Consistency Conditions for Chiral Symmetry and Scale Invariance. II. Scale Noninvariance of the Chiral-Symmetry Limit*

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If it is required that matrix elements of the $SU(3)\times SU(3)$ symmetry-violating operator in the Hamiltonian be gently varying, and that the limits of chiral symmetry and/or scale invariance be smoothly attainable, then scale invariance must be broken in the chiral-symmetry limit.

I. INTRODUCTION

In order to develop a model for the breaking of scale invariance and chiral symmetry, it is necessary to know whether or not scale invariance is violated by the same mechanism which destroys chiral symmetry, or whether an additional chiral singlet is present which prevents the dilatation current from being conserved, even in the chiral-symmetry limit.[1] Qualitative arguments may be given to support the latter hypothesis: It is generally believed that nucleon masses, in the chiral-symmetry limit, are nonvanishing, due to the Nambu-Goldstone mechanism. On the other hand, in the scale-invariant limit, all masses should vanish. Furthermore, the analysis of von Hippel and Kim[2] indicates that diagonal matrix elements between nucleon states of u,[3] the $SU(3)\times SU(3)$ symmetry-breaking part of the Hamiltonian density, vanish. The divergence of the dilatation current D^μ is proportional to Θ, the trace of the energy-momentum tensor $\Theta^{\mu\nu}$.[4] The relevant matrix elements of Θ measure the fermion masses. Therefore Θ cannot be proportional to u.

Neither of the above two reasons is completely convincing. There may be a Goldstone particle associated with dilatation invariance, which prevents the nucleon masses from vanishing. The analysis of von Hippel and Kim requires many unreliable extrapolations, and their conclusion could be circumvented. It is therefore desirable to seek other arguments which shed light on this question.

In this paper we assume the absence of a $SU(3)\times SU(3)$-singlet, scale-invariance-breaking term in Θ. We then prove that this hypothesis leads to matrix elements of u which are rapidly varying the low-energy region. Thus gentle variation is violated. We further demonstrate that this implies that the limit of chiral symmetry and scale invariance cannot be achieved smoothly. Finally it is shown that the diagonal pion matrix element of u is $\frac{2}{3}\mu^2$, where μ is the pion mass. All these undesirable results may be avoided if the hypothesis is abandoned. Thus we conclude that there is a $SU(3)\times SU(3)$ singlet which breaks scale invariance.

The paper is organized as follows. After various preliminary discussions in Sec. II, we derive PCAC (partially conserved axial-vector current) Ward identities in Sec. III. In Sec. IV, low-energy theorems are obtained from which we draw all our results about the matrix element of u. Finally, concluding remarks comprise Sec. V. In the Appendix we present a model calculation which makes use of the tree approximation for a chiral Lagrangian. All our results are verified.

II. PRELIMINARIES

Consider the neutral axial-vector current A^α associated with the neutral pion. Apart from a Schwinger term, the equal-time commutator of A^0 with A^β vanishes:

$$[A^0(0,\mathbf{x}),A^\beta(0)]=0. \tag{2.1}$$

The divergence is defined as

$$\partial_\alpha A^\alpha = F\mu^2\pi\,, \tag{2.2}$$

where the normalization factor F is chosen so that $\langle\pi|\pi(0)|0\rangle=1$ and μ is the pion mass. The dilatation current is given by $D^\mu(x)=x_\nu\Theta^{\mu\nu}(x)$, $\partial_\mu D^\mu(x)=\Theta(x)$.[4] For Θ we may take the expression $\Theta=(4-d)u$; d is the scale dimension of u.[5] However it has been previously argued that this formula yields rapidly varying matrix elements of Θ, regardless of whether a $SU(3)\times SU(3)$ singlet is present.[6] A different formula for Θ can be given in the context of the $(\bar{3},3)\oplus(3,\bar{3})$ model.[7] The modification removes this rapid variation: $\Theta=(4-d)u$

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[1] G. Mack, Nucl. Phys. **B5**, 499 (1968); G. Mack and A. Salam, Ann. Phys. (N. Y.) **53**, 174 (1969); K. Wilson, Phys. Rev. **179**, 1499 (1969); M. Gell-Mann, Cal. Tech. Report No. CALT-68-244 (unpublished); P. Carruthers, Phys. Rev. D **2**, 2265 (1970). The first two authors advocate the former point of view; the remaining ones discuss the likelihood of the latter. Their reasons are given below.

[2] F. von Hippel and J. K. Kim, Phys. Rev. Letters **22**, 740 (1969); Phys. Rev. D **1**, 151 (1970).

[3] The notational conventions in this paper are the same as in the related investigation: R. Jackiw, preceding paper, Phys. Rev. D **3**, 1347 (1971). One exception is that here we call the $SU(3)\times SU(3)$-breaking term u, while in the above reference the $SU(2)\times SU(2)$-breaking term was called u.

[4] C. G. Callan, Jr., S. Coleman, and R. Jackiw, Ann. Phys. (N. Y.) **59**, 42 (1970).

[5] We assume that the entire $SU(3)\times SU(3)$ symmetry-breaking term has the unique dimension d.

[6] R. Jackiw (Ref. 3).

[7] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 244 (1968); M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

$+\frac{1}{2}F\Box\sigma$. Here σ is defined by

$$[A^0(0,\mathbf{x}),\pi(0)]=-2i\sigma(0)\delta(\mathbf{x}). \tag{2.3}$$

This formulation has the additional advantage that d can be determined to be 1.[6] However, all but the last of our present conclusions are independent of this modification and of the value for d. Hence we take for Θ t he following expression:

$$\Theta=(4-d)u+\tfrac{1}{2}cF\Box\sigma. \tag{2.4}$$

It will be seen that c plays no role in most of the argument. We shall also need the commutator which expresses the fact that u breaks $SU(2)\times SU(2)$ in a fashion such that (2.2) is true:

$$[A^0(0,\mathbf{x}),u(0)]=iF\mu^2\pi(0)\delta(\mathbf{x}). \tag{2.5}$$

The assumption that A^α has scale dimension 3 in all its components requires that u, π, and σ all have the same dimension d:

$$[D^0(0,\mathbf{x}),\pi(0)]=-id\pi(0)\delta(\mathbf{x}), \tag{2.6a}$$

$$[D^0(0,\mathbf{x}),\sigma(0)]=-id\sigma(0)\delta(\mathbf{x}). \tag{2.6b}$$

III. WARD IDENTITY

The following Green's functions are defined:

$$\Gamma^{\alpha\beta}(p,q)=\frac{(p^2-\mu^2)}{iF\mu^2}\frac{(q^2-\mu^2)}{iF\mu^2}\int d^4x d^4y\, e^{ipx}e^{iqy}\times\langle 0|TA^\alpha(x)A^\beta(y)u(0)|0\rangle, \tag{3.1}$$

$$\Gamma(p^2,q^2,k^2)=\frac{(p^2-\mu^2)}{i}\frac{(q^2-\mu^2)}{i}\int d^4x d^4y\, e^{ipx}e^{-iqy}\times\langle 0|T\pi(x)\pi(y)u(0)|0\rangle, \tag{3.2}$$

$$g(p^2)=\frac{(p^2-\mu^2)}{i}\int d^4x\, e^{ipx}\langle 0|T\pi(x)\pi(0)|0\rangle,$$

$$k=q-p. \tag{3.3}$$

Note that

$$\Gamma(\mu^2,\mu^2,0)=\langle\pi|u(0)|\pi\rangle, \tag{3.4}$$

$$g(\mu^2)=1. \tag{3.5}$$

Equation (3.5) is a consequence of our normalization for π. A completely straightforward calculation, which uses only the chiral relations (2.1)-(2.3) and (2.5), yields the Ward identity:

$$p_\alpha q_\beta\Gamma^{\alpha\beta}(p,q)=\Gamma(p^2,q^2,k^2)+(q^2-\mu^2)g(p^2)+(p^2-\mu^2)g(q^2)+\frac{(p^2-\mu^2)(q^2-\mu^2)}{\mu^2}g(0)+2\frac{(p^2-\mu^2)(q^2-\mu^2)}{F\mu^2}M(k), \tag{3.6}$$

$$M(k)=i\int d^4x\, e^{-ikx}\langle 0|T\sigma(x)u(0)|0\rangle. \tag{3.7}$$

In offering (3.6), we have replaced $\langle 0|\sigma(0)|0\rangle\equiv\langle\sigma\rangle$ by $-\frac{1}{2}Fg(0)$. This is correct, as is seen from the following chain of equations:

$$\Gamma^\alpha(p)=\frac{(p^2-\mu^2)}{iF\mu^2}\int d^4x\, e^{-ipx}\langle 0|TA^\alpha(x)\pi(0)|0\rangle,$$

$$ip_\alpha\Gamma^\alpha(p)=g(p^2)-2\frac{(p^2-\mu^2)}{F\mu^2}\langle\sigma\rangle, \tag{3.8}$$

$$\langle\sigma\rangle=-\tfrac{1}{2}Fg(0).$$

IV. LOW-ENERGY THEOREMS

Two low-energy theorems may be derived from (3.6), one at $p=q=0$, the other at $p^2=\mu^2$, $q=0$[8]:

$$\Gamma(\mu^2,0,\mu^2)=\mu^2, \tag{4.1a}$$

$$\Gamma(0,0,0)=\mu^2 g(0)-(2\mu^2/F)M(0). \tag{4.1b}$$

In the usual chiral argument, it is assumed that $\Gamma(p^2,q^2,k^2)$ is smooth in the region $0\leqslant p^2, q^2, k^2\leqslant\mu^2$ and that $\Gamma(\mu^2,\mu^2,0)\approx\Gamma(0,0,0)\approx\Gamma(\mu^2,0,\mu^2)\approx\mu^2$. Upon examining (4.2), it is seen that this assumption can be correct only if $(2/F)M(0)=O(\mu^2)$. [$g(0)\approx 1$ according to (3.5).] Without any information to the contrary, this is not impossible. A part of $M(0)$ is the matrix element of the $SU(2)\times SU(2)$ breaking term; this is explicitly $O(\mu^2)$. The remainder of $M(0)$ cannot be estimated without further knowledge about u.

In the present context, however, additional information about u is obtained from (2.4). We now show that (2.4) implies that $M(0)$ is $O(1)$, and that the conclusion that $\langle\pi|u(0)|\pi\rangle\approx\mu^2$ cannot be established:

$$\begin{aligned}M(k)&=\frac{i}{4-d}\int d^4x\, e^{ikx}\langle 0|T\sigma(0)[\Theta(x)-\tfrac{1}{2}cF\Box\sigma(x)]|0\rangle\\&=\frac{i}{4-d}\int d^4x\, e^{ikx}\langle 0|T\sigma(0)\partial_\mu D^\mu(x)|0\rangle\\&\quad+\frac{ck^2}{4-d}\tfrac{1}{2}Fi\int d^4x\, e^{ikx}\langle 0|T\sigma(0)\sigma(x)|0\rangle\\&=\frac{k_\mu\mathfrak{D}^\mu(k)}{4-d}+\frac{d}{4-d}\tfrac{1}{2}Fg(0)+\frac{ick^2}{4-d}\tfrac{1}{2}FG_\sigma(k),\end{aligned} \tag{4.2a}$$

$$M(0)=\frac{d}{4-d}\tfrac{1}{2}Fg(0)=O(1). \tag{4.2b}$$

Equation (4.2a) is derived with the help of (2.6b), (3.8),

[8] Equation (4.1) is derived under the assumption that no particles, degenerate in mass with the pion, exist.

and the definitions[9]

$$\mathfrak{D}^\mu(k)=\int d^4x\, e^{ikx}\langle 0|\,T\sigma(0)D^\mu(x)\,|0\rangle\,, \qquad (4.3)$$

$$G_\sigma(k)=\int d^4x\, e^{ikx}\langle 0|\,T\sigma(0)\sigma(x)\,|0\rangle\,. \qquad (4.4)$$

Equation (4.2b) determines $\Gamma(0,0,0)$ completely, in terms of d. We also know the value of $\Gamma(\mu^2,\mu^2,\mu^2)$ from (2.4) since $\langle\pi|\,\Theta(0)\,|\pi\rangle=2\mu^2$. Therefore, an evaluation of $\Gamma(p^2,q^2,k^2)$ at three points can be given:

$$\Gamma(\mu^2,\mu^2,0)=\frac{2\mu^2}{4-d}\,, \qquad (4.5)$$

$$\Gamma(0,0,0)=\mu^2 g(0)\frac{4-2d}{4-d}\,, \qquad (4.6)$$

$$\Gamma(\mu^2,0,\mu^2)=\mu^2\,. \qquad (4.7)$$

It is now clear that $\Gamma(p^2,q^2,k^2)$ is not a smooth function in the region $0\leqslant p^2,\ q^2,\ k^2\leqslant\mu^2$. As the arguments range in that domain, Γ changes by terms of the same order as its magnitude. One consequence of this is that one can no longer extrapolate from one point to another with any assurance. If one were to decide that an extrapolation *should* be effected, it is more plausible to say that it is $\Gamma(0,0,0)$ rather than $\Gamma(\mu^2,0,\mu^2)$ which goes over smoothly to $\Gamma(\mu^2,\mu^2,0)$. The reason for this is that the extrapolation is being performed symmetrically in the symmetric variables p^2 and q^2, while k^2 remains constant at zero. On the other hand, to extrapolate $\Gamma(\mu^2,0,\mu^2)$ to $\Gamma(\mu^2,\mu^2,0)$, a very unsymmetric path must be taken. If the first extrapolation is performed, we get $d=1$, and $\langle\pi|\,u(0)\,|\pi\rangle=\frac{2}{3}\mu^2$. This value for d is precisely the one which was obtained in our previous argument based on smoothness of matrix elements.[6] Hence even if we do not wish to extrapolate, but accept the previous analysis, which did not depend on the presence or absence of the $SU(3)\times SU(3)$-singlet scale-breaking term, we again conclude that $\langle\pi|\,u(0)\,|\pi\rangle=\frac{2}{3}\mu^2$.

Another related consequence of the low-energy theorems is that the limit of chiral symmetry and scale invariance cannot be achieved smoothly. To see this, we parametrize $\Gamma(p^2,q^2,k^2)$ as follows [$g(0)$ is set equal to 1]:

$$\Gamma(p^2,q^2,k^2)=\frac{4-2d}{4-d}\mu^2+(p^2+q^2)\frac{d-1}{4-d}+k^2\frac{1}{4-d}+\tilde{\Gamma}(p^2,q^2,k^2)\,. \qquad (4.8)$$

Here $\tilde{\Gamma}$ is taken to be of higher order in μ^2 and/or in the kinematical variables, and vanishing at the points given by (4.5)–(4.7), which are reproduced by the explicitly exhibited terms in (4.8). Let us pass to the $SU(2)\times SU(2)$ limit by letting $\mu^2\to 0$:

$$\Gamma(p^2,q^2,k^2)\,|_{\mu^2\to 0}=(p^2+q^2)\frac{d-1}{4-d}+k^2\frac{1}{4-d}+\tilde{\Gamma}(p^2,q^2,k^2)\,|_{\mu^2=0}\,. \qquad (4.9)$$

But now it is seen that there is no way to achieve the $SU(3)\times SU(3)$ limit or the scale-invariance limit. In either limit, Γ must vanish identically, since it involves u which by hypothesis is the only scale and $SU(3)\times SU(3)$-breaking effect. However, the term $k^2/(4-d)$ in (4.9) has no parametric dependence and cannot be made to vanish. (The term in p^2+q^2 disappears for $d=1$.)

V. CONCLUSION

Although there is no intrinsic contradiction in the rapid variation which we have discovered, it nevertheless remains an unpleasant feature. In particular, no low-energy theorem can be reliably proved and the approach to scale and $SU(3)\times SU(3)$ invariance is singular. Furthermore, $\langle\pi|\,u(0)\,|\pi\rangle=\frac{2}{3}\mu^2$, which disagrees with the accepted formula $\langle\pi|\,u(0)\,|\pi\rangle=\mu^2$. The latter expression is favored by conventional $SU(3)$ ideas and a quadratic mass formula.[10] Since it is possible to circumvent this unfortunate turn of events by accepting the presence of a $SU(3)\times SU(3)$ scalar which violates scale invariance even in the chiral-symmetry limit, we believe that this is a sufficiently compelling reason to discard (2.4) and replace it by

$$\Theta=(4-d)u+\tfrac{1}{2}F\Box\sigma+(4-d'')u''\,. \qquad (5.1)$$

Here, u'' is the required scalar with dimension d''. Once (5.1) is accepted, the undesirable result that $M(0)=O(1)$ can no longer be established. The principle of maximal smoothness now imposes a consistency requirement which replaces (4.2b):

$$\tfrac{1}{2}Fg(0)\approx(4-d'')i\int d^4x\langle 0|\,T\sigma(0)u''(x)\,|0\rangle\,. \qquad (5.2)$$

The scale dimension d has been set to 1, as derived previously. Note that even though u'' is a scalar and σ is not, the right-hand side of (5.2) need not vanish in the $SU(3)\times SU(3)$ limit since the symmetry remains spontaneously broken. The original Gell-Mann–Oakes–Renner[7] result that $\langle\pi|\,u(0)\,|\pi\rangle=\mu^2$ may again be accepted, and (5.1) implies that

$$\begin{aligned}\langle\pi|\,\Theta(0)\,|\pi\rangle=2\mu^2&=\langle\pi|\,[(4-d)u(0)+\tfrac{1}{2}F\Box\sigma(0)+(4-d'')u''(0)]\,|\pi\rangle\\&=(4-d)\mu^2+(4-d'')\langle\pi|\,u''(0)\,|\pi\rangle\,,\\(4-d'')\langle\pi|\,u''(0)\,|\pi\rangle&=-\mu^2\,.\end{aligned} \qquad (5.3)$$

[9] In offering (4.2a), we have replaced T products of operators with the d'Alembertian operating on them by $-k^2$ times the corresponding T product without the d'Alembertian. This commutation of $\Box$ with the time ordering is justified by the addition of appropriate seagull terms. Similar seagull terms have been implicitly introduced to cancel any Schwinger terms which may be present in the local commutators with which we are dealing. Although we do not emphasize this here, it is also true that much of the present argument depends only on integrated commutators.

[10] See, for example, P. Carruthers (Ref. 1).

However, since $\langle\pi|u''(0)|\pi\rangle$ is unknown, we cannot evaluate d''.

In conclusion we wish to comment on a result of Gell-Mann.[11] He asserted that $d=2$ if $u''=0$. The argument was based on Eq. (2.4) with $c=0$. Taking the pion matrix element, one gets $2\mu^2=(4-d)\langle\pi|u(0)|\pi\rangle$. If the chiral result $\langle\pi|u(0)|\pi\rangle\approx\mu^2$ is used, the indicated value for d follows. In our argument we circumvented this conclusion by showing that $\langle\pi|u(0)|\pi\rangle\neq\mu^2$ when (2.4) is true.

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This argument was developed in conversations with Professor G. Segrè. I am also grateful to Professor M. Gell-Mann for explaining to me his approach to this problem. Communications concerning these ideas from Professor P. Carruthers and R. Crewther are acknowledged.

APPENDIX

A model calculation, in the tree approximation, is presented which verifies the conclusions in the text. Consider the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(\partial\pi)^2+\tfrac{1}{2}(\partial\sigma)^2-\tfrac{1}{2}\mu^2(\pi^2+\sigma^2)-\lambda(\pi^2+\sigma^2-\tfrac{1}{4}F^2)^2-\tfrac{1}{2}\mu^2F\sigma. \quad \text{(A1)}$$

This is the σ model of Gell-Mann and Lévy,[12] except that fermions and isospin have been suppressed as they are inessential for present purposes. The axial-vector current is defined by

$$A^\mu=2\sigma\partial^\mu\pi-2\pi\partial^\mu\sigma\,, \quad \text{(A2)}$$

$$\partial_\mu A^\mu=F\mu^2\pi\,. \quad \text{(A3)}$$

For $\Theta^{\mu\nu}$ we chose[4]

$$\Theta^{\mu\nu}=\partial^\mu\pi\partial^\nu\pi+\partial^\mu\sigma\partial^\nu\sigma-g^{\mu\nu}\mathcal{L}-\tfrac{1}{6}(\partial^\mu\partial^\nu-g^{\mu\nu}\Box)(\pi^2+\sigma^2)\,, \quad \text{(A4)}$$

$$\Theta=(\mu^2-\lambda F^2)(\pi^2+\sigma^2)+\tfrac{3}{2}\mu^2F\sigma\,. \quad \text{(A5)}$$

This is *not* the energy-momentum tensor introduced in Ref. 3; comparing with (5.1), it is seen that the term $\frac{1}{2}F\Box\sigma$ is absent. However, since our conclusions here do not depend on the use of the modified tensor introduced in Ref. 3, for simplicity we remain with (A5). In this model the scale dimension of all fields is one; hence we deduce that

$$u=\tfrac{1}{2}F\mu^2\sigma\,, \quad \text{(A6)}$$

$$u''=\tfrac{1}{2}(\mu^2-\lambda F^2)(\pi^2+\sigma^2)\,. \quad \text{(A7)}$$

Thus the choice $\mu^2=\lambda F^2$ makes u'' vanish.[13] However, for the moment we do not enforce this equality.

To compute in the tree approximation, the σ field is shifted so that the term linear in σ disappears from $\mathcal{L}$:

$$\sigma=\hat\sigma-\tfrac{1}{2}F\,, \quad \text{(A8)}$$

$$\mathcal{L}=\tfrac{1}{2}(\partial\pi)^2+\tfrac{1}{2}(\partial\hat\sigma)^2-\tfrac{1}{2}\mu^2\pi^2-\tfrac{1}{2}m_\sigma^2\hat\sigma^2-\lambda[(\pi^2+\hat\sigma^2)^2-2F\hat\sigma(\hat\sigma^2+\pi^2)]\,, \quad \text{(A9)}$$

$$m_\sigma^2=\mu^2+2\lambda F^2\,.$$

Inessential constant terms have been dropped.

It is now easy to show that

$$\Gamma(p^2,q^2,k^2)=\frac{-2\lambda F^2\mu^2}{k^2-m_\sigma^2}=\mu^2\frac{\mu^2-m_\sigma^2}{k^2-m_\sigma^2}\,. \quad \text{(A10)}$$

The low-energy theorems (4.1) and (4.2) are satisfied with

$$M(0)=\tfrac{1}{2}F\mu^2/m_\sigma^2\,. \quad \text{(A11)}$$

Therefore, in the general case, when $\mu^2/m_\sigma^2\ll1$, $\langle\pi|u(0)|\pi\rangle\approx\mu^2$, and $\Gamma(p^2,q^2,k^2)$ is smooth. However, when the choice $\mu^2=\lambda F^2$ is made, which eliminates u'' in (A5), then we have $m_\sigma^2=3\mu^2$ and (A10) and (A11) become replaced by[14]

$$\Gamma(p^2,q^2,k^2)=\mu^2\frac{2\mu^2}{3\mu^2-k^2}\,, \quad \text{(A12)}$$

$$\Gamma(\mu^2,\mu^2,0)=\langle\pi|u(0)|\pi\rangle=\tfrac{2}{3}\mu^2\,, \quad \text{(A13)}$$

$$M(0)=\tfrac{1}{6}F\,. \quad \text{(A14)}$$

These formulas verify (4.2b) and (4.5)–(4.7).

Finally we remark that (5.2) is also verified when u'' is nonvanishing. From (A7) and (A9) we have

$$\begin{aligned}(4-d'')i\int d^4x\langle0|T\sigma(0)u''(x)|0\rangle &=i(\mu^2-\lambda F^2)\int d^4x\langle0|T\hat\sigma(0)[\pi^2(x)+\hat\sigma^2(x)-F\hat\sigma(x)]|0\rangle\\ &=-\frac{(\mu^2-\lambda F^2)}{m_\sigma^2}F=\tfrac{1}{2}F-\tfrac{3}{2}F\left(\frac{\mu^2}{m_\sigma^2}\right)\approx\tfrac{1}{2}F\,. \quad \text{(A15)}\end{aligned}$$

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[11] M. Gell-Mann (Ref. 1).

[12] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).

[13] This observation is due to Mack and Salam (Ref. 1).

[14] Note that this choice for λ does not remove the Nambu-Goldstone phenomenon in this model. In the $SU(2)\times SU(2)$ limit, $\mu^2\to0$, σ retains its nonzero vacuum expectation value. Hence if nucleons were present, they would still possess a nonvanishing mass. Indeed in the dilation limit, which here is equivalent to the $SU(2)\times SU(2)$ limit, scale invariance also remains spontaneously broken, since $\langle\sigma\rangle\neq0$. Because $m_\sigma^2=3\mu^2$ vanishes as $\mu^2\to0$, the σ plays the role of a dilaton.