Scale-Invariance and Chiral-Symmetry Breaking for Nucleon States*

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(Received 19 October 1970)

Assuming that chiral invariance is only broken by terms in the energy-momentum tensor Θ^{00} belonging to the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times SU(3)$, we derive a sum rule for the matrix element of Θ (the trace of the energy-momentum tensor) between nucleon and pion-nucleon states which may suggest that the divergence of the axial-vector current has dimension 1. We then go on to consider an analogous sum rule which suggests that the energy-momentum tensor contains a scale-invariance-breaking chiral singlet term, and finally we point out some implications for $SU(3)$ breaking of the form of matrix elements of Θ.

I. INTRODUCTION

THERE has recently been interest shown in the study of scale invariance as an approximate symmetry in particle physics.[1] Among the questions which have been considered are the decomposition of the energy-momentum tensor into terms which are scale-invariant and chiral symmetric and those which break these symmetries.

Let the energy density be written in the form[2]

$$\Theta^{00}=\bar{\Theta}^{00}+\delta+u, \tag{1}$$

where $\bar{\Theta}^{00}$ is invariant under scale and chiral transformations, δ is a chiral singlet which breaks scale invariance, and u, belonging to the $(3,\bar{3})+(\bar{3},3)$ representation of $SU(3)\times\mathrm{SU}(3)$, breaks both chiral symmetry and scale invariance. We then have[1,2]

$$\Theta_\mu{}^\mu=(4-d_\delta)\delta+(4-d_u)u, \tag{2}$$

where d_δ and d_u are, respectively, the dimensions of d and u under scale transformations.

The problem to which we address ourselves in Sec. II is a determination of d_u. Since

$$[F_i{}^5,u_j]=-id_{ijk}v_k, \tag{3}$$

where $F_i{}^5$ is an axial-vector charge which we assume has dimension zero, we also have $d_u=d_v$, where d_v is the dimension of $\partial_\alpha\mathfrak{F}^{5,\alpha}$, the divergence of the axial-vector current. We shall do this by making use of the matrix element between neutron and proton of the commutator of the axial-vector charge and $\Theta_\mu{}^\mu$:

$$-i\langle p|[F_+{}^5(0),\Theta(0)]|n\rangle=(4-d_u)\langle p|\partial_\alpha\mathfrak{F}_+{}^{5,\alpha}(0)|n\rangle, \tag{4}$$

where henceforth we denote d_u by d and $\Theta_\mu{}^\mu$ by Θ. We then use a low-energy theorem for the matrix element of Θ between nucleon and pion-nucleon states and find from it, under certain assumptions, that $d=1$.

In Sec. III we make a similar analysis starting from the commutator

$$i-\langle p|[F_+{}^5(0),u(0)]|n\rangle=\langle p|\partial_\alpha\mathfrak{F}_+{}^{5,\alpha}(0)|n\rangle \tag{5}$$

and conclude that a treatment similar to that of Sec. II suggests that Θ^{00} does contain a chiral singlet δ and that the matrix of u between nucleon states is small. These results are complementary to those of Jackiw.[3]

In Sec. IV we point out some of the implications for $SU(3)$ breaking provided by the form, derived in Sec. II, of matrix elements of Θ.

II. DIMENSION OF CURRENT DIVERGENCE DENSITIES

Let us begin by considering, between proton and neutron states of equal momentum p, the quantity $\Gamma^\alpha(q^2,q\cdot p)$, where

$$-\Gamma^\alpha(q^2,q\cdot p)=\int d^4x\, e^{iq\cdot x}\langle p|T\{\mathfrak{F}_+{}^{5,\alpha}(x)\Theta(0)\}|N\rangle. \tag{6}$$

We then have, using the commutator given in Eq. (4),

$$\begin{aligned} q_\alpha\Gamma^\alpha(q^2,q\cdot p) &= -i\int d^4x\, e^{iq\cdot x}\langle p|T\{\partial_\alpha\mathfrak{F}_+{}^{5,\alpha}(x)\Theta(0)\}|N\rangle \\ &\qquad +(4-d)\langle p|\partial_\alpha\mathfrak{F}_+{}^{5,\alpha}(0)|N\rangle \\ &= -i\Gamma(q^2,q\cdot p)+(4-d)(M_nM_p/E_nE_p)^{1/2} \\ &\qquad \times i(M_p+M_n)D(q^2)\bar{u}_p\gamma_5u_n, \end{aligned} \tag{7}$$

where $D(0)=G_A$.[4] As $q\to 0$, we separate out as usual the contribution of the one-nucleon state (keeping $M_p\neq M_n$ until the end of the calculation) and find

$$\lim_{q\to 0} i\tilde{\Gamma}(q^2,q\cdot p)(E^2/M^2)^{1/2}=(4-d-1)2iMG_A \times\bar{u}_p\gamma_5u_n, \tag{8}$$

* Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT (30-1)-2098.

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[1] P. Carruthers, Nucl. Phys. (to be published) has a rather complete survey of recent work on the subject.

[2] M. Gell-Mann, Caltech. Report No. 68-244, 1969 (unpublished), discusses this point.

[3] R. Jackiw, preceding papers, Phys. Rev. D **3**, 1347 (1971); **3**, 1356 (1971).

[4] Our notation and normalization is that of, e.g., S. Adler and R. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

where $\tilde{\Gamma}(q^2,q\cdot p)$ is $\Gamma(q^2,q\cdot p)$ with the nucleon pole removed. Now there are several ways to calculate $\Gamma(q^2,q\cdot p)$: One can use Feynman diagrams or dispersion relations, or else start out from the matrix element

$$-i\langle p|\left[F_+{}^5(0),\int d\mathbf{y}\,\Theta(\mathbf{y},0)\right]|n\rangle = (4-d)\langle p|\int \partial_\alpha \mathfrak{F}_+{}^{5,\alpha}(\mathbf{y},0)d\mathbf{y}|n\rangle \quad (9)$$

and evaluate it in the infinite-momentum limit.

We do have a t-channel pion pole contributing to $\Gamma(q^2,q\cdot p)$. Its contribution is

$$\lim_{q\to 0} i\tilde{\Gamma}(q^2,q\cdot p)|_{\text{pion pole}} = f_\pi T_{\Theta\pi\pi}(0,0,0)\Delta(0)T_{pn\pi^+}(0)\,,$$

$$T_{\Theta\pi\pi}(k_1{}^2,k_2{}^2,k_1\cdot k_2) = \frac{(k_1{}^2-\mu^2)}{i}\frac{(k_2{}^2-\mu^2)}{i}\int\int d^4x d^4y \times e^{i(k_1\cdot x-k_2\cdot y)}\langle 0|T\{\pi(x)\pi(y)\Theta(0)\}|0\rangle\,, \quad (10)$$

where $T_{pn\pi^+}(0)$ is the off-mass-shell pion-nucleon matrix element, which (using the Goldberger-Treiman relation) is given by[4]

$$f_\pi T_{pn\pi^+}(0) = (M/E)2iMG_A\bar{u}_p\gamma_5 u_n\,, \quad (11)$$

$\Delta(0)$ is the zero-momentum pion propagator which we set equal to $-i/\mu^2$ (M is, of course, the nucleon mass and μ is the pion mass), and finally $T_{\Theta\pi\pi}(0,0,0)$ is the off-mass-shell $\Theta\pi\pi$ vertex. We may now either assume smoothness in the extrapolation of external pion momenta to zero and assume $T_{\Theta\pi\pi}(0,0,0)=2\mu^2$ or else use Jackiw's[3] exact, but unphysical, low-energy theorem and set

$$T_{\Theta\pi\pi}(0,0,0) = (4-2d)\mu^2\,. \quad (12)$$

Neglecting all other contributions to $\tilde{\Gamma}(0,0)$, we obtain from (8)

$$2iMG_A(4-2d)\bar{u}_p\gamma_5 u_n = 2iMG_A(4-d-1)\bar{u}_p\gamma_5 u_n\,, \quad (13)$$

which implies $d=1$. Of course, $T_{\Theta\pi\pi}(0,0,0)$ equal to $2\mu^2$ also implies $d=1$. Before turning to the question of the other contributions to $\tilde{\Gamma}(0,0)$, let us show how the alternate derivations of (13) would proceed.

Dispersively, we would consider a function $A(\nu,q^2)$ defined by (with $\nu=q\cdot p/M$)

$$\tilde{\Gamma}(q^2,q\cdot p) = (M/E)\bar{u}_p\gamma_5 u_n A(q^2,\nu) \quad (14)$$

and then disperse in ν at fixed $q^2=0$. If the pion is elementary, $A(q^2,\nu)$ approaches a constant as $\nu\to\infty$, and consequently one either makes a subtraction or disperses $A(q^2,\nu)$ minus the contribution of the pion pole, calculated as before. In the latter case the t-channel pion pole contributes as before and the s- and u-channel intermediate states (other than the nucleon) are evaluated dispersively by first using partial conservation of axial-vector current (PCAC) to say

$$A(0,\nu) = f_\pi R_{\pi^+n\to\Theta p}(\nu) = f_\pi R(\nu)\,, \quad (15)$$

where $R_{\pi^+n\to\Theta p}(\nu)$ is the invariant scattering amplitude for $\pi^+n\to\Theta p$ and f_π the constant of proportionality between $\partial_\alpha\mathfrak{F}_+{}^{5,\alpha}$ and π^+ (see Ref. 4), and then dispersing R in ν,

$$R(\nu) = \frac{1}{\pi}\int_\mu^\infty d\nu'\left[\frac{\operatorname{Im}R(\nu')}{\nu'-\nu}+\frac{\operatorname{Im}R(-\nu')}{\nu'+\nu}\right]. \quad (16)$$

Suppose now that the pion lay on a Regge trajectory instead of being elementary. Asymptotically $R(\nu)$ would behave as $\nu^{\alpha_\pi(0)}$, where $\alpha_\pi(t)$ is the pion Regge trajectory with $\alpha_\pi(\mu^2)=0$ and a slope $1/m^2$. For $\nu\to\infty$, one may write

$$R(\nu) \sim \frac{\pi}{2}\frac{\mathcal{G}_1\mathcal{G}_2}{m^2}\left(\frac{\nu}{\nu_0}\right)^{\alpha_\pi(0)}\left(\frac{1-e^{-i\pi\alpha_\pi(0)}}{\sin\pi\alpha_\pi(0)}\right), \quad (17)$$

where $\mathcal{G}_1$ and $\mathcal{G}_2$ are the Regge residues for the $\Theta\pi\pi$ and πnp couplings and ν_0 is a constant introduced for dimensional reasons. In the dispersion relation (16), the contribution of the large ν' region, to $R(0)$, is estimated by using (17) and crossing symmetry:

$$R(0)|_{\pi\text{ Regge pole}} = \int_{\nu_a}^\infty \frac{\mathcal{G}_1\mathcal{G}_2}{m^2}\left(\frac{\nu'}{\nu_1}\right)^{\alpha_\pi(0)}\frac{d\nu'}{\nu'} = \frac{\mathcal{G}_1\mathcal{G}_2}{m^2\alpha_\pi(0)}\left(\frac{\nu_a}{\nu_0}\right)^{\alpha_\pi(0)}, \quad (18)$$

where ν_a is some large value of ν.

Putting $\mathcal{G}_1=2\mu^2$, $\mathcal{G}_2=g_{Np\pi^+}(0)$, and $\alpha_\pi(0)=0+\mu^2/m^2+O((\mu^2/m^2)^2)$, we see that neglecting terms of order μ^2/m^2 in the contribution of the pion Regge pole to $R(0)$ and, by (15) (and the Goldberger-Treiman relation), $\tilde{\Gamma}(0)$ equals

$$2iMG_A\times 2\bar{u}_p\gamma_5 u_n \quad (19)$$

and hence, once again we fine $d=1$. If one takes $\nu_a=\nu_0$ and assumes the pion trajectory is linear [i.e., $\alpha_\pi(t)=1+t/m^2$], there are no corrections of order μ^2/m^2, and indeed there should not be any, for the dimensions of an operator may be changed by the presence of terms which break scale invariance,[5] but should not depend on mass ratios if there is any sense to a scale-invariant limit (when all masses are zero). Of course there is always the possibility of corrections proportional to μ^2/m^2 being cancelled by other contributions to the sum rule. Finally there is the question of the Regge residues. The pion trajectory has zero momentum and perhaps $\mathcal{G}_1\neq 2\mu^2$. If we use Jackiw's[3] result for the $\Theta\pi\pi$ vertex where one of the pions has zero four-momentum, we would put

$$\mathcal{G}_1 = T_{\Theta\pi\pi}(\mu^2,0,0) = (4-d-1)\mu^2 \quad (20)$$

and then, neglecting other contributions to $\tilde{\Gamma}(0,0)$, Eq. (8) becomes an identity. We would then have to go to Jackiw's[3] argument for smoothness of T in the momenta variables to obtain $d=1$; of course the PCAC

[5] K. Wilson, Phys. Rev. D 2, 1473 (1970); SLAC Report No. SLAC Pub. 737 (unpublished).

extrapolation in Eq. (15) is not valid in the first place if $\mathcal{G}_1$ is a rapidly varying function of momentum.

Let us now turn to the question of other contributions to $\Gamma(0,0)$. Whereas the matrix elements of Θ between diagonal states at zero-momentum transfer are known, being proportional to the mass of the particle, the off-diagonal matrix elements are not determined. Consider, for simplicity, two spin-zero particles with masses m_1, m_2 and momenta p_1, p_2, where $p_1^2=m_1^2$, $p_2^2=m_2^2$. The energy-momentum tensor $\Theta^{\mu\nu}$ has matrix elements[6]

$$\langle p_1|\Theta^{\mu\nu}(0)|p_2\rangle=(4E_1E_2)^{1/2}\{(g^{\mu\nu}k^2-k^\mu k^\nu)F_1(k^2)+[P_\mu P_\nu k^2+k\cdot P(g^{\mu\nu}k\cdot P-k^\mu P^\nu-P^\mu k^\nu)]F_2(k^2)\}\,, \quad (21)$$

where $P^\mu=p_1^\mu+p_2^\mu$, $k^\mu=p_1^\mu-p_2^\mu$, $k\cdot P=m_1^2-m_2^2$. It is equal to zero when dotted into either k_μ or k_ν; taking the trace, we find the matrix element of Θ

$$\langle p_1|\Theta(0)|p_2\rangle=(4E_1E_2)^{-1/2}[3k^2F_1(k^2)+(P^2k^2+(k\cdot P)^2]F_2(k^2)]. \quad (22)$$

As $k^2\to 0$, we find

$$\langle p_1|\Theta(0)|p_2\rangle=(4E_1E_2)^{-1/2}[(m_1^2-m_2^2)^2F_2(0)]\,, \quad (23)$$

where, for dimensional reasons, $F_2(0)$ must be inversely proportional to (mass)^2. It appears *a priori* that intermediate states other than the nucleon will contribute to our sum rule; if this is so, the dimension d will depend on ratios of masses because of the form of the matrix elements of Θ. If we require, however, that the limit of scale invariance be approached smoothly, we must demand, as stated earlier, that the dimensions of operators be independent of masses (these vanish in the scale-invariant limit). Our sum rule forces us then to the conclusion that the off-diagonal matrix elements of Θ vanish, since otherwise the sum rule of Eq. (9) leads to mass-dependent dimensions. Hence, even for theory including interaction, we have $d=1$.

There is a loophole in this proof, other than the possibility of the scale-invariant limit not being approached smoothly, and that involves what we mean by dependence on particle masses. If we consider all our baryon states as existing even when dimensionless coupling constants tend to zero, i.e., in some sense treat the particles as elementary, then our argument given above is quite strong, does lead to $d=1$, and has other interesting consequences, which we discuss in Secs. III and IV. If, however, there is only one basic mass in the theory and all the other states are generated dynamically, it would probably be correct to write that the mass of the ith state was

$$m_i=M(1+f_i(g))\,, \quad (24)$$

where $f_i(g)$ is a function only of one or more dimensionless coupling constants g. Our situation would then be that

$$d=1+f(f_1(g),f_2(g),\ldots,f_n(g))\,, \quad (25)$$

where f is a function of the various f_i, and hence the dimension d would be modified in the presence of interaction,[5] with $f(f_i,\ldots,f_n)$ approaching zero as $g\to 0$.

The value $d=1$ is that suggested by operator PCAC where $\partial_\alpha\mathfrak{F}^{5,\alpha}$ is proportional to the pion field. It is amusing to note that neglect of the t-channel pion pole (certainly correct if there were no pion) would lead to $d=3$, the naive dimension of $\partial_\alpha\mathfrak{F}^{5,\alpha}$ when its operator form is $\bar\psi\gamma_5\psi$.

III. SUM RULES FOR CHIRAL-SYMMETRY BREAKING TERM IN Θ

Let us begin by defining, analogously to Eq. (6), a function

$$-B^\alpha(q^2,q\cdot p)=\int d^4x\, e^{iq\cdot x}\langle p|\{\mathfrak{F}_+^{5,\alpha}(0)u(0)\}|n\rangle. \quad (26)$$

We then proceed by considering $q_\alpha B^\alpha$ as $q\to 0$, separating out the one-nucleon pole and using the commutator of Eq. (5), defining B as we defined Γ. The matrix element of $u(0)$ between nucleon states is unknown if the energy-momentum tensor contains a chiral singlet δ; the analysis of von Hippel and Kim[7] suggests it is small, but for the moment let us assume that it is given by a constant α such that

$$\langle n|u(0)|n\rangle=(M/E)\alpha\bar{u}u. \quad (27)$$

The analog of Eq. (8) is then

$$\lim_{q\to 0} i\tilde{B}(q^2,q\cdot p)\frac{E}{M}=\left(1-\frac{\alpha}{M}\right)2iMG_A\bar{u}_p\gamma_5u_n\,, \quad (28)$$

where $\tilde{B}$ is B without the N pole. To estimate the contribution of the pion pole we need to know $T_{u\pi\pi}$, the analog of $T_{\Theta\pi\pi}$ discussed in Sec. II, i.e., the matrix element of $u(0)$ between pions. Using the PCAC derivation of Ref. 8 or the consideration in Ref. 3, we find

$$T_{u\pi\pi}(0,0,0)\approx T_{u\pi\pi}(\mu^2,0,0)=\mu^2\,, \quad (29)$$

in which case the contribution of the pion pole to the left-hand side of (28), calculated exactly as in Sec. II, turns out to be

$$\lim_{q\to 0}\frac{iE}{M}\tilde{B}(q^2,q\cdot p)|_{\text{pion pole}}=2iMG_A\bar{u}_p\gamma_5u_n. \quad (30)$$

Limiting ourselves to keeping in B only the nucleon and pion poles, Eq. (28) then requires $\alpha=0$, so that u has a vanishing nucleon matrix element and Θ^{00} must contain a scale-invariance-breaking chiral singlet δ which is primarily responsible for the nucleon mass.

[6] I would particularly like to thank Dr. R. Jackiw for help with the following arguments.

[7] F. von Hippel and J. Kim, Phys. Rev. D 1, 151 (1970).

[8] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968).

If we assume $\delta=0$, then Eq. (2) implies $u=\Theta/(4-d)$. Now the sum rule leads to a contradiction if we keep (29),

$$T_{u\pi\pi}=\mu^2=\frac{T_{\Theta\pi\pi}}{4-d}=\frac{2\mu^2}{4-d}, \tag{31}$$

which is only valid if $d=2$, but then we must still have the condition that the matrix element of u between nucleon states vanishes, which in this case implies a vanishing nucleon mass. Compatibility is achieved only when $T_{u\pi\pi}=\frac{2}{3}\mu^2$ and $d=1$, which contradicts Ref. 8; if, however, we allow a $\delta\neq0$ term in Θ^{00}, there is no contradiction (see also the discussion in Ref. 3, which arrives at the same results), and we may keep $T_{u\pi\pi}=\mu^2$.

IV. IMPLICATIONS FOR $SU(3)$ BREAKING

We would like to point out here some of the implications for $SU(3)$ breaking of the assumed form of the nondiagonal matrix elements of Θ given in Eq. (22). Let us take u, defined in (2), to be given by

$$u=u_0+cu_8, \tag{32}$$

where u_8 is the eighth component of the octet of scalar densities and u_0 is the $SU(3)$ singlet. In Sec. II we showed that to maintain $d=1$, we required that $\langle p_1|\Theta(0)|p_2\rangle$ vanish as $(p_1-p_2)^2=k^2\rightarrow0$. The relation between Θ and u given in (2) must hold for any value of c in (32) and this presumably could only be true if

$$\lim_{k^2\rightarrow0}\langle p_1|u_8(0)|p_2\rangle=0, \tag{33}$$

where $|p_1\rangle$ and $|p_2\rangle$ are different one-particle states; this implies that the coupling of u to states belonging to different $SU(3)$ multiplets (i.e., nondiagonal matrix elements) must have a kinematical zero for $k^2\rightarrow0$. This in turn leads us to conclude that, in the $SU(3)$ limit, the matrix elements of $u_{K^+}(0)$ [obtained from $u_8(0)$ by an $SU(3)$ rotation] between different $SU(3)$ multiplet states must also vanish as $k^2\rightarrow0$.

Now consider a typical $SU(3)$ relation obtained by current commutator techniques, as for instance the Gell-Mann–Okubo mass formula. The relevant commutator is[9]

$$[F_{K^+}(t),[F_{K^+}(t),H]]_{t=0}=0, \tag{34}$$

where F_i are the generators of $SU(3)$. The matrix element of (34) between, e.g., K^+ and K^- states leads to the desired mass formula for pseudoscalar mesons when only π^0 and η intermediate states are included; other intermediate states are of second order in the breaking of $SU(3)$, i.e., of order c^2 using the notation of Eq. (32). To calculate such a correction, say that due to a one-particle intermediate state r of mass m_r, we need to evaluate the matrix element

$$\begin{aligned}\langle r|[F_{K^+}(t),H]_{t=0}|K^-\rangle&\\ &=i\langle r|\int\partial_0\mathfrak{F}_{K^+}{}^0(\mathbf{x},0)d\mathbf{x}|K^-\rangle\\ &=i(2\pi)^3\delta(\mathbf{p}_r-\mathbf{p}_{K^-})\langle r|\partial_\alpha\mathfrak{F}_{K^+}{}^\alpha(0)|K^-\rangle\\ &\quad ic(2\pi)^3\delta(\mathbf{p}_r-\mathbf{p}_{K^-})\langle r|u_{K^+}(0)|K^-\rangle\end{aligned} \tag{35}$$

and, letting $p_{K^-}\rightarrow\infty$, we have $(E_r-E_{K^-})\rightarrow0$ and hence $k^2=(p_r-p_{K^-})^2\rightarrow0$, also. By the previous arguments the matrix element $\langle r|u_{K^+}(0)|K^-\rangle$ must then vanish, at least in the limit of $SU(3)$. We conclude then that part of the success of the $SU(3)$ relations which can be obtained by saturation of current commutators at infinite momentum may be due to the vanishing of correction terms, for purely kinematical reasons. All this of course is predicated on the vanishing of $F_2(0)$, defined in Eq. (21).

V. CONCLUSION

We have presented arguments suggesting that the dimension of $\partial_\alpha\mathfrak{F}^{5,\alpha}$ is 1, that the energy-momentum tensor contains a scale-invariance-breaking chiral singlet δ, and that the matrix elements of u between nucleons are small. These problems have already been studied,[10] but our approach differs from previous ones and, we feel, clarifies the issues involved.

As we stated in the text, our results depend crucially on the vanishing of nondiagonal one-particle matrix elements of Θ, as the (momentum transfer)^2 approaches zero and this may well not be true. It does appear that our results are true to lowest order in μ^2 and in $g_{NN\pi}$.

Finally we have shown that, if the nondiagonal matrix elements of Θ vanish as $k^2\rightarrow0$, there are nontrivial predictions for $SU(3)$ breaking that follow.

Note added in proof. To ensure the absence in Γ of $\sigma nn\pi$ contact terms, we should use the Θ which has no σ pole as introduced in the preceding paper by Jackiw.

ACKNOWLEDGMENTS

I would like to thank Dr. R. Jackiw for his continued encouragement and assistance, and Dr. M. Gell-Mann for a helpful discussion of the problem in Sec. II at the Aspen Center for Theoretical Physics.

[9] G. Furlan, F. Lannoy, C. Rossetti, and G. Segrè, Nuovo Cimento **40**, 497 (1965); S. Fubini, G. Furlan, and C. Rossetti, *ibid.* **40**, 1171 (1965).

[10] See, e.g., S. P. de Alwis and P. J. O'Donnell, Phys. Rev. D **2**, 1023 (1970); University of Toronto Reports, 1970 (unpublished).