations yield for the spin-$\frac{3}{2}$ daughters

$$\begin{aligned} &\pm\omega_S^{3/2^\pm}=\tfrac{3}{2}\omega_S^{5/2^\pm},\\ &\pm\omega_A^{3/2^\pm}=\tfrac{3}{2}\omega_A^{5/2^\pm},\\ &\pm\omega_{10}^{3/2^\pm}=\tfrac{3}{2}\omega_{10}^{5/2^\pm}. \end{aligned}\tag{4.11}$$

The $PB_8\to PB_8$ equations along with Eqs. (4.3) give

$$\begin{aligned} &\omega_S^{1/2^\pm}=(\tfrac{1}{3}\sqrt{5})\omega_A^{1/2^\pm},\quad \omega_{10}^{1/2^\pm}=0,\quad \omega_1^{1/2^\pm}=0,\\ &(\omega_A^{1/2^+})^2=(\omega_A^{1/2^-})^2\\ &\qquad=(9/8)[(\omega_1^{3/2^\pm})^2-(9/4)(\omega_1^{5/2^\pm})^2]. \end{aligned}\tag{4.12}$$

The above states are decoupled from the rest of the system. Therefore, in order to specify a solution to (4.12), one more parameter is needed in addition to the present couplings $\omega_1^{5/2^+}$ and $\omega_1^{5/2^-}$.

(c) The leading $SU(3)$ multiplets are 8^+, 8^-, 10^+, 10^-, and $\omega_A{}^{5/2\pm}\neq-(5)^{-1/2}\omega_S{}^{5/2\pm}$. This is a particular case of (b) and corresponds to solution (b) (ii) [Eq. (3)] of Ref. 2. We obtain the χ couplings in terms of one parameter $\chi_8{}^{5/2^+}$:

$$\chi_8{}^{5/2^-}=-\left(\sqrt{\frac{5}{21}}\right)\left(\frac{6+\gamma}{2+\gamma}\right)\chi_8{}^{5/2^+},$$

$$\chi_{10}{}^{5/2^-}=-\left(\sqrt{\frac{4}{21}}\right)\left(\frac{6+\gamma}{2+\gamma}\right)\chi_8{}^{5/2^+}, \tag{4.13}$$

$$\chi_{10}{}^{5/2^+}=(\sqrt{\tfrac{4}{5}})\chi_8{}^{5/2^+},$$

with $\chi_R{}^{3/2\pm}$ given by Eqs. (3.14) and (3.15). The PB_8 couplings break down into two sets as follows:

$$\frac{\omega_A{}^{5/2^+}}{\omega_S{}^{5/2^+}}\frac{\omega_A{}^{5/2^-}}{\omega_S{}^{5/2^-}}=\frac{1}{5},\quad \omega_S{}^{5/2^-}=(\sqrt{5})\omega_A{}^{5/2^+},$$

$$\omega_{10}{}^{5/2^+}=-2\sqrt{2}\omega_A{}^{5/2^+}, \tag{4.14}$$

$$\omega_{10}{}^{5/2^-}=-(8/5)^{1/2}\omega_S{}^{5/2^+},$$

$$\pm\omega_{A,S,10}{}^{3/2\pm}=\tfrac{3}{2}\omega_{A,S,10}{}^{5/2\pm}.$$

Here, $\omega_A{}^{5/2^+}$ and $\omega_S{}^{5/2^+}$ may be taken as arbitrary parameters. Again the spin-$\frac{1}{2}$ and the singlet states decouple. They satisfy the relations

$$\omega_S{}^{1/2\pm}=\tfrac{1}{3}(\sqrt{5})\omega_A{}^{1/2\pm},\quad \omega_{10}{}^{1/2\pm}=\omega_1{}^{1/2\pm}=0,$$

$$(\omega_A{}^{1/2^+})^2=(\omega_A{}^{1/2^-})^2=(9/8)(\omega_1{}^{3/2\pm})^2. \tag{4.15}$$

The states $\omega_1{}^{3/2\pm}$ take on the role of parent trajectories in (4.15), where the spin-$\frac{1}{2}$ couplings are expressed in terms of them.

We would now like to discuss briefly the comparison of the different solutions with experiments. For this purpose, we consider nonstrange πN resonances for which considerable information exists through phase-shift analysis.[12] However, the precise positions of the resonances, their total and partial widths are subject to large uncertainties[13] and hence we do not attempt a quantitative comparison in the present investigation. In addition to $F_{15}(1688)$ and $D_{15}(1670)$, the resonance $P_{11}(1780)$, $S_{11}(1700)$, $S_{31}(1650)$, and $D_{33}(1670)$ are believed to be well established.[14] The masses of these resonances are close enough to be considered as a set of degenerate particles in our idealized model. If we assume that these are members of $J^P=\frac{5}{2}^\pm,\frac{1}{2}^\pm$ octets and $J^P=\frac{1}{2}^-,\frac{3}{2}^-$ deciments, we expect that the corresponding PB_8 couplings, namely, $\omega_S{}^{5/2\pm}$, $\omega_A{}^{5/2\pm}$,$\omega_S{}^{1/2\pm}$,$\omega_A{}^{1/2\pm}$, $\omega_{10}{}^{1/2^-}$, and $\omega_{10}{}^{3/2^-}$ should all exist. Hence we examine different solutions from the point of view of accommodating nonvanishing values for the above couplings with as few additional particles as possible.

Solution (a):

If we set

$$\omega_{10}{}^{5/2^+}=0, \tag{4.16}$$

it follows from (4.4) and (4.6) that

$$\omega_A{}^{5/2^+}/\omega_S{}^{5/2^+}=\omega_A{}^{5/2^-}/\omega_S{}^{5/2^-}=3/\sqrt{5} \tag{4.17}$$

and

$$\omega_{10}{}^{3/2^+}=0.$$

From (4.7) one can easily verify that $\omega_S{}^{1/2\pm}$, $\omega_A{}^{1/2\pm}$, and $\omega_{10}{}^{3/2^-}$ can have nonvanishing values provided $\omega_1{}^{3/2\pm}$, $\omega_{10}{}^{1/2^+}$ are also nonvanishing. Also, they imply

$$\omega_S{}^{1/2^-}=0\quad\text{or}\quad \omega_A{}^{1/2^-}/\omega_S{}^{1/2^-}=3/\sqrt{5},$$

and

$$\omega_A{}^{1/2^+}\neq(3/\sqrt{5})\omega_S{}^{1/2^+}. \tag{4.18}$$

Solutions (b) and (c): Both these solutions give

$$\omega_{10}{}^{1/2^-}=0 \tag{4.19}$$

contrary to our requirement.

Further in (b),

$$\omega_{10}{}^{3/2^-}\neq0\quad\text{implies}\quad \omega_{10}{}^{5/2^-}\neq0. \tag{4.20}$$

Similarly solution (c) requires $\omega_{10}{}^{5/2^-}\neq0$ if $\omega_{S,A}{}^{5/2^+}\neq0$.

From these considerations it appears reasonable that solution (a) is preferable at present. The $SU(3)$ multiplets and their spins and parities that appear in this solution are listed in Table II. It is worth noting that the $J^P=\frac{5}{2}^+$ and $J^P=\frac{3}{2}^+$ decimets in this table have nonvanishing couplings only to the PB_{10} system. The only multiplets that are predicted to have nonvanishing PB_8 couplings which were not observed are $J^P=\frac{3}{2}^+$ octet and singlet and $J^P=\frac{1}{2}^+$ decimet.

It should be remarked that nothing in our considerations so far prohibits more than one multiplet of given

TABLE II. Baryon resonances [$SU(3)$ multiplets] as predicted in solution (a) of the text. Italicized entries represent those octets and decimets for which there exists experimental evidence of PB_8 couplings. No attempt has been made to compare the predictions for the singlets with experiment. Entries in parentheses for them represent theoretically allowed singlets. However, their couplings can be set to zero without affecting the positivity requirements on the other residues.

$-J^p$	$SU(3)$
$\frac{5}{2}^+$	*8*, 10
$\frac{5}{2}^-$	*8*, 1
$\frac{3}{2}^+$	8, 10, 1
$\frac{3}{2}^-$	*8*, *10*, 1
$\frac{1}{2}^+$	*8*, 10, (1)
$\frac{1}{2}^-$	*8*, *10*, (1)

[12] See, for example, A. Donnachie, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[13] See, for example, R. J. Plano, in *Proceedings of the Boulder Conference on High Energy Physics, 1969*, edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Associated U. P., Boulder, Colo., 1970).

[14] Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970).

spin and parity. If there is more than one degenerate multiplet, the considerations apply to effective residues and these effective residues still must satisfy positivity restrictions. These remarks assume more relevance in the case of mesons where such degeneracy has been demonstrated from duality considerations.[15] Also when we attempt to extend global and local duality requirements to mesonic reactions, parity and angular momentum conservation provide additional constraints. Thus in the PP system only $J^P=2^+$ is the spin-2 contribution at a given position. In order to satisfy the equations corresponding to $(t\,10,10^*)$ and $(t\,27)$ in the crossed channel, it is necessary that the couplings $\omega_S{}^{2^+}$ and $\omega_1{}^{2^+}$ separately vanish. This obviously is in contradiction with the existence of a single A_2 meson. The splitting of A_2 does not provide a solution unless one imagines a split ρ trajectory. These are well-known difficulties for which we have no satisfactory answer at present.

In $PV\to PV$, however, there are contributions from both $J^P=2^+$ and $J^P=2^-$ states. The requirement that they cancel among themselves in all the amplitudes corresponding to (4.3) can be met if

$$\chi_{1,S}{}^{2^-}(3)/\chi_{1,S}{}^{2^-}(1)=\sqrt{\tfrac{2}{3}}\,,$$
$$\chi_1{}^{1^+}(2)/\chi_1{}^{1^+}(0)=1/\sqrt{2}\,. \tag{4.21}$$

Consequently from (2.21), the quantities $A_1{}^{2^-}$ and $A_1{}^{1^+}$ appearing in Table I become

$$A_1{}^{2^-}=5/3\,,\quad A_1{}^{1^+}=\tfrac{3}{2}\,. \tag{4.22}$$

The comparison of the relations above as well as the predicted mass spectrum with experiment has to await more definitive information. The A_2 splitting and the presence of states in the 1300-MeV region whose spins and parities are not as yet well established makes it difficult to arrive at any firm conclusions.

V. CONCLUDING REMARKS

In the previous section we considered an idealized model in which a set of particles degenerate in mass, but differences in spins, parities, and internal quantum numbers are required to satisfy local duality in reactions whose crossed channels are exotic. We found that, in general, parity doubling and the occurrence of daughter states are necessary when spin considerations are included. If the results can be generalized to a set of particles lying on Regge trajectories, the picture that emerges is that of parent and daughter trajectories. Like the parent trajectories,[1,2] the daughter trajectories exhibit specific exchange-degeneracy patterns with constraints relating some of the parent and daughter residues at resonance positions.

Barring a few low-lying states[16] on the known hadronic trajectories, such a picture is not qualitatively inconsistent with the presently available experimental information. It is also essential for models based on Veneziano-type considerations. We should remark here, however, that in addition to local duality, certain specific assumptions concerning the couplings, namely, a given resonance coupled dominantly to the lower of the two allowed partial waves, have played an important role in obtaining the solutions. These assumptions are motivated by the available information concerning some resonances in the energy region of interest, but they can be relaxed and will not be true at higher energies. In the present investigation, comparison of the theoretical results with experiments has been limited to qualitative considerations mainly concerning the particle mass spectrum. Some new baryon (meson) multiplets are inevitably predicted, but their presence is not ruled out because their couplings are predominantly to the B_{10} (PV) system. Precise experimental information is lacking at present for these couplings.

In summary, the spin considerations in dual models and the examination of the constraints on the s-channel resonances do provide interesting information concerning the particle spectrum and the coupling patterns. For a more detailed comparison with experiments, it is necessary to consider the effect of nondegeneracy in mass and $SU(3)$ symmetry breaking. Also, more precise experimental information concerning total widths and partial widths of different decay modes is needed. Further theoretical investigation along similar lines should be useful in putting constraints on satellite terms in Veneziano-type models. One can also extend the present study to reactions whose crossed channels are not exotic and compare the s-channel description in terms of resonances with the crossed-channel Regge amplitude.[17] These considerations as well as the extension to higher-energy regions and higher-spin external particles are currently under investigation.

[15] S. Fubini and G. Veneziano, Nuovo Cimento **64A**, 811 (1969); Y. Nambu, in Conference on Symmetries and Quark Models, Wayne State University, (unpublished).

[16] For example, $\Delta(1238)$ has no parity partner or daughter state. Likewise ρ and ω have no parity partners.

[17] P. G. Tomlinson, R. A. Sidwell, D. B. Lichtenberg, R. M. Heinz, R. R. Crittenden, and E. Predazzi, Indiana University Report No. C00-2009-16, 1970 (unpublished).