is identical to the ratio given in Eq. (21). Since our Veneziano models for both $\pi N \to \rho N$ and $\gamma N \to \pi N$ are asymptotic, the conclusion is that, at least at small $|t|$, with increasing energy, the two ratios are expected to tend to the same value.[11]

Finally, it is readily seen that the ratio ρ_{1-1}/ρ_{11} in the Jackson or Donohue-Högaasen frame is completely different from (27). Thus, in agreement with other considerations[6,12] our final conclusion is that the helicity frame offers the only meaningful choice for the definition and test of vector-dominance relations.

[11] The same conclusion is obtained in the Born model (Ref. 6). Note that the present experimental data for $(\sigma_\perp - \sigma_{\parallel})/(\sigma_\perp + \sigma_{\parallel})$ at 3.4 GeV and for $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$ at 4 GeV [R. Diebold and J. Poirier, Phys. Rev. Letters **22**, 255 (1969)] show a significant difference between the two ratios. This may be due to the following reasons: (i) finite ρ mass corrections important at relatively low s, (ii) nonleading terms in s left out in our asymptotic expressions, and (iii) significant experimental error in the present results for $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$.

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One of us (A.P.C.) would like to thank Dr. M. Jacob and Dr. V. Müller for discussions and Professor W. Thirring and Professor J. Prentki for the hospitality extended to him at the Theoretical Study Division of CERN.

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Towards Inclusion of Spin in the Generalized Veneziano Model for $K^-p \to \pi^-\pi^+\Lambda$

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By combining the $U(6,6)$ supermultiplet scheme with the Veneziano model, we are able to remove some of the arbitrariness of the Petersson-Törnqvist model. The model is applied to $K^-p \to \pi^-\pi^+\Lambda$. The results are encouraging.

INTRODUCTION

Recent studies[1] of three-particle production data using a model proposed by Petersson and Törnqvist have had considerable success. In this model a reaction is described by an amplitude of the form

$$A = K \sum_1^{12} B_5 .$$

The sum is over the 12 five-point Veneziano terms corresponding to noncyclic permutations of the external lines. K is a kinematic factor which, for processes with over-all abnormal parity, is taken as

$$K = \epsilon_{\alpha\beta\gamma\delta} P_1{}^\alpha P_2{}^\beta P_3{}^\gamma P_4{}^\delta ,$$

where the P_i are any four of the external particle momenta. This kinematic factor takes no account of the spin or isospin of the external particles. In this paper we take an amplitude of the form

$$A = \sum_{i=1}^{12} F_i B_5{}^i .$$

Following the suggestion of Mandelstam[2] and the recent work of Delbourgo and Rotelli,[3] we extend the latter's prescription for including spin and isospin in

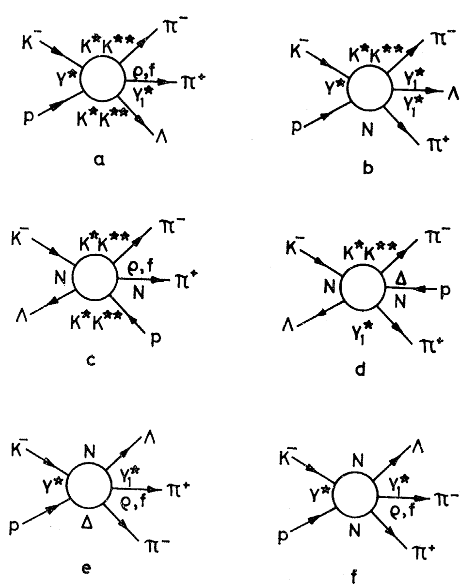

FIG. 1. The permutations remaining after excluding those involving exotic meson exchanges.

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[1] See, for example, (a) B. Petersson and N. A. Törnqvist, Nucl. Phys. **B13**, 629 (1969); (b) N. A. Törnqvist, *ibid.* **B18**, 530 (1970); (c) P. A. Collins, B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Phys. Rev. D **1**, 3195 (1970); (d) P. A. Collins, B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Nucl. Phys. **B22**, 150 (1970).

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[3] R. Delbourgo and P. Rotelli, Phys. Letters **30B**, 192 (1969).

the five-point amplitude. This enables us to determine the F_i for each B_5 term. We apply the model to the process $K^- p \to \pi^- \pi^+ \Lambda$ which was considered in the original paper of Petersson and Törnqvist. In their

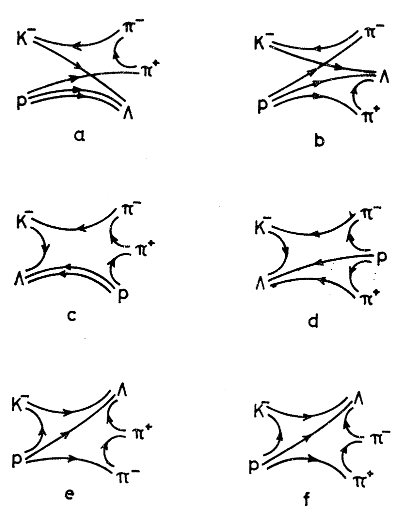

FIG. 2. The duality diagrams corresponding to the permutations of Fig. 1.

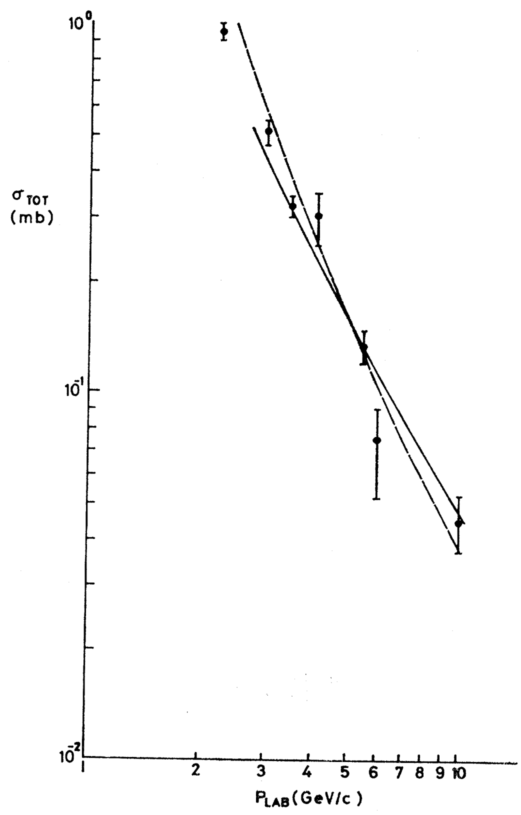

FIG. 3. The energy dependence of the total cross section. Dashed line represents theoretical distributions for A_1; solid line represents theoretical distributions for A_2.

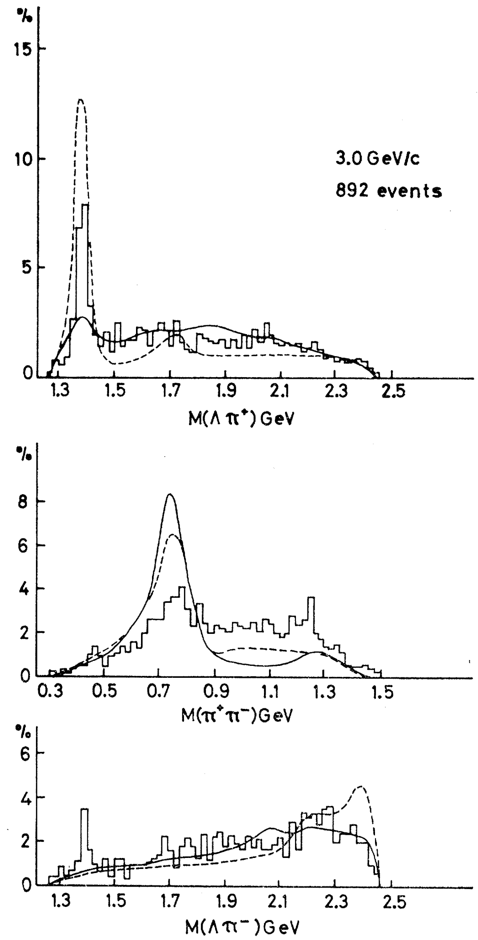

FIG. 4. Mass distributions at 3.0 GeV/c.

paper it was found that good fits to the data could only be obtained by using the permutations corresponding to "illegal" Harari-Rosner[4] duality diagrams. By choosing this process for an initial application, we can make two interesting tests of our model.

1. Using the same B_5 terms as in Ref. 1(a), we can see how our attempt to include spin in the model modifies the previous predictions.

2. By doing a similar calculation using only terms corresponding to "legal" Harari-Rosner diagrams, it is possible to find out if our model in any way resolves the apparent contradiction of the suggestion of Harari and Rosner found in Ref. 1(a).

FORMALISM

As we have already stated, we write the amplitude as

$$A=\sum_{i=1}^{12} F_i B_5{}^i(x_{12},x_{23},x_{34},x_{45},x_{51}).$$

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[4] H. Harari, Phys. Rev. Letters 22, 562 (1969); J. L. Rosner, *ibid.* 22, 689 (1969).

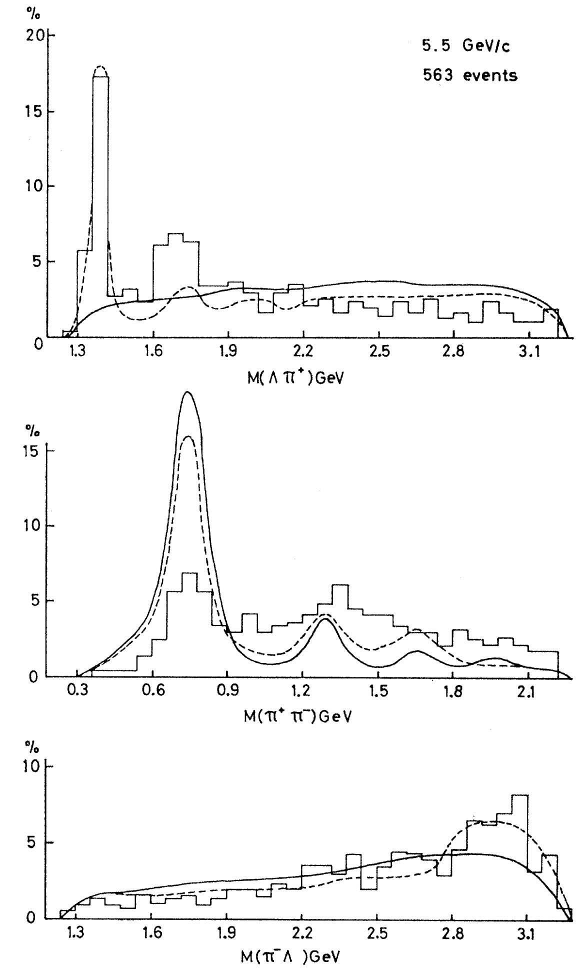

FIG. 5. Mass distributions at 5.5 GeV/c.

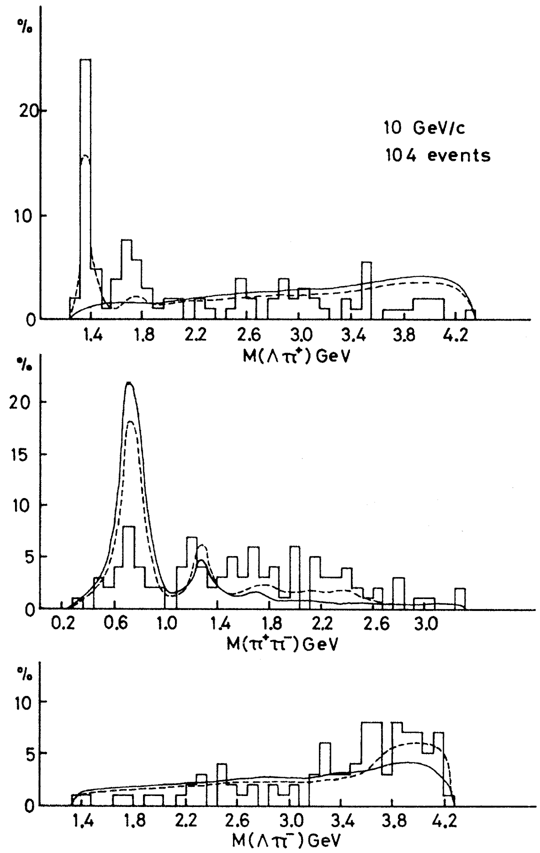

FIG. 6. Mass distributions at 10.0 GeV/c.

The arguments of the five-point Veneziano functions are of the form $x_{jk}=1-\alpha_{jk}$, where α_{jk} are linear Regge trajectories coupling to the external particles j and k

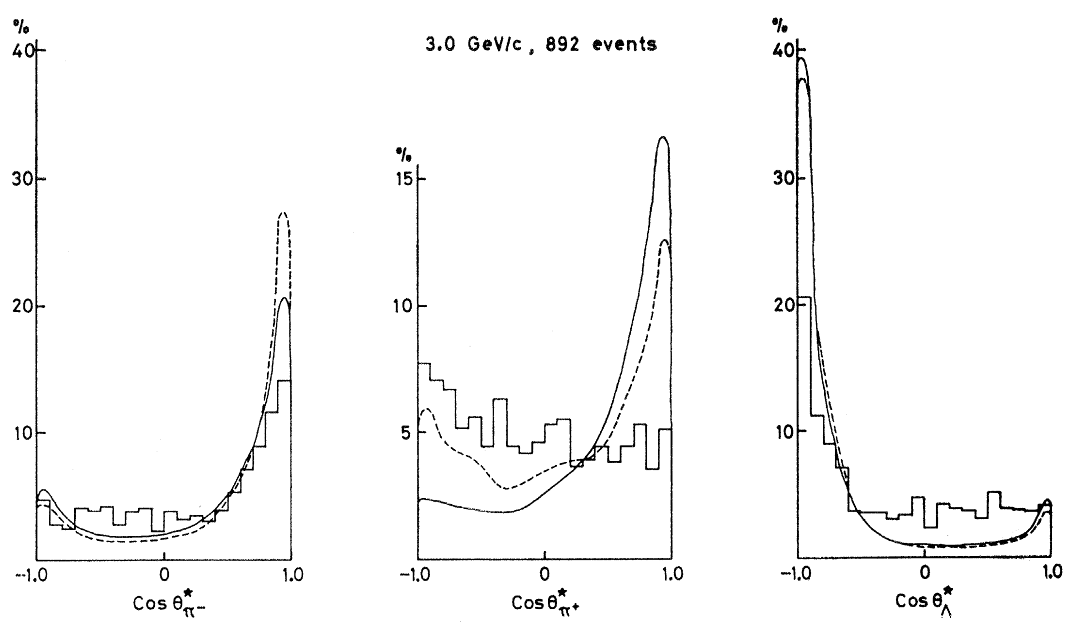

FIG. 7. c.m. production angle distributions at 3.0 GeV/c.

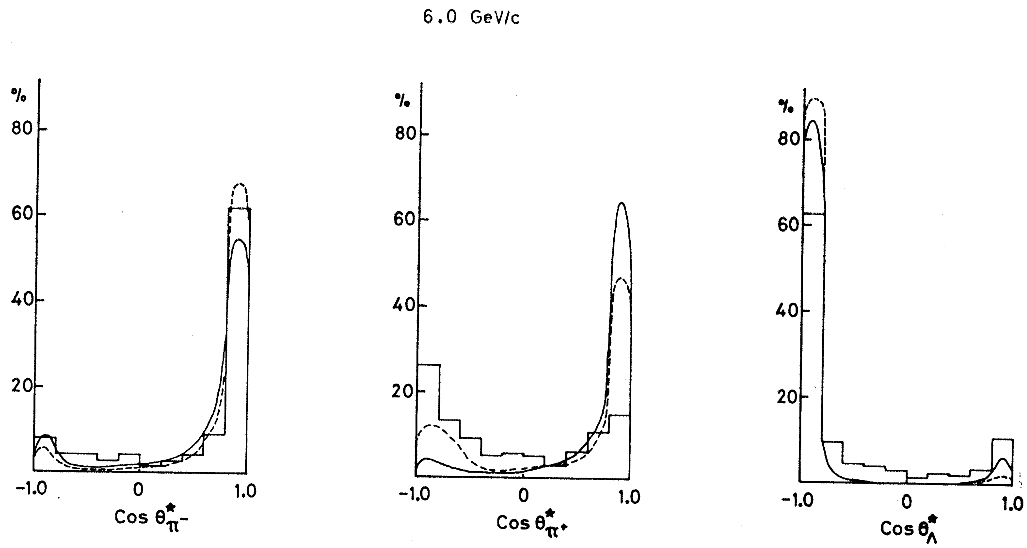

FIG. 8. c.m. production angle distributions at 6.0 GeV/c.

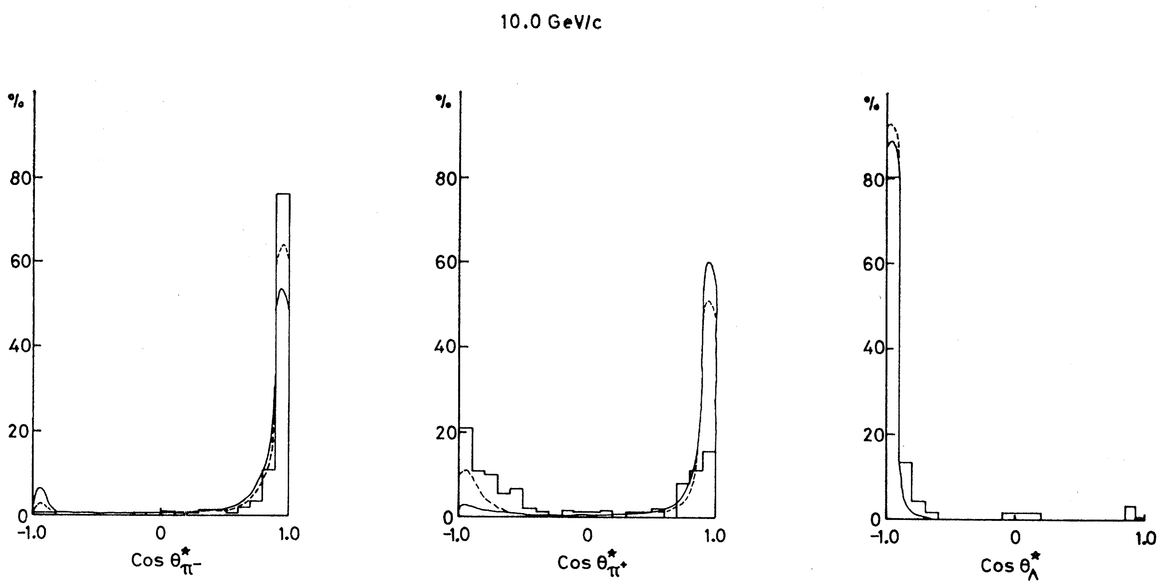

FIG. 9. c.m. production angle distributions at 10.0 GeV/c.

For fermion trajectories the arguments are shifted by a half-integer in order to get correct resonance structure and asymptotic behavior.

To evaluate the F_i, we employ the $U(6,6)$ multispinor formalism[5] in which the $(56,1)^+$ baryonic supermultiplet is described by the wave function

$$\psi_{(ABC)}(P) \equiv \psi_{(\alpha a,\beta b,\gamma c)}(P) = (1/2m)(\sqrt{\tfrac{2}{3}})[(\boldsymbol{P}+m)\gamma_\mu C]_{\alpha\beta} \times D_{\mu\gamma}(abc) + (1/2m)(\sqrt{\tfrac{1}{6}})\{[(\boldsymbol{P}+m)\gamma_5 C]_{\alpha\beta} \times \epsilon_{abs} N_{\gamma c}{}^{s} + (\text{cyclic perms. in } \alpha a, \beta b, \gamma c)\}$$

and the $(6,\bar{6})^-$ meson supermultiplet by the wave function

$$\Phi_A{}^B(\mathrm{P}) \equiv \Phi_{\alpha a}{}^{\beta b}(P) = (1/2\mu)[(\boldsymbol{P}+\mu)(\gamma_\mu \phi_{\mu a}{}^{b} - \gamma_5 \phi_{5a}{}^{b})]_\alpha{}^\beta.$$

For each term in the amplitude we draw the corresponding duality diagram and evaluate F_i by simply contracting over quark indices in the specific way suggested by the flow of the quark lines (see the Appendix for a detailed calculation of a typical F_i).

[5] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **284A**, 146 (1965); M. A. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); B. Sakita and K. C. Wali, *ibid.* **14**, 404 (1965); Phys. Rev. **139**, B1355 (1965); R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, *Seminar on High-Energy Physics and Elementary Particles, Trieste* (International Atomic Energy Agency, Vienna, 1965), p. 455.

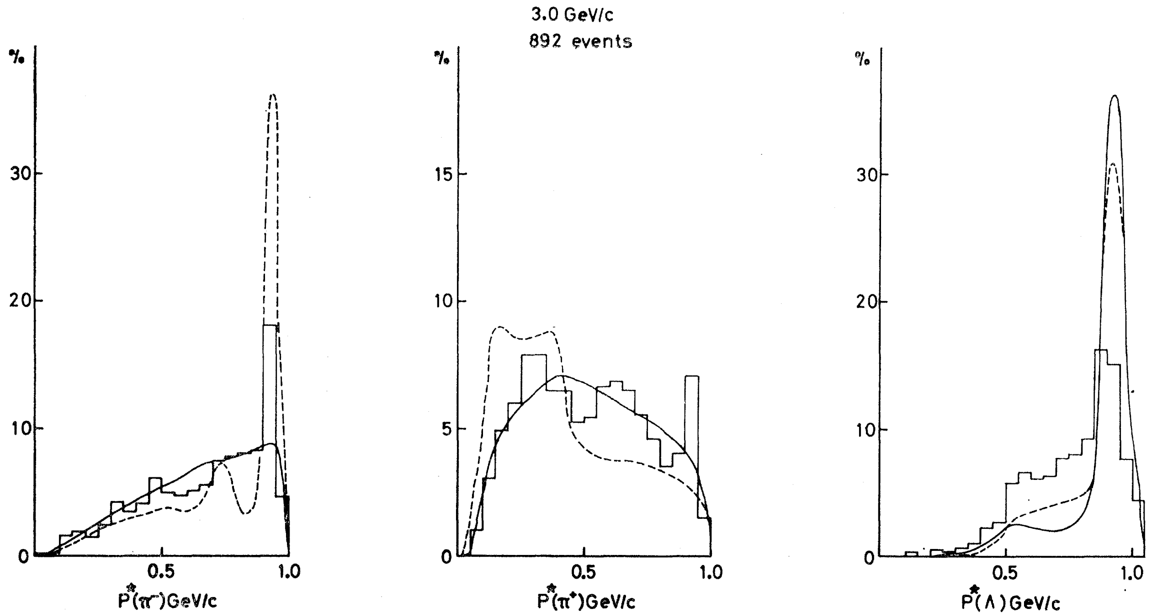

FIG. 10. c.m. $|P^*|$ distribution at 3.0 GeV/c.

We note that among other terms the F_i contain the "kinematic factor" of Petersson and Törnqvist. This was used because of the presence of spin-1 meson and P-wave baryon intermediate state at the first pole.

The extra terms in our case arise from the fact that we have contracted over $U(6,6)$ group-theoretic fields. This leads to (a) a Toller-like exchange structure, and thus gives a parity doubling independent of the parity doubling present in the Veneziano terms, and (b) an infinite set of daughter trajectories under the $U(6)$ decomposition of $U(6,6)$. These daughters are due solely to the internal symmetry and are in no way connected with Freedman-Wang daughters.

APPLICATION TO $K^-p \to \pi^-\pi^+\Lambda$

As in Ref. 1 we assume that there are no doubly charged mesons. This reduces to six the number of permutations contributing to the amplitude. These are shown in Fig. 1. We have used the same trajectories as in Ref. 1(a). In Fig. 2 we show the corresponding Harari-Rosner duality diagrams. Diagrams (a) and (b) in Fig. 2 disobey the Harari-Rosner rules and, as determined by the corresponding diagrams in Fig. 1, are not unique. The duality diagrams we have taken are those for which meson trajectories correspond to two quark lines, which is consistent with our assumption that there are no exotic mesons.

In common with all previous work in this field, we assume degeneracy of the ρ, f^0 and K^*, K^{**} trajectories. Thus, we can include only one of the permutations of the amplitudes which differs from another in having the K^- and π^- or p and Λ lines permuted. When permutations of this type are added, every second resonance on the exchange-degenerate trajectory will be canceled. This applies to pairs of diagrams [(a),(c)], [(b),(d)], and [(e),(f)] in Fig. 1. Hence our amplitude can only contain three terms. Because they only proceed by double baryon exchange in any Regge limit, diagrams (e) and (f) give small contributions and in Ref. 1(a) they are ignored. We will consider two possible amplitudes:

$$A_1=C_1[F_aB_5(a)+F_bB_5(b)+F_eB_5(e)]\,,$$
$$A_2=C_2[F_cB_5(c)+F_dB_5(d)+F_eB_5(e)]\,.$$

A_1 corresponds to choosing the "illegal" Harari-Rosner terms [as in Ref. 1(a)]. A_2 contains only "legal" Harari-Rosner terms. C_1 and C_2 are normalization constants. We use the same trajectory parameters as in Ref. 1(a). For mass splitting in $U(6,6)$, we take $\mu=0.417$ GeV/c^2, the average of the 0^- nonet, and $m=1.150$ GeV/c^2, the average of the $\frac{1}{2}^+$ octet.

The theoretical distributions are calculated using the program FOWL[6] and using the program[7] by Hopkinson for B_5. The predictions are compared with data[8] at 3, 5.5, 6, and 10 GeV/c.

[6] F. James, FOWL, CERN Program Library No. W505 (unpublished).

[7] J. F. L. Hopkinson, Daresbury Report No. DNPL/P21, 1969 (unpublished).

[8] G. W. London, R. R. Rau, N. P. Samios, S. S. Yamamoto, M. Goldberg, S. Lichtman, M. Primer, and J. Leitner, Phys. Rev. **143**, 1034 (1966); Amsterdam-Ecole Polytechnique-Saclay collaboration (unpublished); Birmingham-Glasgow-London (I.C.)-Oxford-Rutherford collaboration, Phys. Rev. **152**, 1148 (1966); J. Mott, R. Ammar, R. Davis, W. Kropac, D. Slate, B. Werner, S. Dagan, M. Derrick, T. Fields, J. Loken, and F. Schweingruber, *ibid.* **177**, 1966 (1969); Birmingham-Glasgow-London (I.C.)-Oxford-Munich-Rutherford collaboration, Phys. Letters **24B**, 489 (1967) and (unpublished); Aachen-Berlin-CERN-London (I.C.)-Vienna collaboration, Nucl. Phys. **B5**, 606 (1968).

DISCUSSION OF RESULTS

In Figs. 3–11 we show the predictions for amplitudes A_1 and A_2. We note that among other terms the F_i contain the "kinematic factor" of previous works.

In Fig. 3 we show the energy dependence of the total cross section. Here we have fixed the normalization parameters C_1 and C_2 using the data at 5.5 GeV ($C_1/C_2=1.44$). Agreement with the data is good.

In general we note that the distributions for the "illegal" term amplitude (A_1) are similar in shape to those of Ref. 1(a). However, the fits are not as good. This is most noticeable as an exaggeration of the peaks in the distributions. This is presumably due to the terms in the F_i other than $\epsilon_{\alpha\beta\gamma\delta}P_1{}^{\alpha}P_2{}^{\beta}P_3{}^{\gamma}P_4{}^{\delta}$. The most significant failings of amplitude A_1 are a tendency to give too much ρ production and for the π^+ to have a peak in the forward direction which is in contradiction with the data. Also we get too much $Y^*(1385)$ production at low energies and too little at high energies.

For the amplitude A_2 the predictions are in general similar to those of A_1. However, in this case we do not obtain the peak in the $P_L{}^*(\pi^-)$ distribution and the Y^* peak is far too small in the $\Lambda\pi^+$ distribution. The latter problem is to be expected, as for A_2 the Y^* trajectory only appears in the $\Lambda\pi^+$ channel in Fig. 1(e) which we expect to be small.

CONCLUSIONS

In our prescription for determining the "kinematic factors" in front of the B_5 terms, we obtain among other terms the kinematic factor of previous works.[1] The price we pay for removing some of the arbitrariness in this factor, is a deterioration in the "goodness" of the fits. Of course, this may be improved by a different $U(6,6)$ mass-splitting prescription for the m and the μ. Except for the absence of a large $Y^*(1385)$ peak in the $\Lambda\pi^+$ distribution, which is in fact really only put into A_1 by hand, the results for amplitude A_2 seem as acceptable as those for A_1. It is true to say that all the qualitative features of the structure in the amplitudes comes from B_5 and the net effect of the kinematic factor is to accentuate or attenuate this structure. Thus, in the choice of which diagrams are dominant we seek those which contain the resonances observed experimentally.

For A_1, whenever "illegal" diagrams are used, the quark content of some of the channels will not be consistent with that usually assigned to the trajectory coupling to the channel. This difficulty would be avoided in an application of the model to a process such as $K^-p\rightarrow\bar{K}^0\pi^-p$, where only legal diagrams contribute.

We regard the present calculation as a preliminary report on the amalgamation of $U(6,6)$ and the Veneziano model. We are presently looking at the spin-density matrices for four-body subprocesses of our five-point function and these should provide a stringent test of the supermultiplet Veneziano model.

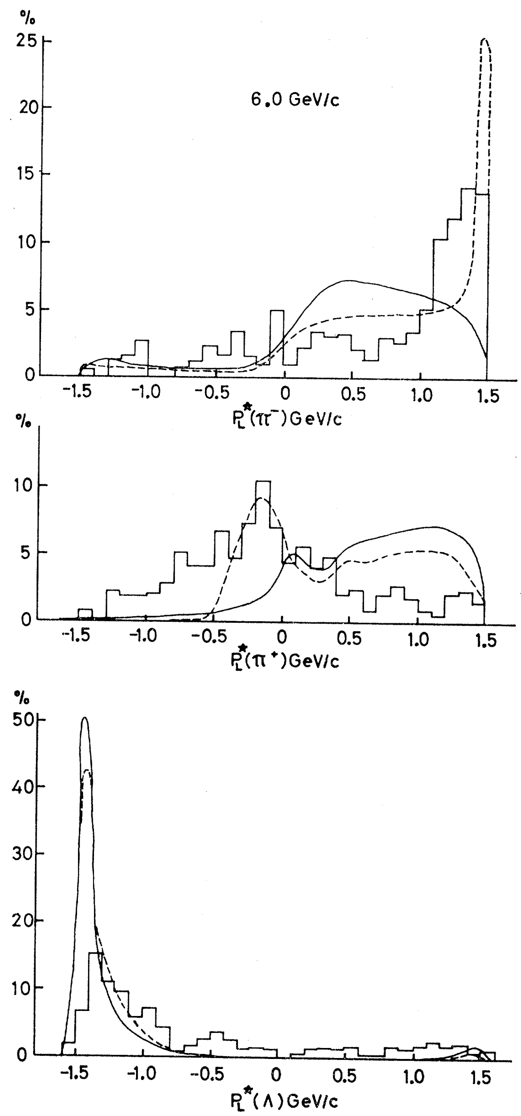

FIG. 11. Longitudinal momentum distributions at 6.0 GeV/c.

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We wish to thank Professor P. T. Matthews for encouragement in this work and for a critical reading of the manuscript. One of us (B. J. H.) wishes to thank the University of London for the award of the George William Britt (Junior) Studentship, two others (P. A. C. and R. W. M.) wish to thank the Science Research Council for Research Studentships, and another (S. A. A.) wishes to thank the University of Ghana for a Postgraduate Scholarship.

APPENDIX

In order to illustrate the evaluation of the F_i, we evaluate F_c in detail. F_c is obtained by contracting quark indices of the particle wave functions as indicated in the duality diagram Fig. 2(c). We obtain

$$F_c=\bar{\psi}^{ABC}(P_\Lambda)\psi_{ABD}(P_p)\Phi_E{}^D(-P_{\pi^+})\times\Phi_F{}^E(-P_{\pi^-})\Phi_C{}^F(P_{K^-})\,.$$

The $U(2,2)$ trace contribution to F_c is given by

$$F_c{}^{U(2,2)}=\{\bar{N}^\gamma(P_\Lambda)[C^{-1}\gamma_5(\boldsymbol{P}_\Lambda+m)]^{\alpha\beta}+\text{(cyclic perms.)}\}\times\{[(\boldsymbol{P}_p+m)\gamma_5 C]_{\alpha\beta}N_\delta+\text{(cyclic perms.)}\}\times\{(-\boldsymbol{P}_{\pi^+}+\mu)\gamma_5(-\boldsymbol{P}_{\pi^-}+\mu)\gamma_5(\boldsymbol{P}_{K^-}+\mu)\gamma_5\}_\gamma{}^\delta\,.$$

The $SU(3)$ contribution is given by

$$F_c{}^{SU(3)}=(\epsilon^{abs}\bar{N}_{2s}{}^c+\epsilon^{bcs}\bar{N}_{2s}{}^a+\epsilon^{cas}\bar{N}_{2s}{}^b)\times(\epsilon_{abr}N_{1d}{}^r+\epsilon_{bdr}N_{1a}{}^r+\epsilon_{dar}N_{1b}{}^r)\times(\tau_{\pi^-})_e{}^d(\tau_{\pi^+})_f{}^e(\tau_{K^-})_c{}^f\,,$$

where $\bar{N}_2$, N_1 are the $SU(3)$ isospin matrices for the outgoing Λ and incoming proton, and the τ matrices are for the mesons all considered as ingoing.

The final expression for F_c is

$$\begin{aligned}F_c&=F_c{}^{SU(3)}F_c{}^{U(2,2)}\\&=\frac{1}{192\mu^3m^2}\bigg[\frac{8i(2m+3\mu)}{\sqrt{6}}\epsilon_{\alpha\beta\gamma\delta}P_\alpha\Lambda_\beta\pi_\gamma{}^+\pi_\delta{}^-\langle\bar{N}_2N_1\rangle\\&\quad+\frac{24(2P\cdot\Lambda+m^2)}{\sqrt{6}}\{[(2m+3\mu)\pi^+\cdot\pi^- -2\mu P\cdot\pi^+ +2\mu P\cdot\pi^- -2\mu^2 m-\mu^3]\langle\bar{N}_2\gamma_5N_1\rangle\\&\quad+[(\mu^2+2\pi^+\cdot\pi^- -2P\cdot\pi^-)\pi_\nu{}^+ +(2P\cdot\pi^+ -3\mu^2)\pi_\nu{}^-]\langle\bar{N}_2\gamma_\nu\gamma_5N_1\rangle-\tfrac{1}{2}i(2m+3\mu)\epsilon_{\nu\lambda\rho\kappa}\pi_\nu{}^-\pi_\lambda{}^+\langle\bar{N}_2\sigma_{\rho\kappa}N_1\rangle\}\\&\quad+(2/\sqrt{6})\{[P\cdot\Lambda(4\mu P\cdot\pi^- +4m\mu^2-10\mu\pi^+\cdot\pi^- -4\mu P\cdot\pi^+ -2\mu^3+6m\pi^+\cdot\pi^-)+P\cdot\pi^+(12\mu\Lambda\cdot\pi^- -8m\Lambda\cdot\pi^- -4m^2\mu)\\&\quad+P\cdot\pi^-(12\mu\Lambda\cdot\pi^+ -8m\Lambda\cdot\pi^+)+4m^3\mu^2-2m^2\mu^3-4m^2\mu\pi^+\cdot\pi^- -2m^3\pi^+\cdot\pi^-]\langle\bar{N}_2\gamma_5N_1\rangle\\&\quad+[2(P\cdot\Lambda+m^2)(\mu^2+2\pi^+\cdot\pi^- -2P\cdot\pi^-)\pi_\nu{}^+ +(4(P\cdot\Lambda)(P\cdot\pi^+)\\&\qquad+12m\mu P\cdot\pi^+ -6\mu^2P\cdot\Lambda-6m^2\mu^2+2m^3\mu)\pi_\nu{}^-]\langle\bar{N}_2\gamma_5\gamma_\nu N_1\rangle\}\bigg]\end{aligned}$$

where $\Lambda_\alpha\equiv P_{\alpha\Lambda}$, etc. $\langle\bar{N}_2N_1\rangle$, etc., are $U(2,2)$ traces.